the runs. Finally, it is our pleasure to acknowledge the interest and support of Dr. Ralph P. Shutt.

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[3]Results of a preliminary analysis were presented earlier by T. G. Schumann, D. J. Crennell, G. R. Kalbfleisch, K. W. Lai, J. M. Scarr, I. O. Skillicorn, and M. S. Webster, Bull. Am. Phys. Soc. 12, 46 (1967).

[4]Ionization for each event was examined visually in order to check consistency with predictions of kinematics. We demanded positive identification for the Λ decay. The ambiguity between K^+ and π^+ interpretation for the positively charged track was resolved in about 80% of events. It is important to note, however, that the values of the $(K\pi)^+$ and $(K\pi\pi)^0$ mass from the final state $\Lambda(K\pi\pi)^0$ used here are unaffected by this ambiguity, since we use the fitted value for the lambda momentum but measured values for the other tracks at the production vertex. We assigned half-weight to events ambiguous between the two possible $\Lambda(K\pi\pi)^0$ final states.

[5]See the compilation by A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, P. Soding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. 39, 1 (1967).

[6]See, for instance, J. D. Jackson, Nuovo Cimento 34, 1644 (1964).

CONDENSATION OF REGGE CUTS, VANISHING TOTAL CROSS SECTIONS, AND TWISTING TRAJECTORIES*

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A very crucial problem in a Reggeized theory of strong interactions is that of Regge cuts. Various authors have advanced strong theoretical reasons to justify the presence and possible importance of Regge cuts.[1] The angular-momentum singularities important for crossed-channel asumptotic behavior have trajectories of the form

$$\alpha^{(n)}(t)=n\alpha(t/n^2)-n+1, \qquad (1)$$

due to an intermediate state containing n (identical) Regge poles, $\alpha(t)$. Here $n=1$ corresponds to the "parent" Regge pole $\alpha(t)$ and integer $n>1$ to Regge cuts.

From Eq. (1), one notices the ugly feature that a Pomeranchuk pole (called P pole, hereafter) with $\alpha_{\rm P}(0)=1$, generates an infinite number of Regge cuts all condensing to $J=1$ (at $t=0$), which would give rise to an essential singularity. Such a phenomenon is universal, in the sense that every trajectory of whatever quantum number would have a similar condensation at its intercept at $t=0$. This is due to the fact that the P pole has the quantum numbers of the vacuum and hence it can mix with any trajectory to produce the infinite sequence of cuts which, with $\alpha_{\rm P}(0)=1$, condense at $\alpha_c^{\,n}(0)=\alpha(0)$. (See Fig. 1.)

In this Letter, we propose to resolve this dilemma by a simple mechanism. We find some very interesting consequences (twist effect, vanishing total cross sections, etc.) and also provide some experimental tests for our pro-

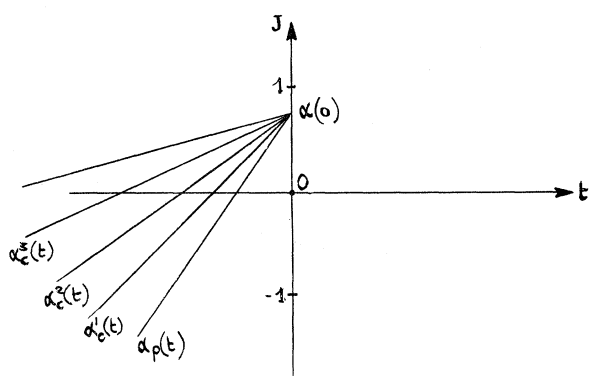

FIG. 1. The pole and cut trajectories with the Pomeranchuk intercept $\alpha_{\rm P}(0)=1$.

posal.

The argument is simply this: One introduces some parent Regge poles to provide the most dominant singularity in the J plane (for a given process). However, iteration through unitarity produces an infinite number of other singularities (cuts) which have slopes which are higher than the parent pole, a fact which may invalidate the pole-dominance idea. If, however, $\alpha_P(0)=1-\epsilon$ ($\epsilon>0$), then all the successive cuts are displaced to $\alpha_c^{(n)}(0)=\alpha(0)-n\epsilon$, which tend to $-\infty$ as $n\to\infty$.[2] (See Fig. 2.)

