for many informative discussions and for his aid with some of the calculations. M. J. Whippman also contributed to our better understanding of the basis of current predictions of the form factors.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]G. H. Trilling, Argonne National Laboratory Report No. ANL-7130, 1965 (unpublished); University of California Radiation Laboratory Report No. UCRL-16473 (unpublished). N. Cabibbo, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967), p. 29; U. Camerini and C. T. Murphy, ibid., p. 40; C. T. Murphy, University of Michigan Research Note No. 58/66 (unpublished).

[2]The transverse component P_T lies in the plane of the muon and pion momenta. It is in the direction defined by the unit vector $\hat{n}=\hat{t}\times\hat{k}$, where $\hat{k}$ is a unit vector in the direction of the muon momentum and $\hat{t}=\hat{k}\times\hat{p}_\pi$ where $\hat{p}_\pi$ is a unit vector along the pion momentum direction. The other transverse component is normal to the plane determined by $\hat{k}$ and $\hat{p}_\pi$, i.e., in the direction defined by $\hat{t}$. Recent measurements of that component [D. Bartlett, C. Friedberg, K. Goulianos, and D. Hutchinson, Phys. Rev. Letters 16, 282 (1966); K. K. Young, M. J. Longo, and J. A. Helland, Phys. Rev. Letters 18, 806 (1967)] yield values for Imξ of 0.11 ± 0.35 and -0.014 ± 0.066, respectively. In what follows, we have everywhere taken Im$\xi\equiv0$.

[3]L. B. Auerbach, J. M. Dobbs, A. K. Mann, W. K. McFarlane, D. H. White, R. Cester, P. T. Eschstruth, G. K. O'Neill, and D. Yount, Phys. Rev. 155, 1505 (1967).

[4]Trilling (Ref. 1); T. Devlin, private communication.

[5]The fraction of the muon energy spectrum corresponding to each of the quoted experiments was estimated from the measured muon spectrum of V. Bisi et al., Phys. Rev. 139, B1068 (1965). We necessarily assume that in all other respects, the experiments sample the Dalitz plot without bias.

[6]Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967), p. 40; A. Wattenberg, private communication.

[7]R. L. Imlay, P. T. Eschstruth, A. D. Franklin, E. B. Hughes, D. H. Reading, D. R. Bowen, A. K. Mann, and W. K. McFarlane, Phys. Rev. (to be published).

[8]See, for example, S. W. MacDowell, Phys. Rev. 116, 1047 (1959); P. Dennery and H. Primakoff, Phys. Rev. 131, 1334 (1963); C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 153 (1966); V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters 16, 947 (1966); T. Das, Phys. Rev. Letters 17, 671 (1966).

ELECTRON INTERFERENCE EFFECTS INDUCED BY LASER LIGHT*

John F. Dawson and Zoltan Fried
Lowell Technological Institute, Lowell, Massachusetts
(Received 22 June 1967)

We propose an electron-interference experiment involving laser light. It is estimated (roughly) that presently available (cw) laser sources can yield detectable "fringe" displacement in the electron interference pattern.

The interaction of laser light with free electrons has been the subject of numerous papers.[1,2] All of these involve the effect of the background laser light on Compton scattering. With the exception of the Kapitza-Dirac effect,[1] the deviations from the Klein-Nishina formula due to the presence of the laser light[2] are characterized by the dimensionless parameter ξ. In terms of the fundamental parameters of the problem,

$$\xi=(1/137)\rho\lambda_C{}^2\lambda,$$

where ρ is the photon density, λ_C is the electron Compton wavelength, and λ is the wavelength of the laser radiation. Presently available lasers yield small values for the dimensionless parameter ξ ($\sim10^{-4}$); hence any deviation from the Klein-Nishina formula[2] is difficult to detect. In retrospect, the fact that the change in the phase shift of the electron-photon systems as result of the background light is so small is not really surprising. The change in the Klein-Nishina amplitude[2] as a consequence of the background external field comes about (a) because the incident and final-state electron wave functions contain external-field-dependent phase terms, and (b) because these phases are momentum dependent. This relative phase, which can be detected in a scattering experiment, thus depends on the amount of the momentum transfer. Since in optical experiments with slow electrons the momentum transfer is of the order $\hbar\vec{k}/mc$, the net phase change is small. ($\vec{k}$ is the wave vector of the laser light.)

