ghellini, Nuovo Cimento 30, 193 (1963); S. Mandelstam, Nuovo Cimento 30, 1127, 1148 (1963); V. N. Gribov, I. Ya. Pomeranchuk, and K. A. Ter-Martirosyan, Phys. Rev. 139, B184 (1965); J. C. Polkinghorne, J. Math. Phys. 6, 1960 (1965).

[2]With $\alpha_P(0)=1$, Mandelstam (Ref. 1) pointed out that this sequence of cuts would invalidate the Mandelstam representation with a finite number of subtractions. For any fixed s, the number of subtractions in a dispersion relation in the t variable is finite, but this number increases indefinitely as $s\rightarrow\infty$. Of course, if the pole trajectories keep rising indefinitely, as they seem to be doing so far, it is hard to see how a finitely subtracted Mandelstam representation can be valid irrespective of these cuts.

[3]Such a bold hypothesis, that the total cross section may be vanishing (albeit slowly), was first put forward, in a simple current-algebra model, by N. Cabibbo, L. Horwitz, J. J. J. Kokkedee, and Y. Ne'eman, Nuovo Cimento 45A, 275 (1966). See also lectures by N. Cabibbo at the 1966 International School of Physics "Ettore Majorana," Erice, 1966 (Academic Press, Inc., New York, 1967).

[4]It should be noted that if one agrees to include these Regge cuts and allows for $\alpha_P(0)=1$, one has a very difficult problem to decide just what the asymptotic behavior of the total cross section should be. The popular choice, of course, is to keep $\alpha_P(0)=1$, ignore the infinite sequence of cuts, and thus get a constant asymptotic cross section. Certainly, this convenience deserves some explanation!

[5]A similar phenomenon is known in potential theory, where if two Regge surfaces $\alpha_1(s)$ and $\alpha_2(s)$ intersect, then they exchange tails producing a branch point in the trajectory functions. See, for instance, R. G. Newton, The Complex j-plane (W. A. Benjamin, Inc., New York, 1966). Thus, our hypothesis would lead to an infinite number of branch points (producing complex cuts) in the trajectory function. Hence, the possibility of writing down simple dispersion relations for the trajectory functions seems remote. Also, the "residues" are exchanged. Phenomenologically, one uses exponentially decreasing (as a function of t) residues [see, for instance, B. R. Desai, Phys. Rev. Letters 11, 59 (1963); T. O. Binford and B. R. Desai, Phys. Rev. 138, B1167 (1965); V. de Lany, D. Gross, I. Muzinich, and V. Teplitz, Phys. Rev. Letters 18, 149 (1967); K. Huang, C. Jones, and V. Teplitz, Phys. Rev. Letters 18, 146 (1967); R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965)]. In this picutre, such a quick decline of the effective residue functions may be a manifestation of the fact that the discontinuities of the higher cuts are decreasing for larger (negative) t, and not necessarily that of the parent pole.

[6]Desai; Binford and Desai; de Lany, Gross, Muzinich, and Teplitz; and Huang, Jones, and Teplitz, Ref. 5.

[7]We may reiterate that if $\epsilon=0$, there is a priori no reason to expect only the parent pole and the first cut to be important for $-t\approx 0$.

[8]Phillips and Rarita, Ref. 5. It has been kindly pointed out to me by S. Mandelstam that it is possible to find an appropriate set of residue parameters if one constrains the trajectory to go through the ρ on the positive side. See, e.g., F. Arbab and N. Bali, to be published. However, as has been noted later by F. Arbab, N. Bali, and J. Dash, Phys. Rev. (to be published), from charge-exchange data alone there are still unresolved ambiguities in a determination of ρ and A_2 parameters. It is for this reason that I have only considered the Phillips-Rarita analysis since they collect data from a variety of reactions (πp, Kp, elastic, charge-exchange, etc.) and find the best fit without any particular bias to constrain the trajectory to go through ρ (or A_2). Since we are interested in the "effective" leading singularity, this seems to me more appropriate. I would like to thank Dr. Mandelstam for raising this issue.

[9]R. J. N. Phillips and W. Rarita, Phys. Rev. Letters 15, 807 (1965).

[10]If the 2^+ singlet Pomeranchukon with a mass of 850-950 MeV really exists, the positive-side slope of α_P would be ~ 1 (BeV)$^{-2}$. Again, the twisting mechanism can account nicely for its slope being only $\approx\frac{1}{3}$ (BeV)$^{-2}$ on the negative t side.

