form according to conjugate representations the rest of the proof is as above.

(5) It cannot be denied that the machinery brought to bear previously has been sufficient to obtain this general result. However, by using $CPT$ (which can be obtained from that machinery) directly, the relation to the intuitive argument that if $N$ fields have dependent adjoints they can be replaced by $N$ Hermitian fields is made rather clearer. The Hermitian fields are just $T\varphi(x)$ in the case of local conjugation.

It is satisfying that the use of $CPT$ invariance leads so directly to the results of the various previous investigations, since the existence and properties of the operator $\theta$ summarize concisely the relation between particle and antiparticle.

I have enjoyed discussions with Professor C. N. Yang, Dr. W. Bardeen, and Professor B. W. Lee.

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[6]This argument is also applicable to the case of reducible representations of the Lorentz group provided dynamical equations are given so that the particle creation operators can be fully specified. Without dynamical equations a doubled representation may contain distinct particles and antiparticles and still satisfy an equation of the same form as (2).

## DETERMINATION OF RELATIVE SIGN OF AMPLITUDES IN $\Lambda^0$ DECAY*

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It has been realized for some time[1] that a certain admixture of $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ has the same physical consequences in nonleptonic $\Lambda^0$ decay as the $\Delta I=\frac{1}{2}$ rule, and only differs from it in the sign of the $p\pi^-$ amplitudes relative to the $n\pi^0$ amplitudes. Moreover, it is known that this sign does have physical significance in the nonmesonic decays of $\Lambda^0$ hypernuclei,[2] although it has not been thought to have any physical consequences in free $\Lambda^0$ decay. Thus, while it can be argued that it is rather implausible that this particular admixture should occur in $\Lambda^0$ decay, still an uneasiness remains that apparent agreement between prediction and experiment may not be verifying the $\Delta I=\frac{1}{2}$ rule after all. It is the intent of this note to show how the relative sign of the amplitudes can be experimentally determined by observing the effects of the strong interaction in free $\Lambda^0$ decay, namely in the final-state interaction between the outgoing nucleon and pion. Our recent experiment on $\Lambda^0$ decay[3] detects this final-state interaction and as shown here indicates that the required admixture of $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ which duplicates the predictions of the $\Delta I=\frac{1}{2}$ rule does not occur.

The amplitudes in $\Lambda^0$ decay are, for $\Lambda^0\to p+\pi^-$,

$$S^-=-(\tfrac{2}{3})^{\frac{1}{2}}S_1e^{i\delta_1}+(\tfrac{1}{3})^{\frac{1}{2}}S_3e^{i\delta_3}, \tag{1a}$$

$$P^-=-(\tfrac{2}{3})^{\frac{1}{2}}P_1e^{i\delta_{11}}+(\tfrac{1}{3})^{\frac{1}{2}}P_3e^{i\delta_{31}}, \tag{1b}$$

and, for $\Lambda^0\to n+\pi^0$,

$$S^0=(\tfrac{1}{3})^{\frac{1}{2}}S_1e^{i\delta_1}+(\tfrac{2}{3})^{\frac{1}{2}}S_3e^{i\delta_3}, \tag{1c}$$

$$P^0=(\tfrac{1}{3})^{\frac{1}{2}}P_1e^{i\delta_{11}}+(\tfrac{2}{3})^{\frac{1}{2}}P_3e^{i\delta_{31}}, \tag{1d}$$

where $\delta_1$, $\delta_3$ are the pion-nucleon $s$-wave scattering phase shifts, and $\delta_{11}$, $\delta_{31}$ are the $p$ wave, for $I=\frac{1}{2}$ and $\frac{3}{2}$ at 37 MeV. Under time-reversal invariance $S_1$, $S_3$, $P_1$, and $P_3$ are all real. Since experiment[3] has not detected any violation of time-reversal invariance in the decay $\Lambda^0\to p+\pi^-$, we will assume these quantities are real in this analysis.