This has various interesting consequences apart from removing the essential singularity at the "parent" intercept $\alpha(0)$. (i) Since the cuts produced by iteration are successively pushed lower and lower, the parent Regge pole dominates (for $-t\approx 0$). A priori, there is no reason to believe why this should be true when $\epsilon\equiv 0$. (ii) The total cross section behaves as

$$\sigma_T(S)\xrightarrow[S\to\infty]{} S^{-\epsilon},$$

and hence would go to zero asymptotically ($\epsilon\neq 0$).[3,4]

Certainly in the diffraction-scattering region there is ample reason to believe in the Regge-pole dominance. The experimental situation regarding the asymptotic behavior of the total cross section is, however, far from clear. (For a discussion, see Ref. 3.) It is perhaps an even bet between constancy and a mildly decreasing cross section. Thus, we are led to investigate some further manifestations of our hypothesis.

From Fig. 1, it is clear that the cuts intersect with the pole only at $t=0$. However, in Fig. 2 ($\epsilon\neq 0$), the situation is remarkably different. Here $\alpha(t)$ crosses $\alpha_c^1(t)$ at t_1 (point b), $\alpha_c^1(t)$ crosses $\alpha_c^2(t)$ at t_2 (point c), and so on. The leading behavior as t is decreased is thus not given by $\alpha(t)$ but by the "effective" trajectory, $\tilde{\alpha}(t)$, which connects the points $abcde\cdots$. One may visualize this situation by regarding this as an "exchange-of-tails" effect between the singularity surfaces.[5] Anyway, the leading effective singularity $\tilde{\alpha}(t)$ is therefore constantly "twisting" (i.e., changing its curvature) as a function of t as it encounters other Regge cuts—even when the "parent" pole $\alpha(t)$ is taken to be a straight line.

From the diagram it is clear that on the positive-t side, however, the trajectory remains a straight line, since all the extrapolated cuts are systematically lower. Thus, the known part of the trajectory spectrum, where the particles and resonances lie, remains in accordance with the experimental straight-line behavior.

To get an idea regarding the magnitude of twisting, let us look at a specific example of

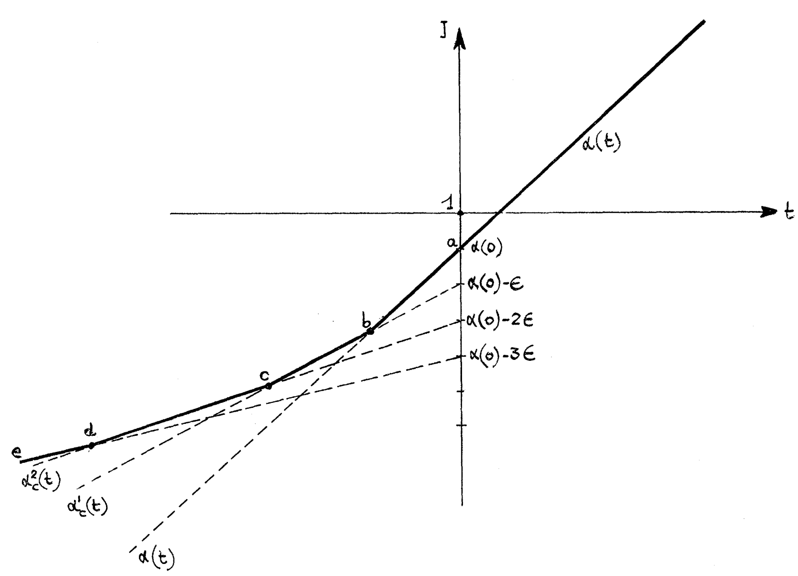

FIG. 2. The pole and cut trajectories with the Pomeranchuk intercept $\alpha_P(0)=1-\epsilon$. The effective leading singularity (in the negative-t region) is the curve $abcde\cdots$. On the positive-t side, pole trajectory $\alpha(t)$ dominates.