DIFFICULTY IN THE INFINITE-MOMENTUM-LIMIT METHOD RELATED TO LOCALITY

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The proof of the Fubini–Dashen–Gell-Mann sum rule from the infinite-momentum limit is studied again taking the locality of the currents into account. It is shown that the sum rule cannot be derived from the $p\rightarrow\infty$ method proposed by Dashen and Gell-Mann.

Some time ago, Fubini[1] and Dashen and Gell-Mann[2] derived the same sum rule by two completely different methods. Fubini made use of commutators involving one space component of the current in such a way that the derivation deals only with covariant quantities all through the proof. Dashen and Gell-Mann, on the other hand, only introduced the commutator of two time components, thus working with noncovariant expressions. In this last method (see also Amati, Juengo, and Remini[3]), the covariant result is obtained by taking the so-called infinite-momentum limit as first proposed by Fubini and Furlan.[4]

In this paper, we study again the proof of Dashen and Gell-Mann, taking into account the locality properties of the currents. For that purpose, we will make use of the so-called Jost-Lehmann-Dyson representation. It will then appear that the sum rule of Fubini[1] cannot be derived by the infinite-momentum limit considered in Refs. 2 and 3.

In a recent paper by Le Bellac and the author[5] the covariant derivation of sum rules and the equal-time limits were also studied by introducing the Jost-Lehmann-Dyson representation following a method first proposed by Stichel and Schröer.[6] We will use essentially the same approach as in Refs. 5 and 6 to which we refer for more details.

Let us introduce

$$t_{\mu\nu}{}^{\alpha\beta}(q)=\frac{1}{i}\int d^4x\, e^{iqx}\langle p|[j_\mu{}^\alpha(x),j_\nu{}^\beta(0)]|p\rangle. \quad (1)$$

As in Ref. 5, we consider only the case of spinless particles and nonconserved currents. In this case, one usually expands $t_{\mu\nu}{}^{\alpha\beta}$ in a set of invariants by writing (we suppress the indices α and β unless they are explicitly needed)

$$t_{\mu\nu}=ap_\mu p_\nu+b_1p_\mu q_\nu+b_2p_\nu q_\mu+cq_\mu q_\nu+dg_{\mu\nu}. \quad (2)$$

The sum rule derived by Fubini[1] and Dashen and Gell-Mann[2] writes (the integration is performed for fixed q^2)

$$(1/2\pi)\int_0^{+\infty}d\nu\,(a^{\alpha\beta}-a^{\beta\alpha})=f^{\alpha\beta\gamma}G^\gamma, \quad (3)$$

where $f^{\alpha\beta\gamma}$ is the usual SU(3) coefficient, G^γ is defined by

$$\langle p|v_\mu{}^\gamma(0)|p\rangle=p_\mu G^\gamma,$$

and

$$\nu=(p\cdot q).$$

The usual proof of (3) from the infinite-momentum limit is essentially equivalent to the following: First one integrates (1) over q^0 for fixed $\vec{q}$ obtaining

$$\int_{-\infty}^{+\infty}dq^0t_{00}{}^{\alpha\beta}(q^0,\vec{q})$$
$$=(2\pi/i)\int\delta(x^0)d^4x\,e^{iqx}\langle p|[j_\mu{}^\alpha(x),j_\nu{}^\beta(0)]|p\rangle. \quad (4)$$

The second member is evaluated from the standard equal-time commutation relations. This leads to

$$\int_{-\infty}^{+\infty}dq^0t_{00}{}^{\alpha\beta}(q^0,\vec{q})=2\pi p_0f^{\alpha\beta\gamma}G^\gamma. \quad (5)$$

One introduces the expansion (2) inside the integral and goes to the limit where $|p|\to\infty$ with $\vec{p}\cdot\vec{q}=0$. The usual argument is now that in (2) the term a will dominate at the limit since it is multiplied by $p_0{}^2$ in the expression for t_{00}. Then one gets

$$\lim_{p\to\infty}\int_{-\infty}^{+\infty}d\nu\,a^{\alpha\beta}(\nu,\vec{q})=2\pi f^{\alpha\beta\gamma}G^\gamma. \quad (6)$$

On the other hand, for fixed ν and $p\to\infty$

$$\vec{q}^2=\nu^2/p_0{}^2-q^2\to-q^2,$$

so that, if one could interchange the limit and the integral in (6), one would obtain the sum rule (3).