Applying the $\Delta I=\frac{1}{2}$ rule, then $S_3=P_3=0$ and

$$\frac{S^0}{S^-}=\frac{P^0}{P^-}=-\frac{1}{\sqrt{2}}. \tag{2}$$

This leads to the predictions (1) that the branching ratio

$$B_\lambda = \frac{\Gamma(\Lambda^0 \to p + \pi^-)}{\Gamma(\Lambda^0 \to p + \pi^-) + \Gamma(\Lambda^0 \to n + \pi^0)} = \frac{2}{3}, \quad (3a)$$

and (2) that the decay parameters for the neutral and charged modes are equal, i.e.,

$$\alpha_0 = \alpha_-,$$
$$\beta_0 = \beta_-,$$
$$\gamma_0 = \gamma_-, \quad (3b)$$

where

$$\alpha = \frac{2\,\mathrm{Re}S^*P}{|S|^2 + |P|^2}, \quad \beta = \frac{2\,\mathrm{Im}S^*P}{|S|^2 + |P|^2}, \quad \text{and } \gamma = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2}.$$

Experimentally it is known that $B_\lambda = 0.675 \pm 0.027$,[4] $\alpha_0/\alpha_- = 1.10 \pm 0.27$,[5] and $\gamma_0/\gamma_- = 1.04_{-0.21}^{+0.33}$.[3,6]

However, it is also true that the opposite choice of sign for the amplitudes, namely

$$\frac{S^0}{S^-} = \frac{P^0}{P^-} = +\frac{1}{\sqrt{2}}, \quad (4)$$

gives the same predictions as (3) although this choice would correspond to large $\Delta I = \frac{1}{2}$ violation. Neglecting the final-state-interaction phase shifts, the choice of sign (4) implies

$$S_3 = -2\sqrt{2}\,S_1,$$
$$P_3 = -2\sqrt{2}\,P_1. \quad (5)$$

If we include the final-state-interaction phase shifts, both choices of relative sign predict $B_\lambda = \frac{2}{3}$ if now the relations (5) become

$$S_3 = -2\sqrt{2}\,S_1 \cos(\delta_1 - \delta_3),$$
$$P_3 = -2\sqrt{2}\,P_1 \cos(\delta_{11} - \delta_{31}). \quad (6)$$

Including the phase shifts also affects the relationships between the decay parameters but since the phase shifts are small, $\alpha$ and $\gamma$, which depend on the cosines of the phase shifts, are insensitive to them. However, the $\beta$ parameter is very sensitive to the phase shifts. In fact, it differs from zero only because of the final-state interactions, assuming time-reversal invariance is valid in the decay. We find then, for condition (2)

$$\beta_-/\alpha_- = \tan(\delta_{11} - \delta_1),$$

while for condition (6)

$$\beta_-/\alpha_- = [\sin(\delta_{11}-\delta_1) + 4\sin(\delta_{31}-\delta_3)\cos(\delta_{11}-\delta_{31})\cos(\delta_1-\delta_3) + 2\sin(\delta_{31}-\delta_1)\cos(\delta_{11}-\delta_{31})$$
$$+ 2\sin(\delta_{11}-\delta_3)\cos(\delta_1-\delta_3)][\cos(\delta_{11}-\delta_1) + 4\cos(\delta_{31}-\delta_3)\cos(\delta_{11}-\delta_{31})\cos(\delta_1-\delta_3)$$
$$+ 2\cos(\delta_{11}-\delta_{31})\cos(\delta_{31}-\delta_1) + 2\cos(\delta_1-\delta_3)\cos(\delta_{11}-\delta_3)]^{-1}.$$

Evaluating these using $\delta_1 = 5.7°$, $\delta_3 = -3.4°$, $\delta_{11} = -0.95°$, and $\delta_{31} = -0.60°$,[7] we find for

$$\frac{S^0}{S^-} = \frac{P^0}{P^-} = -\frac{1}{\sqrt{2}}, \quad \frac{\beta_-}{\alpha_-} = -0.12$$

and for

$$\frac{S^0}{S^-} = \frac{P^0}{P^-} = +\frac{1}{\sqrt{2}}, \quad \frac{\beta_-}{\alpha_-} = -0.006,$$

to be compared with our experimental value[3] of $\beta_-/\alpha_- = -0.16 \pm 0.10$. To show the insensitivity of the $\alpha$ parameter in this regard, we find under condition (4), including the final-state interactions gives $\alpha_0/\alpha_- = 1.004$.

Thus the measurement of the final-state strong interaction in $\Lambda^0$ decay provides good evidence that the accidental conditions (4) and (6) do not occur and that experimental verification of the predictions (3) constitute a check on the applicability of the $\Delta I = \frac{1}{2}$ rule without this reservation. If time-reversal invariance is not valid in this decay, the effective result is that the $\delta$'s of (1) are not given by the pion-nucleon phase shifts. It is clear that a small violation of time-reversal invariance with phases ~1° would not alter the conclusions of this paper.