the ρ trajectory and the cuts generated in collusion with the P pole. If we choose for the parent ρ and P poles a linear shape, then the nth cut-trajectory function has the form

$$\alpha_c^{(n)}(t)=\alpha_\rho(0)-n\epsilon+\frac{\alpha_{\mathrm{P}}'\alpha_\rho'}{\alpha_{\mathrm{P}}'+n\alpha_\rho'}t, \qquad (2)$$

where $\alpha_\rho(0)$ is the ρ intercept at $t=0$, and α_P', α_ρ' denote the P and ρ slopes, respectively. If we choose $\alpha_P'\approx\frac{1}{3}$ (BeV)$^{-2}$, $\alpha_\rho'\approx 1$ (BeV)$^{-2}$,[6] we obtain for the slopes of the first and the second cuts $\alpha_c^{(1)\prime}\approx\frac{1}{4}$ and $\alpha_c^{(2)\prime}\approx 0.14$. The "crossings" between the ρ pole and $\alpha_c^{(1)}$, and between $\alpha_c^{(1)}$ and $\alpha_c^{(2)}$, occur at energies $t_1=-\frac{4}{3}\epsilon$ and $t_2=-9\epsilon$, respectively. In Ref. 3, $\epsilon\approx 0.07$–in any case, $\epsilon\lesssim 0.1$. Thus, $-t_1\approx 0.09$ to 0.13 (BeV)^2 and $-t_2\approx 0.6$ to 1 (BeV)^2. In the diffraction-scattering region [$|t|\lesssim 0.5$ (BeV)^2], therefore, one needs consider only the ρ pole and $\alpha_c^{(1)}$.[7] The "average" slope in this region should be $\approx\frac{1}{2}$ (BeV)$^{-2}$, which is in accord with the phenomenological trajectory as obtained by Phillips and Rarita.[8]

Now is there any experimental confirmation of such a twisting behavior for the leading trajectory function (for negative t) from high-energy phenomenology? We would like to think the answer is yes. We shall present the following pieces of evidence in support of this:

(1) In charge-exchange πp scattering, $\pi^-+p\to\pi^0+n$, only the ρ exchange need be considered. There the extensive data analysis by Phillips and Rarita[8] gives for a linear ρ trajectory the form

$$\alpha_\rho(t)=(0.53\pm 0.003)+(0.47\pm 0.02)t\cdots. \qquad (3)$$

This slope is about half of the ρ slope on the positive side. [$\alpha_\rho'\approx 1$ (BeV)$^{-2}$ in the resonance region.]

(2) In another charge-exchange scattering, $\pi^-+p\to\eta+n$, only the A_2 trajectory can be exchanged. Again one finds that A_2 has a lot of curvature in the negative-t region.[9] Their "best" A_2 trajectory seems to be

$$\alpha_{A_2}(t)=(0.37\text{ to }0.43)+(0.50\text{ to }0.80)t+\cdots. \qquad (4)$$

If one extrapolates this to the positive side, at the A_2 mass (≈ 1.3 BeV), the trajectory is only around 1 instead of the now established value of spin 2 for A_2.

In general, it seems to be true that the slopes on the negative side are much lower than on the positive side giving support to our hypothesis.

(3) The phenomenon of "dip" in the differential cross section where the exchanged trajectory is at a "nonsense" (unphysical) value has been widely hailed as a triumph of Regge poles. However, in the Phillips-Rarita analysis,[8] the value of t at which the ρ trajectory is at 0 (its unphysical value) is not the value one would obtain by extrapolating a straight line from the resonance side. Thus, it is the slope of the twisted trajectory which gives the correct t at which the dip phenomenon occurs. The same remark holds for the t value where $\alpha_\rho(t)=-\frac{1}{2}$, where again the pole contribution vanishes.

The numbers for our slopes seem consistent with the phenomenological analysis. As a byproduct, we are provided with a "natural" explanation why only the parent pole and the first (few) cut(s) should in general be dominant in the diffraction region.

To summarize: We conjecture that the simplest way to avoid the $J=\alpha(0)$ disease (condensation of infinite Regge cuts there) is to have the Pomeranchuk intercept slightly less than 1. Thus, cross sections would asymptotically tend to 0. Experimental evidence in this regard is quite unclear. However, with this hypothesis, the leading singularity generated by the parent Regge pole in collusion with the Pomeranchuk pole (which is assumed straight) is constantly changing its curvature ("twist effect").[10] This is perhaps easier to test in suitably chosen bins of t values for the charge-exchange differential cross section data. We have presented above some evidence in support of a twisted leading singularity.

A detailed analysis of the differential cross-section data along these lines is currently under preparation.

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[2]With $\alpha_P(0)=1$, Mandelstam (Ref. 1) pointed out that this sequence of cuts would invalidate the Mandelstam representation with a finite number of subtractions. For any fixed s, the number of subtractions in a dispersion relation in the t variable is finite, but this number increases indefinitely as $s\rightarrow\infty$. Of course, if the pole trajectories keep rising indefinitely, as they seem to be doing so far, it is hard to see how a finitely subtracted Mandelstam representation can be valid irrespective of these cuts.

