Zatsepin show a spectral index 3.0 ± 0.08 for high-energy electrons at the energy range 3.5 BeV $<E<$ 70 BeV. Bleeker et al. reported a spectral index 2.6 ± 0.2 for 2 BeV $<$ 30 BeV. N_{s1} and N_{s2} calculated in this paper agree with these new data within the uncertainties of measurement.) On the other hand, in a recent article Verma[12] pointed out that the east-west asymmetry observed by Daniel and Stephens may be due to re-entrant albedo electrons instead of positron excess. If this is the case, and if further experiment indicates that the positron fraction in the >10-BeV energy region is less than 10% (the value at 1- to 5-BeV energy range), then the third model that sources of primary electrons are concentrated in the disk would be compatible with the observed spectrum. It is hoped that a direct measurement on the positron-to-negatron ratio, as well as further observations in the even higher energy range, will clarify the situation and determine the origin of cosmic-ray electrons.

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PREDICTION OF REGGE PARAMETERS OF ρ POLES FROM LOW-ENERGY πN DATA*

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Using finite-energy sum rules we predict important features of the Regge structure of ρ poles from low-energy πN data. The combined effect of N^* resonances in generating (via the sum rules) properties of the exchanged ρ poles is discussed, thus starting a new type of bootstrap calculation.

We use finite-energy sum rules[1,2] (FESR) to calculate the Regge parameters (as functions of t) of the high-energy πN charge exchange (CEX) amplitudes from the low-energy πN data (phase shifts). Thus we carry out a new type of bootstrap: Given the low-energy πN data (the N^* states), we calculate the exchanged ρ trajectories (masses) and coupling constants. We show how several interesting features of the N^* states "cause" (via FESR, i.e., via analyticity) corresponding features of the exchanged Regge poles.

Using the low-energy data and the FESR, we predict the following high-energy features of the $A'^{(-)}$ and $B^{(-)}$ amplitudes of πN CEX[3]: (1) The spin-flip amplitude $\nu B^{(-)}$ is larger than the nonflip amplitude $A'^{(-)}$ by an order of magnitude at $t=0$. This explains the near-forward peak in πN CEX. (2) $B^{(-)}$ has a zero near $t\approx-0.5$ BeV^2. This explains the observed dip in πN CEX. (3) $A'^{(-)}$ has a zero near $t\approx-0.1$ BeV^2. (4) In an effective one-pole model, we predict the ρ mass and a trajectory $\alpha_{\rm eff}$ which is 0.1 to 0.2 lower than the one measured at high energies. (5) Using high-energy fits as an additional input, we find some evidence for a second ρ trajectory, 0.4 lower than the ρ. This may be the manifestation of a cut. (6) There is strong evidence for an (approximately) fixed pole in $B^{(-)}$ at $j=0$.

The results (1) to (3) are "caused" (via FESR) by the following features (1′) to (3′) of the N^* states: All prominent resonances (1′) enter with the same sign in $B^{(-)}$, but with alternating signs in $A'^{(-)}$ at $t=0$, (2′) have their first zero[4] "simultaneously" in the narrow interval $-0.6<t<-0.4$ BeV^2 in $B^{(-)}$, and (3′) have their first zero at $-0.2<t<-0.1$ BeV^2 in $A'^{(-)}$. For negative t, we find large cancellations between the Born term and the lowest resonances in $A'^{(-)}$. This can be useful for determining coupling constants of these resonances, thus checking symmetry predictions.

Finite-energy sum rules are consistency conditions imposed by analyticity. If a function F can be expanded at high energies ($\nu \geq N$) as a sum of Regge poles $\beta[\pm 1-e^{-i\pi\alpha}][\Gamma(\alpha+1)\times\sin\pi\alpha]^{-1}\nu^{\alpha}$, then the following expansions (FESR) are equally valid[5]:

$$S_n \equiv \frac{1}{N^{n+1}}\int_0^N \nu^n \,\mathrm{Im}F d\nu = \sum \frac{\beta N^{\alpha}}{(\alpha+n+1)\Gamma(\alpha+1)}. \quad (1)$$

It is crucial to note that the relative importance of successive terms in the finite-energy sum rules (including higher moments) is the same as in the usual Regge expansion, i.e., if a secondary pole or a cut is unimportant in a high-energy fit above N, then this singularity is unimportant to exactly the same extent in the low-energy sum rules[1] (for the various moments). Note also that it is irrelevant in the FESR whether a singularity is above or below a certain point, say $\alpha=-1$.