It is important to emphasize that even if the $\Delta I = \frac{1}{2}$ rule is not valid in $\Lambda^0$ decay, the measurement of $\beta_0$, $\alpha_0$, $\beta_-$, and $\alpha_-$ allows complete determination of $S_1$, $P_1$, $S_3$, and $P_3$, including relative signs, when the final-state interactions are considered as done here. Although we have only discussed $\Lambda^0$ decay, a similar sign problem exists in $\Sigma$ decay,[8] and the above remarks are applicable to those decays as well.

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# $N^*$ PRODUCTION IN $\pi^{\pm}$-$p$ AND $p$-$p$ INTERACTIONS*†

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We have observed a peak in the pion momentum spectrum in inelastic $\pi^{\pm}$-$p$ scattering corresponding to a recoiling baryon of mass 1.4 BeV. Previous investigations have demonstrated the existence of this peak in $p$-$p$ collisions.[1-4] In the present investigation, production of this mass peak and the 1.24-, 1.52-, and 1.69-BeV isobars have been observed in $\pi^{\pm}$-$p$ as well as $p$-$p$ interactions in the range of four-momentum transfer $0.01 < |t| < 0.2$ (BeV/$c$)$^2$ at incident lab momenta from 10 to 26 BeV/$c$.

The apparatus has been described previously,[5] and consisted of a magnetic spectrometer of scintillation-counter hodoscopes. The average momentum resolution was ~0.4 %.

Figure 1 shows typical momentum spectra $d^2\sigma/dpdt$ at various angles and incident momenta. The dominant feature of the data at small angles is the large peak corresponding to a missing mass of 1.4 BeV. In order to extract production cross sections, the data were fitted with Briet-Wigner line shapes broadened by the resolution (deduced from the width of the elastic peak) plus various simple forms for the unknown background. The tail of the elastic peak was fitted by a Gaussian. The mass which best fitted the 1.4 peak was $1.40 \pm 0.03$ BeV, in good agreement with $1.405 \pm 0.015$ BeV from Anderson et al.,[2] and $1.410 \pm 0.015$ BeV from Blair et al.[4] Within the error, the mass observed was independent of incident energy, four-momentum transfer, and incident particle. The other isobar masses used in the fit were fixed at 1.24, 1.52, 1.69, and 1.92 BeV.[6] The spectra did not include measurements corresponding to masses greater than 2 BeV. The widths of the isobars were chosen as 0.12 BeV for the 1.24-, 1.52-, 1.69-, and 1.92-BeV isobars and 0.15 BeV for the 1.4-BeV isobar. Various background shapes were tried including flat (independent of $P/P_{el}$) and polynomial in $P/P_{el}$ up to the fourth power, with the background vanishing at the elastic peak. However, since the more complex functions did

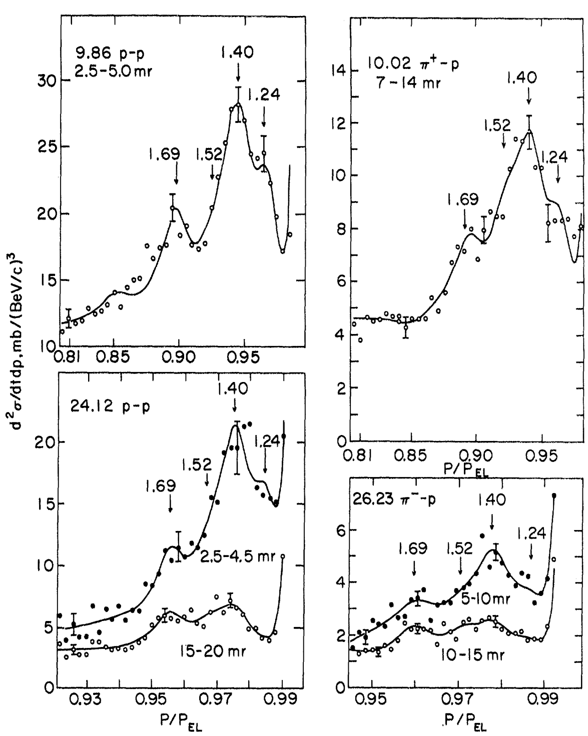


FIG. 1. Typical inelastic momentum spectra $d^2\sigma/dpdt$ plotted versus the ratio of particle momentum to that of elastically scattered particles at the same angle. The arrows show the expected locations of the isobars. The solid lines are least-squares fits described in the text.