SELF-CONJUGATE BOSONS, INTERNAL SYMMETRY, AND CPT INVARIANCE

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(1) In recent weeks a variety of proofs[1-4] have been offered for a theorem concerning the possible representations D for a system of self-conjugate bosons under the transformations of an internal symmetry group G. The conclusion—which is that there must be a basis in which the representation is real—is somewhat obscured by the complexity of the auxiliary hypotheses. The conclusion follows most simply from the existence of the CPT operation. We rely on the following lemma: If for some matrix A such that $A\bar{A}=1$ we have for the complex conjugate matrices $\bar{D}(g)$

$$\bar{D}(g)=AD(g)A^{-1}, \tag{1}$$

then the representation D can be made real. The proof is immediate, since such an A can always be represented as $\bar{T}^{-1}T$ for some matrix T. Thus TDT^{-1} is a real representation.

We recall here some definitions. By "internal symmetry" we mean that there exists a representation of G by unitary operators $U(g)$ such that for every point x,

$$U^{-1}(g)\varphi_\alpha(x)U(g)=\sum_\beta D_{\alpha\beta}(g)\varphi_\beta(x).$$

The field φ is presumed to transform according to an irreducible representation of the Lorentz group.[5] We denote by $\varphi_\alpha{}^\dagger(x)$ the Hermitian adjoint of the operator $\varphi_\alpha(x)$. Note that we do not assume the existence of a unitary transformation connecting φ and $\varphi^\dagger$, so that our result holds even for theories which are not invariant under charge conjugation. The antiunitary CPT operator is, on the other hand, known to exist for every relativistically invariant local field theory. We denote this operator by θ, and its action is as follows[5]:

$$\theta\varphi_\alpha(x)\theta^{-1}=\eta\varphi_\alpha{}^\dagger(-x),$$

$$\eta^2=(-)^{2s}=1.$$

(2) Local self-conjugacy.[2]—To illustrate the line of reasoning we consider first a trivial case in which even CPT invariance is not needed. Suppose

$$\varphi_\alpha{}^\dagger(x)=A_{\alpha\beta}\varphi_\beta(x); \tag{2}$$

then we have the relation

$$\varphi_\alpha(x)=[\varphi_\alpha{}^\dagger(x)]^\dagger=\bar{A}_{\alpha\beta}\varphi_\beta{}^\dagger(x)=\bar{A}_{\alpha\beta}A_{\beta\gamma}\varphi_\gamma(x). \tag{3}$$

Thus $\bar{A}A=1$. On the other hand, under the action of G we have [we suppress matrix indices now]

$$[U(g)^{-1}\varphi(x)U(g)]^\dagger=\bar{D}\varphi^\dagger(x)=ADA^{-1}\varphi^\dagger(x). \tag{4}$$

Hence the lemma applies and no additional assumption is needed.

(3) Nonlocal self-conjugacy.[1,3]—If Eq. (2) is replaced by the apparently more general relation

$$\varphi_\alpha{}^\dagger(x)=\int\tilde{A}_{\alpha\beta}(x-y)\varphi_\beta(y)d^4y, \tag{2$'$}$$

we may proceed most easily using Fourier transforms, $\varphi(n)$:

$$\varphi_\alpha(-n)^\dagger=A_{\alpha\beta}(n)\varphi_\beta(n). \tag{2$''$}$$

We must now invoke CPT which insures us of the existence of an antiunitary operator θ such that[5]

$$\theta\varphi_\alpha(-n)^\dagger\theta^{-1}=\bar{\eta}\varphi_\alpha(n). \tag{5}$$

Applying this to Eq. (2″) we have

$$\varphi(n)=\bar{A}(n)\varphi^\dagger(-n). \tag{6}$$

Substituting this in (2″) we find the relation

$$\varphi(-n)^\dagger=A(n)\bar{A}(n)\varphi(-n)^\dagger. \tag{7}$$

Since there must be some n for which the $\varphi_\alpha(-n)^\dagger$ are independent operators we conclude that, for that n, $A(n)\bar{A}(n)=1$. The action of G upon Eq. (2″) leads then to $\bar{D}=A(n)DA(n)^{-1}$ and we proceed as before.

(4) Particle theory.[4]—In the absence of field operators[6] self-conjugacy means (for example, for particles at rest) that antiparticle states can be expressed in terms of particle states:

$$|\bar{\alpha}\rangle\equiv\theta|\alpha\rangle=A_{\beta\alpha}|\beta\rangle. \tag{8}$$

Since θ is antilinear we again have $A\bar{A}=1$. Since the antiparticle and particle states transform according to conjugate representations the rest of the proof is as above.