The sum rule S_0 was applied by several authors[2,1] to the πN nonflip CEX amplitude at $t=0$, where $\sigma_{\mathrm{tot}}^{(-)}$ can be used for $\mathrm{Im}A'^{(-)}$. One finds that not only S_0 but also S_1 and S_2 are satisfied (within experimental error) with one ρ pole.[6]

The various sum rules can be used either together with the high-energy data to provide a better over-all determination of the Regge parameters, or by themselves to predict the Regge parameters from low-energy data alone. The latter use is a new kind of bootstrap program. Thus, assuming a one-pole fit, one can predict its trajectory by using the property

$$S_n : S_m = (\alpha+m+1):(\alpha+n+1). \quad (2)$$

Once $\alpha(t)$ is determined, one can go on and determine $\beta(t)$ from the various S_i. One works separately with the odd- and the even-moment sum rules, since one of these families contains the wrong-signature nonsense poles that do not affect the observable amplitude.

The crucial point, which allows us to predict the pole in the crossed channel (e.g., the ρ pole in the πN CEX), is the fact that $\mathrm{Im}F$ (the absorptive part in the direct channel) stays regular at the position of this pole, $\mathrm{Im}F \sim \nu^{\alpha}$. The partial wave expansion of $\mathrm{Im}F$ therefore converges (in the approximation that the ρ pole dominates the 2π continuum).

The position of the pole is reached when $\alpha=1$. This algebraic condition [Eq. (2)] is much easier to use than the conventional N/D condition that the solution D of the integral equation vanishes.[7]

Our new type of bootstrap works particularly well for amplitudes like $B^{(-)}$ or $A'^{(+)}$, where all prominent resonances ($\Delta, N_{\alpha}, N_{\gamma}$) enter with the same sign and add up constructively in the FESR. On the other hand, the use of the FESR in the reverse direction[8] (e.g., predicting the relative strength of N and $N_{1238}{}^*$) is particularly suitable for $A'^{(-)}$ and $B^{(+)}$, where the resonances enter with alternating signs and tend to cancel in the FESR. These cancellations become particularly large for negative t (see below).

The fact that the prominent resonances add up in the FESR for $B^{(-)}$ gives an example of the double counting committed in the interference model that regards the Regge term as a background on which one has to superimpose resonances.[9] There are two complete representations of the amplitude: One is the partial-wave series which can be dominated by direct-channel resonances or might have a large non-resonating background, and the other is the Regge asymptotic series consisting of pole terms s^{α} plus a background integral in the j plane. The combined FESR tell us that the sum of Regge terms s^{α} gives a fit to the smoothed-out experimental curve, and only the remaining wiggles are contributed by the background integral in the j plane. The smoothed-out resonance contribution is already included by the Regge-pole terms.[10]

The interference model has led Gatto[11] and Dass and Michael[12] to sum rules which, if applied to $B^{(-)}$, would produce a violent contradiction by equating a sum of positive resonance residues to 0. Another example is the dip in πN CEX which can be explained as the vanishing of $B^{(-)}$ when $\alpha_{\rho}=0$.[13] Recently, Hoff[14] suggested that it is rather due to the behavior of the resonances in the s channel in this region. In our opinion, both descriptions are adequate and equivalent.

In applying the FESR off the forward direction we have used the phase shifts of Bareyre et al.[15] to construct the imaginary parts of the amplitudes plotted in Fig. 1. The error bars indicate the variation obtained using phase shifts of different groups.[16,17]

In Fig. 1(b) we plot $\mathrm{Im}\nu B^{(-)}$ for various values of t. The nucleon Born term gets very large and negative for negative t, and the continuum also decreases rapidly; therefore, when evaluating the integral of this function, we find a 0 between $t=-0.4$ and -0.5 BeV^2 (see Fig. 2). From the dominance of the Born term and the smallness of the continuum, we conclude that such a 0 has to occur also if the cutoff N is chosen around 3 or 4 BeV. Thus we predict from low-energy data that the high-energy amplitude has a 0 as expected from the ρ Regge-pole interpretation

$$\mathrm{Im}\nu B^{(-)} \to d+\nu^{\alpha}=\beta\frac{\alpha\nu^{\alpha}}{\Gamma(\alpha+1)}. \qquad (3)$$

In Fig. 1(a) we plot $A'^{(-)}$ at several t values. In the integral S_0 we find large cancellations, but the net result (Fig. 2) is negative for $t\leqslant -0.2$. For $t\geqslant 0$, there are serious cut-off problems, but at $t=0$ we use σ_{tot} and establish that $A'^{(-)}$ changes sign between $t=0$ and $t=-0.2$.