As already indicated, we want to take into account the fact that the currents $j_\mu{}^\alpha$ are local, i.e., commute for spacelike separation. In fact, one can show that, since $t_{\mu\nu}{}^{\alpha\beta}$ vanishes in x space for spacelike separation, one can choose a, b, c, and d to be also local functions. For that purpose one solves the differential equation obtained by writing (2) in x space and takes the solution which vanishes for large x in any spacelike directions.[7]

Applying the general theorems of Jost et al.,[8] we now write Jost-Lehmann-Dyson representations for a, b, c, and d. The corresponding weight functions will be denoted by $\tilde{a}$, $\tilde{b}$, $\tilde{c}$, and $\tilde{d}$. For instance

$$a(q)=\int d^4u\,ds\,\epsilon(q^0-u^0)\delta((q-u)^2-s)\tilde{a}(u,s).$$

In Ref. 4 we pointed out that, if the left member of (4) makes sense, the contribution of the terms b_1, b_2, and c of Eq. (2) to this integral becomes

$$p_0\int duds\,(\tilde{b}_1+\tilde{b}_2)+2\int duds\,u_0\tilde{c}, \quad (7)$$

for any fixed value of p. For proving this result, one essentially inserts the Jost-Lehmann-Dyson representation of b_1, b_2, and c into (2) and integrates over q_0 first.

On the other hand, one deduces from PT invariance that

$$2\int duds\,u_0\tilde{c}=p_0h, \quad (8)$$

where h is a Lorentz invariant.

In the method of Refs. 2 and 3 which we recalled above, the contribution of a is considered as being first order in p as p goes to infinity since one writes it in the form

$$(p_0)^2\int dq_0\, a = p_0\int d\nu\, a,$$

and assumes that $\int d\nu\, a(\nu)$ has a finite limit as $p\to\infty$. With this assumption, the contribution of a cannot be considered as the leading term in the infinite-momentum limit, since, according to (7) and (8), b_1, b_2, and c also give contributions proportional to p in the infinite-momentum limit [the integrals of $\tilde{b}_1$ and $\tilde{b}_2$ which appear in (7) are Lorentz invariant]. This shows that the sum rule (3) cannot be derived by the method proposed by Dashen and Gell-Mann in Ref. 2. Let us now show that even a stronger result is likely to hold, namely that for all finite p,

$$\int dq^0\, a(q^0, \vec{q}) = 0, \tag{9}$$

if one integrates for fixed $\vec{q}$. In fact, as shown in Ref. 4, Eq. (9) holds if the integral

$$\int du ds\, \tilde{a}(u,s)/s \tag{10}$$

makes sense. For the proof, one integrates, as previously, over q^0 first. This leads to (9) since

$$\int dq_0\, \epsilon(q_0-u_0)\delta((q-u)^2-s) = 0,$$

where the vanishing follows from obvious symmetry considerations. On the other hand, if (10) does not converge, one can choose a test function

$$f_T = 1 \text{ if } |q_0| \leqslant 1/T,$$
$$= 0 \text{ if } |q_0| \geqslant 1/T+\epsilon,$$

and define the left member of (4) by

$$\int dq^0\, t_{00}^{\ \alpha\beta} = \lim_{T\to 0}\int f_T(q^0) t_{00}^{\ \alpha\beta} dq^0.$$

In this case, the limit in T reduces for small T to the integral

$$\int du \int_{(1/T-u_0)^2}^{(1/T+u_0)^2} ds\, \frac{\tilde{a}(u,s)}{\sqrt{s}}.$$

Accordingly it will exist if

$$\tilde{a}(u,s) \sim s^n \text{ as } s\to\infty \text{ with } n \leqslant 0.$$

(There can be logarithmic terms if $n<0$.) However, it is easy to see that Eq. (9) will still hold except if $n=0$.[9]

One can thus see that what one usually considers as the leading term as $p\to\infty$ is in fact identically equal to 0 for all finite p except if $\tilde{a}$ behaves like a constant for large values of s. We believe that this last possibility is very unappealing physically. For instance, in this case, one would not be able to approximate $\tilde{a}$ by a finite number of δ functions in s so that one could not approximately saturate the sum rule by a finite number of one-particle intermediate states. Furthermore, one could not write an unsubtracted Jost-Lehmann-Dyson representation for the retarded function associated to $\tilde{a}$ so that arbitrary constants would appear in the covariant derivation of the sum rule (3) as given in Ref. 5.

