# GENERALIZED WEINBERG SUM RULES AND THE AXIAL-VECTOR CABIBBO ANGLE

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Weinberg's[1] sum rules for the spectral functions of the currents have recently been generalized by Das, Mathur, and Okubo[2] on the basis of the asymptotic properties of the propagation functions for the currents, which are suggested by the symmetries. In this note, we apply their arguments to derive sum rules by considering the propagation functions for the axial-vector currents with the internal quantum numbers of $\pi$ and $K$, respectively. Assuming the dominance of these sum rules by suitable low-lying states,[3] we obtain an expression for the ratio $F_K/F_\pi$, which, in the Cabibbo theory[4] for the semileptonic weak interactions, measures the SU(3)-symmetry-breaking effect on the Cabibbo angle[4] in the two-body leptonic decays of the pseudoscalar mesons. The result is in reasonable agreement with experiment.

The Fourier transforms of the propagation functions for the $\pi$- and $K$-type axial-vector currents are defined, respectively, by

$$\Delta_{\mu\nu}^{\pi}(q) = \int d^4x\, e^{-iq\cdot x}\langle 0|T\{A_{\mu 2}^{1}(x)A_{\nu 1}^{2}(0)\}|0\rangle \quad (1)$$

and

$$\Delta_{\mu\nu}^{K}(q) = \int d^4x\, e^{-iq\cdot x}\langle 0|T\{A_{\mu 3}^{1}(x)A_{\nu 1}^{3}(0)\}|0\rangle. \quad (2)$$

Assuming that the SU(3) symmetry becomes exact in the limit $q\to\infty$, we obtain from the asymptotic condition

$$\lim_{q\to\infty}\{\Delta_{\mu\nu}^{\pi}(q)-\Delta_{\mu\nu}^{K}(q)\}=0 \quad (3)$$

two sum rules analogous to those of Weinberg.[1] The assumption of dominance of these sum rules by the states $A_1(1080)$, $\pi$, $K_A(1320)$, and $K$ leads to the relation[5]

$$G^2(M_{A_1}^{-2}-M_{K_A}^{-2})=F_K^{2}-F_\pi^{2}. \quad (4)$$

Combining this relation with Weinberg's result[1]

$$G^2(M_\rho^{-2}-M_{A_1}^{-2})=F_\pi^{2} \quad (5)$$

gives

$$F_K/F_\pi = [1+(M_{A_1}^{-2}-M_{K_A}^{-2})(M_\rho^{-2}-M_{A_1}^{-2})^{-1}]^{1/2}. \quad (6)$$

Numerically, it is

$$F_K/F_\pi \simeq 1.17. \quad (7)$$

This means that the "renormalized" axial-vector Cabibbo angle $\theta_A^{(M)}$ for pseudoscalar meson decays is related to the "bare" Cabibbo angle $\theta$ by

$$\tan\theta_A^{(M)} \simeq 1.17\tan\theta. \quad (8)$$

Since there exists uncertainty[6] concerning the electromagnetic corrections to the vector coupling constant for the nuclear beta decay, we choose to approximate[7] the "bare" Cabibbo angle $\theta$ by the vector Cabibbo angle $\theta_V^{(M)}$ determined from the three-body leptonic decays of the pseudoscalar mesons. According to the latest experiment of Auberbach et al.,[8,9]

$$\theta_V^{(M)} \simeq 0.22, \quad (9)$$

$$\theta_A^{(M)} \simeq 0.265. \quad (10)$$

The agreement of the relation (8) with experiment is reasonable.

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Note added in proof.—After the submission of this note for publication, a preprint by S. L. Glashow, H. J. Schnitzer, and S. Weinberg was received, in which the result (7) is also obtained.

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[1]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[2]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967).

[3]We use the data compiled by A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

[4]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[5]Our $G^2$ corresponds to $g^2$ of Ref. 1.

[6]N. Cabibbo, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (University of California Press, Berkeley, 1967); T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 15, 381 (1965); E. S. Abers, R. E. Norton, and D. A. Dicus, Phys. Rev. Letters 18, 676 (1967).

[7]This is justified by the Ademollo-Gatto theorem.

[8]L. B. Auberbach, J. M. Dobbs, A. K. Mann, W. K. McFarlane, D. H. White, R. Coester, P. T. Eschstruth, G. K. O'Neill, and D. Yount, Phys. Rev. 155, 1505 (1967). We would like to thank Professor A. K. Mann for communicating the results prior to publication.