An interesting feature occurs for $-0.4>t>-1$ BeV^2: Very large cancellations take place between the Born term and the $\Delta(1238)$, while the higher resonances are suppressed. The reason is that at these values of t, the N and Δ are outside the physical region (their $z_s \ll -1$) while the higher resonances are inside it (note that we are not yet in the region of the double spectral function and the partial-wave series converges). The large values of z_s enhance the lowest terms via the Legendre polynomials. The moral of this story is that if one wants to saturate analogous sum rules by low resonances, than one has to choose appropriate t values by taking into account the kinematics of the problem. Note in particular that $t=0$ is not the right choice here. This may be important for deriving estimates of coupling constants and discussing the validity of higher symmetries.

For a comparison (Fig. 2) of our predictions with high-energy experiments, we assume a one-pole model and take[18] $\alpha(t)=0.57+0.96t$ as an input from high-energy experiments in order to predict the residue functions $c(t)$ and

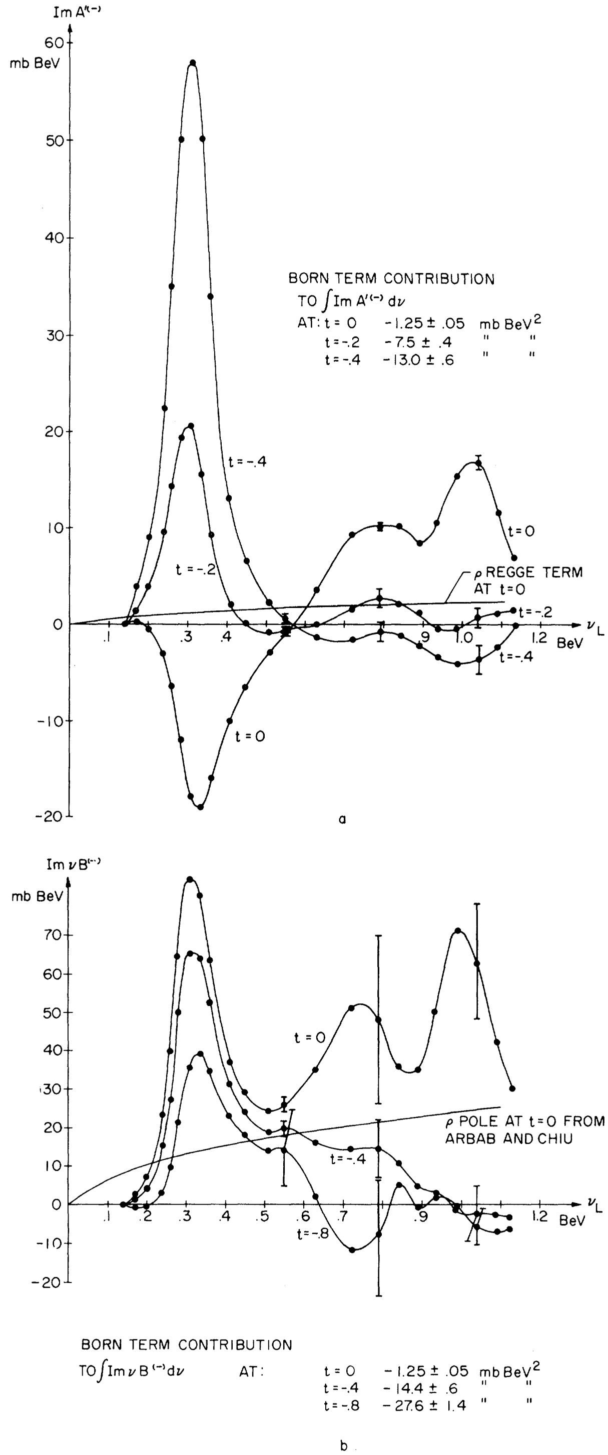

FIG. 1. The imaginary parts of $A'^{(-)}$ and $\nu B^{(-)}$ as determined from Bareyre's phase shifts. Error bars show the variation between different phase-shifts groups.