[3]Such a bold hypothesis, that the total cross section may be vanishing (albeit slowly), was first put forward, in a simple current-algebra model, by N. Cabibbo, L. Horwitz, J. J. J. Kokkedee, and Y. Ne'eman, Nuovo Cimento 45A, 275 (1966). See also lectures by N. Cabibbo at the 1966 International School of Physics "Ettore Majorana," Erice, 1966 (Academic Press, Inc., New York, 1967).

[4]It should be noted that if one agrees to include these Regge cuts and allows for $\alpha_P(0)=1$, one has a very difficult problem to decide just what the asymptotic behavior of the total cross section should be. The popular choice, of course, is to keep $\alpha_P(0)=1$, ignore the infinite sequence of cuts, and thus get a constant asymptotic cross section. Certainly, this convenience deserves some explanation!

[5]A similar phenomenon is known in potential theory, where if two Regge surfaces $\alpha_1(s)$ and $\alpha_2(s)$ intersect, then they exchange tails producing a branch point in the trajectory functions. See, for instance, R. G. Newton, The Complex j-plane (W. A. Benjamin, Inc., New York, 1966). Thus, our hypothesis would lead to an infinite number of branch points (producing complex cuts) in the trajectory function. Hence, the possibility of writing down simple dispersion relations for the trajectory functions seems remote. Also, the "residues" are exchanged. Phenomenologically, one uses exponentially decreasing (as a function of t) residues [see, for instance, B. R. Desai, Phys. Rev. Letters 11, 59 (1963); T. O. Binford and B. R. Desai, Phys. Rev. 138, B1167 (1965); V. de Lany, D. Gross, I. Muzinich, and V. Teplitz, Phys. Rev. Letters 18, 149 (1967); K. Huang, C. Jones, and V. Teplitz, Phys. Rev. Letters 18, 146 (1967); R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965)]. In this picutre, such a quick decline of the effective residue functions may be a manifestation of the fact that the discontinuities of the higher cuts are decreasing for larger (negative) t, and not necessarily that of the parent pole.

[6]Desai; Binford and Desai; de Lany, Gross, Muzinich, and Teplitz; and Huang, Jones, and Teplitz, Ref. 5.

[7]We may reiterate that if $\epsilon=0$, there is a priori no reason to expect only the parent pole and the first cut to be important for $-t\approx 0$.

[8]Phillips and Rarita, Ref. 5. It has been kindly pointed out to me by S. Mandelstam that it is possible to find an appropriate set of residue parameters if one constrains the trajectory to go through the ρ on the positive side. See, e.g., F. Arbab and N. Bali, to be published. However, as has been noted later by F. Arbab, N. Bali, and J. Dash, Phys. Rev. (to be published), from charge-exchange data alone there are still unresolved ambiguities in a determination of ρ and A_2 parameters. It is for this reason that I have only considered the Phillips-Rarita analysis since they collect data from a variety of reactions (πp, Kp, elastic, charge-exchange, etc.) and find the best fit without any particular bias to constrain the trajectory to go through ρ (or A_2). Since we are interested in the "effective" leading singularity, this seems to me more appropriate. I would like to thank Dr. Mandelstam for raising this issue.

[9]R. J. N. Phillips and W. Rarita, Phys. Rev. Letters 15, 807 (1965).

[10]If the 2^+ singlet Pomeranchukon with a mass of 850-950 MeV really exists, the positive-side slope of α_P would be ~ 1 (BeV)$^{-2}$. Again, the twisting mechanism can account nicely for its slope being only $\approx\frac{1}{3}$ (BeV)$^{-2}$ on the negative t side.

DIFFICULTY IN THE INFINITE-MOMENTUM-LIMIT METHOD RELATED TO LOCALITY

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The proof of the Fubini–Dashen–Gell-Mann sum rule from the infinite-momentum limit is studied again taking the locality of the currents into account. It is shown that the sum rule cannot be derived from the $p\rightarrow\infty$ method proposed by Dashen and Gell-Mann.

Some time ago, Fubini[1] and Dashen and Gell-Mann[2] derived the same sum rule by two completely different methods. Fubini made use of commutators involving one space component of the current in such a way that the derivation deals only with covariant quantities all through the proof. Dashen and Gell-Mann, on the other hand, only introduced the commutator of two time components, thus working with noncovariant expressions. In this last method