[9]The value for $\theta_V^{(M)}$ is obtained from $K_{13}$ data by including the $q^2$ dependence of the form factor according to the $K^*$ dominance model. See S. Oneda and J. Sucher, Phys. Rev. Letters 15, 927 (1965).

# EVIDENCE FOR THE $K^*(1300)$ IN $\pi^-p$ INTERACTIONS AT 6 GeV/$c$*

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Evidence for a $K\pi\pi$ enhancement at ~1300 MeV has come from the study of $Kp$ interactions, in particular from the reaction $K^+ + p \to K^+ + \pi^+ + \pi^- + p$ at 4.65 and 5 GeV/$c$.[1] The $K^*(1300)$ has been shown to decay strongly into $\pi + K^*(890)$ and possibly also into $K + \rho(760)$. Since the $K^*(1300)$ mass is not far above the thresholds for these quasi-two-body decays, it has been difficult to exclude the possibility that the enhancement is due to kinematic effects.[2] In this Letter we report independent evidence[3] from the reaction $\pi^- + p \to \Lambda + K^*(1300)^0 \to \Lambda + K + \pi + \pi$ supporting the resonance interpretation for this $K^*$. We find a mass of ~1300 MeV, a width of ~60 MeV, and isospin $\frac{1}{2}$.

The sample of events for this study is obtained from a systematic investigation of $\pi^\pm p$ interactions at 6 GeV/$c$ in the Brookhaven National Laboratory 80-inch hydrogen bubble chamber. This analysis is based on a $\pi^-p$ exposure of 230 000 pictures and a $\pi^+p$ exposure of 80 000 pictures. The events used for this analysis were required to have a visible $\Lambda^0$ decay with seen and unseen $K^0$ included. We present here data on the following channels leading to either neutral or doubly charged $K\pi\pi$ states.[4]:

$$\pi^- + p \to \Lambda + K^0 + \pi^+ + \pi^-, \quad 647 \text{ events}; \qquad (1a)$$

$$\to \Lambda + K^+ + \pi^0 + \pi^-, \quad 401 \text{ events}; \qquad (1b)$$

$$\pi^+ + p \to \Lambda + K^0 + \pi^+ + \pi^+, \quad 127 \text{ events}; \qquad (2a)$$

$$\Lambda + K^+ + \pi^+ + \pi^0, \quad 163 \text{ events}. \qquad (2b)$$

Figures 1(a) and 1(b) show the $(K\pi\pi)$ distributions for Reactions (1) and (2), respectively. Two enhancements are seen in the neutral $(K\pi\pi)^0$ mass spectrum at 1300 and 1440 MeV, whereas no significant structure is observed in the $(K\pi\pi)^{++}$ distribution. The solid curves indicate an estimate of the background togeth-

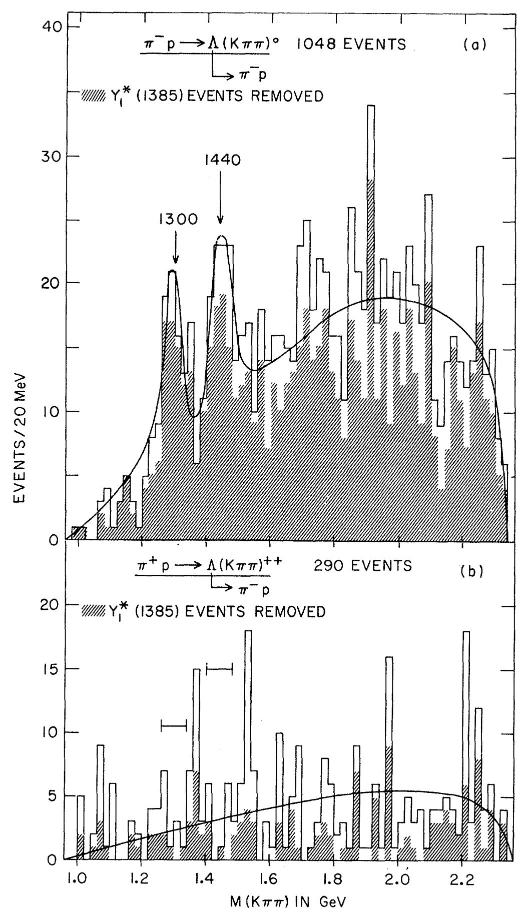


FIG. 1. The $(K\pi\pi)$ mass spectra. The solid curves indicate an estimate of the background, together with two resonance peaks in the case of the $(K\pi\pi)^0$ distribution.