$d(t)$ defined in Fig. 2. Note that the high-energy differential cross sections only measure

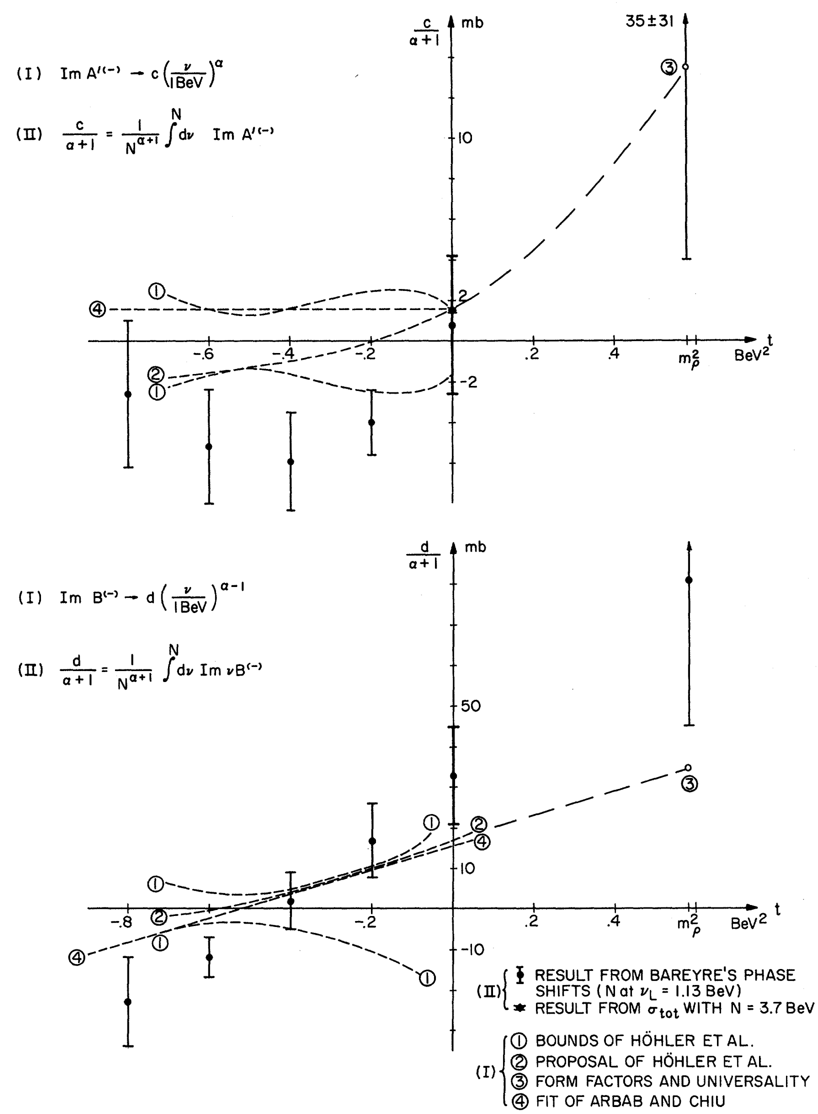

FIG. 2. Comparison of the Regge residue functions from (I) high-energy fits[18,19] and (II) the FESR, under the one-pole assumption ($\alpha_\rho = 0.57 + 0.96t$).

the sum of the squares $c^2 + d^2$ of the residue functions. The ambiguity in choosing c and d separately has been discussed by Höhler, Baacke, and Eisenbeiss,[18] and is shown in Fig. 2 (curves 2 and 4). Experiment gives bounds on c^2 and d^2, shown as curve 1. The FESR allow us to resolve these ambiguities: Figure 2 shows qualitatively that the choice 2 is preferred over 4,[19] i.e., c changes sign near $t = -0.1$ or -0.2 BeV^2.