The purpose of this note is to point out that a much larger effect can be obtained by an electron-interference experiment. Here one compares the change in the phase of the electron wave function, which is accumlated throughout the passage time of the electron through the laser beam, with another (coherent) wave function which propagates outside the electromagnetic field. Schematically (see Fig. 1), the electron wave function is split into amplitudes A and B. Amplitude A passes through the laser light and amplitude B passes through the field-free region. A shift in the location of the interference pattern of the electron should be observed as a result of the cumulative phase change in amplitude A during its long $(\Delta t \gg \lambda/c)$ passage time through the laser beam. An accurate computation of the accumulated phase is hard[3] but not impossible. (We intend to publish this elsewhere.) So let us proceed and estimate this effect by (mis)use of the Volkov solution. Recall that the Volkov[4] solution (ignoring electron spin) is given by

$$\psi_p(x)=\exp\left\{-\frac{ip\cdot x}{\hbar}-\frac{i\hbar}{2n\cdot p}\int_{n\cdot x_1}^{n\cdot x}[2ep\cdot A-e^2A^2]d(n\cdot x')\right\}, \tag{1}$$

where p is the electron four-momentum, A is the vector potential, n is a lightlike four vector, and $(e^2/4\pi\hbar c)$ is 1/137. This solution refers to a one-dimensional wave packet, i.e., a plane electromagnetic wave. Such an arrangement precludes, however, the possibility of doing an interference experiment. Let us make the (perhaps drastic) assumption that an exact solution including the shape of the macroscopic laser field will yield not too different a result,[5] and that the Volkov formula (with suitable interpretation) is applicable to the arrangement shown in Fig. 1. To obtain a number we interpret Eq. (1) as follows (for an alternative derivation, see below):

$$\psi_p(x)\sim\exp\left\{-\frac{ip\cdot x}{\hbar}-\frac{i\hbar}{2n\cdot p}\int_{ct_1-z_1}^{ct_2-z_2}[2ep\cdot A-e^2A^2]d(n\cdot x')\right\}. \tag{2}$$

Here [in Eq. (2)] the space-time point x refers to the electron coordinate (i.e., the center of the electron wave packet) at a time t after it traversed the laser beam; $t_2-t_1=\Delta\tau$ is the transit time of the electron through the diameter (d) of the "cylindrical" laser beam. We further assume that z_2-z_1 is of the size of the transverse spread of the electron wave packet which is at most of the order of 10^3 Å. Therefore, the accumulated phase η is

$$\eta=\frac{\hbar}{2n\cdot p}\frac{e^2}{\hbar c}\frac{\rho\lambda}{2\pi}c\Delta\tau, \tag{3}$$

which for slow electrons can be written as

$$\eta\sim\frac{\hbar}{2mc}\frac{e^2}{\hbar c}\frac{\rho\lambda}{2\pi}c\Delta\tau, \tag{4}$$

v/c being of the order of 10^{-2}. We have omitted the term

$$\frac{\hbar}{2n\cdot p}\int_{ct_1-z_1}^{ct_2-z_2}2ep\cdot A\,d(n\cdot x'),$$

which for physically suitable parameters, namely $\rho\sim10^{16}$ photons/cm^3 and $\lambda\sim10^{-5}$ cm, is always smaller than unity. Similar remarks pertain to the oscillating part of A^2.

For an alternative, more transparent derivation, consider the following Hamiltonian[6]:

$$H(t)=\frac{p^2}{2m},\quad t<t_1 \text{ and } t>t_2,$$

$$=\frac{p^2}{2m}+\left(\frac{e^2}{2mc^2}\right)\langle A^2\rangle,\quad t_1<t<t_2, \tag{5}$$

where $\langle\,\rangle$ denotes the time average over a cycle of the radiation field. We assume that the passage of the electron through the diameter of the "cylindrical" laser beam is equivalent to turning on the $\langle A^2\rangle$ term in the Hamiltonian for the time duration d/v, v being the speed of the electron. The wave function at time $t>t_2$ is given by

$$\psi_p(\vec{x},t)=\exp\left\{\frac{i\vec{p}\cdot\vec{x}}{\hbar}-i\frac{p^2}{2m}\frac{t}{\hbar}-i\frac{e^2}{2mc^2}\langle A^2\rangle\frac{d}{\hbar v}\right\}. \tag{6}$$