Finally, if every weight function $\tilde{a}$, $\tilde{b}$, $\tilde{c}$, and $\tilde{d}$ is well-behaved, one has

$$p_0[\int du ds(\tilde{b}_1+\tilde{b}_2)+h] = \int_{-\infty}^{+\infty} dq_0\, t_{00}^{\ \alpha\beta}(q_0,\vec{q})$$

so that Eq. (5) gives

$$\int du ds(\tilde{b}_1^{\ \alpha\beta}+\tilde{b}_2^{\ \alpha\beta}) + h^{\alpha\beta} = 2\pi f^{\alpha\beta\gamma}G^\gamma,$$

which is independent of the Lorentz frame. Thus going to infinite momentum in Eq. (5) does not lead to any new result. The relation obtained, which we already derived in Ref. 5, merely states that the equal-time limit is the same as in the quark model.

Finally, if $\tilde{a}$ is well behaved for large values of s one sees immediately that

$$\lim_{p\to\infty}\int d\nu\, a(\nu,\vec{q}) \neq \int d\nu \lim_{p\to\infty} a(\nu,\vec{q})$$

contrary to what is assumed in the infinite-momentum method. In fact, the left member is 0 since the integral vanishes for all finite p. The right member is, as expected, different from 0 since there one integrates for fixed q^2 so that no symmetry consideration applies.

In general, it is important to remark that one will not easily see if (9) holds in perturbation theory. In fact, a particular graph will not satisfy (9) alone since it is not local in x space. Equation (9) will result from complicated cancellations between different graphs.

As a conclusion, we have shown that the proof of the Fubini–Dashen–Gell-Mann sum rule from the infinite-momentum limit is not straightforward, since the coefficient of $p_\mu p_\nu$ cannot be considered as the leading term in the limit. Moreover, this term is likely to be equal to 0 for all finite p. Accordingly one is led to believe that in Refs. 2 and 3, one obtains the good result at the end because the error made by taking the infinite-momentum limit inside the integral just compensates the error made by neglecting the other terms in the expansion of $t_{\mu\nu}$. It would be very interesting to show this cancellation explicitly since one can still derive the sum rule by the covariant method of Fubini.[1] However, we have not yet been able to solve that problem.

Finally, it is worthwhile to point out that our discussion does not apply to the infinite-momentum-limit method where one first derives the sum rule completely in a noncovariant way and takes the limit $p \to \infty$ only afterwards, as for instance in Ref. 4. Anyhow this method cannot be used to derive the sum rule of Fubini, Dashen, and Gell-Mann.

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Note added in proof.—After this paper was written, we became aware of a paper by Meyer and Suura[10] in which an equation analogous to (9) is considered to derive new sum rules. However, in Ref. 10, the currents are assumed to be conserved. The situation thus is quite different from the one we considered since, then, one does not know of any expansion analogous to (2) with independent coefficients satisfying a Jost-Lehmann-Dyson representation (see Meyer and Suura[11]). On the other hand, in Ref. 10 one takes the limit $p \to \infty$ inside the integral over q^0. We have found, in our case, a clear-cut example for which this is not permissible.

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SU(6) AND SUPERCONVERGENCE OF PION PHOTOPRODUCTION*

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The superconvergence sum rules of pion photoproduction amplitudes are studied to predict the electromagnetic properties of the nucleon and 3-3 nucleon resonance. Exactly the same results are obtained for the D/F ratio of nucleon magnetic moment and magnetic dipole decay width of the 3-3 resonance as from static SU(6). The problem of saturation of the intermediate-state summation by low-lying resonances is discussed.

It has been pointed out[1] that the static SU(6) properties of strongly interacting particles can be interpreted in terms of the current algebra of U(6)⊗U(6) type. From this viewpoint, particle interactions generated by vector and axial-vector current densities imply some higher symmetry insofar as the interactions are saturated only by bound states and lower lying resonances which turn out to form the basis of a supermultiplet of the higher symmetry.

It has also been suggested[2] that some dynamical requirements, such as superconvergence of dispersion integrals of scattering amplitudes, enable us to reproduce the results of higher symmetry as consequences of the dynamics. In these problems the saturation of the summation over intermediate states by particles belonging to lower lying supermultiplets is essen-