Quantitatively[20] Fig. 2 shows that the one-pole approximation using the conventional values of α is outside the bound 1 established by high-energy experiments. This discrepancy (by a factor of about 2) can be fitted either by introducing a second ρ pole or by taking one "effective ρ pole," whose $\alpha_{\text{eff}}(t)$ has to be 0.3 lower in order to give the correct predictions at 10 BeV. Choosing N small enables us to see the effect of additional singularities although it introduces big errors in the sum rules.[16,20] The higher N gets, the better the one-pole fit will be.

S_3 shows in general a very similar behavior to S_1 for $B^{(-)}$. Its zero occurs at $t = -0.52$. Because of the change in the weight function, this zero is no longer due to the cancellation between the Born term and the rest of the resonances, but stems from the fact that the first zeros of the different $P_l'(z)$ of the higher resonances occur simultaneously in the narrow interval $-0.6<t<-0.4$ BeV^2. B is mainly determined by $P_l'(z)$. We find therefore that the N^* resonances occur with just the right quantum numbers and at the right intervals to vanish at the point where the Regge pole passes through 0. In the B amplitude, we find in the intermediate energy region that the prominent resonances (1520-2190) have a forward peak of the same width and roughly the same height as the Regge amplitude.

A similar behavior is found for the first zeros of the $P_l(z)$ that occur for the various resonances almost simultaneously at $-0.2<t<-0.1$ BeV^2. This is the reason that S_2 of $A'^{(-)}$ has a 0 at about the same place S_0 has one.

We now consider $B^{(-)}$ and use S_1 and S_3 together to determine $\alpha_{\text{eff}}(t)$ from Eq. (2). We get an intercept $\alpha_{\text{eff}}(0)=0.4\pm0.2$ and we predict $\alpha_{\text{eff}}(n_\rho^2)=1.0\pm0.3$. For $\alpha=0$ we consider the zeros of S_1 and S_3 directly and find $\alpha_{\text{eff}}=0$ at $t=-0.5\pm0.1$ BeV^2. We get a surprisingly good agreement with high-energy determination,[18,19] but we note that for $-0.6\leq t\leq 0$ the effective trajectory seems to be lower by 0.2.

Let us now use S_1 and S_3 of $B^{(-)}$ together with the high-energy fit of Arbab and Chiu[19] to determine the location of a possible second ρ trajectory, which is an explanation of the discrepancy found in Fig. 2.[20] It turns out that such a trajectory would lie approximately 0.4 below $\alpha_\rho(t)$.

An amazing result comes from evaluating S_0 for $B^{(-)}$. We find it to violate dramatically the one-pole fit (by a factor of 5 to 9 for $0>t>-0.8$). There must therefore be an additional pole that affects very strongly this even moment of B and is weak in the other moments. We use S_0 and S_1 together with the ρ-pole parameters of Arbab and Chiu[19] to determine the location of this pole. We find its position to vary from 0.12 to -0.15 as t varies from 0 to -0.8 BeV^2. In other words, it is very near $\alpha=0$ and has an unusually flat slope. The effect is consistent with the existence of a fixed pole at $\alpha=0$ which cannot affect the physical amplitude because it is at a wrong-signature nonsense point.[6] The absence of such poles would lead to a Schwarz-type sum rule.[21] However, Mandelstam and Wang[22] have recently shown that fixed poles arise because of effects of the third double spectral function. Note that the pole found in our treatment is an additive one and will not change the conclusions about the dip of the $B^{(-)}$ amplitude.

A more detailed presentation of our analysis and results will be published elsewhere.

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[4]Except the nucleon and the 1238, but they are strongly suppressed in the higher sum rules.

[5]The integration is over the right-hand cut in s and is always defined to include the Born term even if the latter occurs at negative values of ν.

[6]The sum rule S_1 excludes the possibility of a strong fixed pole at $j=-2$ in $A'^{(-)}$.

[7]Our bootstrap program avoids several other difficulties of the conventional N/D bootstrap; determinantal approximation, use of unmeasurable left-hand cut (forces), many-channel calculations, overemphasis of elastic unitarity.

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[10]This implies that the explanation of the high-energy πN polarization by interference of the ρ pole with resonances or resonance tails is incorrect.