Again the change in phase is

$$\eta=(e^2/2mc^2)\langle A^2\rangle(d/\hbar v). \tag{7}$$

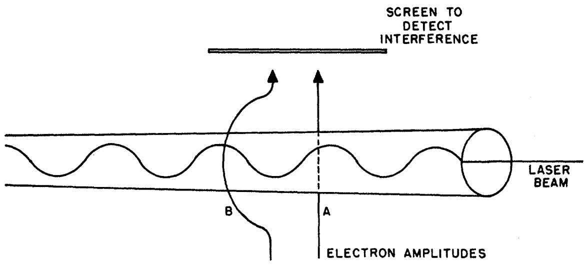

FIG. 1. Schematic for electron-interference experiment.

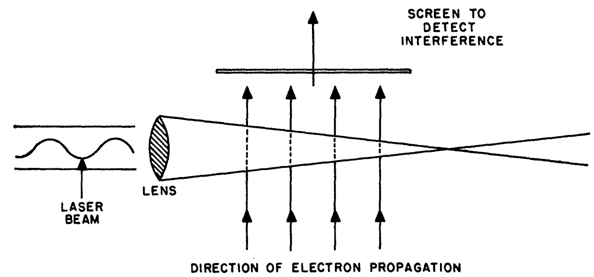

FIG. 2. An alternative way to construct interference.

For a focused (i.e., focused to a diameter of the order of 10^{-4} cm) cw argon laser, this parameter can be as large as unity. All other competing effects, such as Compton scattering (even by induced emission) and harmonic production, are negligible.[7]

Finally, an alternative way to do the experiment (to be sure, there are many more ways) is indicated in Fig. 2. Here the complete electron amplitude propagates through the tapering conical section of the focused laser beam. The interference arises due to the change in the effective "optical" path length of the electron wave function. For a fixed power emanating from the laser, the phase change as a function of the position-dependent diameter is

$$\eta(d)=\frac{e^2}{2mc^2}\frac{\langle A^2\rangle}{\hbar\nu}\left(\frac{d_0}{d}\right)^2 d, \tag{8}$$

where d_0 is the diameter of the laser beam at the position of the lens.

In summary, we wish to stress that although interference experiments with electrons are harder than scattering experiments (such as Kapitza-Dirac effect), the proposed scheme has the advantage of requiring much smaller intensities.

It is a pleasure to thank Professor Rainer Weiss of MIT for discussions concerning the feasibility of this proposed experiment.

---

*Work supported by U. S. Army Research Office (Durham) and administered by Lowell Technological Institute Research Foundation.

[1]P. L. Kapitza and P. A. M. Dirac, Proc. Cambridge Phil. Soc. 29, 297 (1933).

[2]T. W. B. Kibble, Phys. Rev. 150, 1060 (1966); N. D. Sen Gupta, Phys. Letters 21, 642 (1966); Z. Fried, A. Baker, and D. Korff, Phys. Rev. 151, 1040 (1966); P. Stehle and P. G. De Baryshe, Phys. Rev. 152, 1135 (1966), and numerous references therein.

[3]The difficulty stems from the fact that the exact macroscopic shape of the spatial extent of the laser beam has to be taken into account.

[4]D. M. Volkov, Z. Physik 94, 250 (1936).

[5]The "edge effect," due to the finite spatial extent of the laser beam, may change the value of the phase shift, but not by an order of magnitude.

[6]Admittedly, we are ignoring all fine points here, such as gauge invariance, finite extent of radiation field, etc.

[7]Several authors, notably J. J. Sanderson, Phys. Letters 18, 114 (1965), and T. W. B. Kibble, Phys. Rev. Letters 16, 1054, 1233(E) (1966), have pointed out that drastic "edge effects" take place when a classical electron enters into a region of strong electric field gradient. These considerations do not apply here, however, because the photon density needed for a measurable phase shift is orders of magnitude smaller. Furthermore, in a proposed experiment such as this, where use is made of the wave nature of the electron, there is no substitute, but a detailed semiclassical calculation which we are carrying out.