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[16]B. H. Brandsen, P. J. O'Donnell, and R. G. Moorhouse, Phys. Letters 19, 420 (1965); A. Donnachie, R. G. Kirsopp, A. T. Lea, and C. Lovelace, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967); we used the solutions A,B,H,I. The large error bars were usually caused by the solution B.

[17]We have also carried out the whole program in the resonance model (full amplitude ≃ resonances) [A. H.

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[20]The error bars of the points II in Fig. 2 include (a) the experimental error in the low-energy integral (Born term and phase shifts) and (b) an estimate of the background integral in the j plane, as derived from the size of the wiggles. This latter error would rapidly diminish with a higher limit of integration N.

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[22]S. Mandelstam and L. L. Wang, Phys. Rev. (to be published).

NEW APPROACH TO ALGEBRA OF CURRENTS AND APPLICATION TO $K\to 2\pi$ DECAY*

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Ambiguities arising in some applications of current algebra are overcome by employing the algebra of currents to calculate the subtraction constant in a once-subtracted dispersion relation. The new approach is applied to $K\to 2\pi$ decay and yields corrections to the usual current-algebra method of about 10%. The branching ratio $(K^+\to\pi^++\pi^0)/(K_1^0\to\pi^++\pi^-)$ is also computed and agrees well with experiment.

It has been demonstrated by several authors[1] that the $\Delta I=\frac{1}{2}$ rule governing nonleptonic decays of kaons follows naturally from the algebra of currents, even though the original "current-current" Hamiltonian may contain an intrinsic $\Delta I=\frac{3}{2}$ part. However, one should note that the application of the soft-pion technique to $K\to 2\pi$ decay implies setting $m_K=m_\pi$ because of energy-momentum conservation. Hence, the $\Delta I=\frac{1}{2}$ rule for $K\to 2\pi$ decay is only valid in the approximation of neglecting the K-π mass difference. Thus one may inquire whether a large $\Delta I=\frac{3}{2}$ contribution will result if terms of order $[m(K)-m(\pi)]$ are not neglected. On the other hand, we should like the admixture of $\Delta I=\frac{3}{2}$ to be sufficient to explain the mode $K^+\to\pi^++\pi^0$. In this connection, we recall that Nambu and Hara,[2] using the soft-pion method, derived the relation

$$R=\frac{M(K^+\to\pi^++\pi^0)}{M(K_1^0\to\pi^++\pi^-)}\simeq\frac{m^2(\pi^+)-m^2(\pi^0)}{2m^2(K)}\approx\frac{1}{370}, \qquad (1)$$

which is too small (the experimental value is $R\simeq 1/22$). The theoretical prediction would be much improved if one could justify the replacement of $m(K)$ by $m(\pi)$ in Eq. (1).

In this note, we take a closer look at the class of decays wherein taking the soft-pion limit imposes an unreasonable constraint on the four-momentum of the decaying particle. To this end, we propose a new way of utilizing the algebra of currents in combination with a once-subtracted dispersion relation. In this new approach we do not take the limit $k\to 0$ (k is the pion four-momentum) but instead let $k^2\to 0$ ($k\to 0$ implies $k^2\to 0$ but not the converse). With this modification of the soft-pion technique, one is able to evaluate the corrections to the $K_1^0\to 2\pi$ calculation of Suzuki and Sugawara[1] and, moreover, one finds that Eq. (1) is replaced by

$$|R|=\left|\frac{M(K^+\to\pi^++\pi^0)}{M(K_1^0\to\pi^++\pi^-)}\right|=\left|\frac{m^2(\pi^+)-m^2(\pi^0)}{2m^2(\pi)}\right|\approx\frac{1}{30}. \qquad (2)$$

We proceed to explain the method; let us set

$$M(K(p)\to\pi_\alpha(k)+\pi_\beta(k'))$$
$$=\frac{i}{V^{3/2}}\frac{1}{(8p_0k_0k_0')^{1/2}}T_{\alpha\beta}(k^2,k'^2,p^2). \qquad (3)$$

Note that due to energy-momentum conservation $p=k+k'$; $T_{\alpha\beta}$ is a function of the variables p^2, k^2, and k'^2. Hereafter, we always take the mass value $p^2=m^2(K)$, and hence we shall no longer mention the possible dependence on p^2. Our procedure depends upon taking the succes-