(5) It cannot be denied that the machinery brought to bear previously has been sufficient to obtain this general result. However, by using CPT (which can be obtained from that machinery) directly, the relation to the intuitive argument that if N fields have dependent adjoints they can be replaced by N Hermitian fields is made rather clearer. The Hermitian fields are just $T\varphi(x)$ in the case of local conjugation.

It is satisfying that the use of CPT invariance leads so directly to the results of the various previous investigations, since the existence and properties of the operator θ summarize concisely the relation between particle and antiparticle.

I have enjoyed discussions with Professor C. N. Yang, Dr. W. Bardeen, and Professor B. W. Lee.

[1]P. Carruthers, Phys. Rev. Letters 18, 353 (1967).

[2]Y. S. Jin, Phys. Letters 24B, 411 (1967). This case has also been partially discussed by L. Stodolsky, unpublished.

[3]G. N. Fleming and E. Kazes, Phys. Rev. Letters 18, 764 (1967). The representation which these authors denote by D is denoted by $\overline{D}$ in the present discussion.

[4]Huan Lee, Phys. Rev. Letters 18, 1098 (1967).

[5]See, for example, R. F. Streater and A. S. Wightman, PCT, Spin and Statistics and All That (W. A. Benjamin, Inc., New York, 1964). The existence of θ is discussed on p. 131. [Note that in the case of half-integral spin $\theta^2=-1$ and so we find in Secs. (3) and (4) that $A\overline{A}=-1$. However, A is a direct product, $A=A_{\text{int}}\otimes A_{\text{spin}}$ and since $A_{\text{spin}}\overline{A}_{\text{spin}}=-1$ also, the rest of the argument is as given here.]

[6]This argument is also applicable to the case of reducible representations of the Lorentz group provided dynamical equations are given so that the particle creation operators can be fully specified. Without dynamical equations a doubled representation may contain distinct particles and antiparticles and still satisfy an equation of the same form as (2).

DETERMINATION OF RELATIVE SIGN OF AMPLITUDES IN Λ^0 DECAY*

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It has been realized for some time[1] that a certain admixture of $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ has the same physical consequences in nonleptonic Λ^0 decay as the $\Delta I=\frac{1}{2}$ rule, and only differs from it in the sign of the $p\pi^-$ amplitudes relative to the $n\pi^0$ amplitudes. Moreover, it is known that this sign does have physical significance in the nonmesonic decays of Λ^0 hypernuclei,[2] although it has not been thought to have any physical consequences in free Λ^0 decay. Thus, while it can be argued that it is rather implausible that this particular admixture should occur in Λ^0 decay, still an uneasiness remains that apparent agreement between prediction and experiment may not be verifying the $\Delta I=\frac{1}{2}$ rule after all. It is the intent of this note to show how the relative sign of the amplitudes can be experimentally determined by observing the effects of the strong interaction in free Λ^0 decay, namely in the final-state interaction between the outgoing nucleon and pion. Our recent experiment on Λ^0 decay[3] detects this final-state interaction and as shown here indicates that the required admixture of $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ which duplicates the predictions of the $\Delta I=\frac{1}{2}$ rule does not occur.

The amplitudes in Λ^0 decay are, for $\Lambda^0\to p+\pi^-$,

$$S^-=-(\tfrac{2}{3})^{\frac{1}{2}}S_1e^{i\delta_1}+(\tfrac{1}{3})^{\frac{1}{2}}S_3e^{i\delta_3}, \tag{1a}$$

$$P^-=-(\tfrac{2}{3})^{\frac{1}{2}}P_1e^{i\delta_{11}}+(\tfrac{1}{3})^{\frac{1}{2}}P_3e^{i\delta_{31}}, \tag{1b}$$

and, for $\Lambda^0\to n+\pi^0$,

$$S^0=(\tfrac{1}{3})^{\frac{1}{2}}S_1e^{i\delta_1}+(\tfrac{2}{3})^{\frac{1}{2}}S_3e^{i\delta_3}, \tag{1c}$$

$$P^0=(\tfrac{1}{3})^{\frac{1}{2}}P_1e^{i\delta_{11}}+(\tfrac{2}{3})^{\frac{1}{2}}P_3e^{i\delta_{31}}, \tag{1d}$$

where δ_1, δ_3 are the pion-nucleon s-wave scattering phase shifts, and δ_{11}, δ_{31} are the p wave, for $I=\frac{1}{2}$ and $\frac{3}{2}$ at 37 MeV. Under time-reversal invariance S_1, S_3, P_1, and P_3 are all real. Since experiment[3] has not detected any violation of time-reversal invariance in the decay $\Lambda^0\to p+\pi^-$, we will assume these quantities are real in this analysis.

Applying the $\Delta I=\frac{1}{2}$ rule, then $S_3=P_3=0$ and

$$\frac{S^0}{S^-}=\frac{P^0}{P^-}=-\frac{1}{\sqrt{2}}. \tag{2}$$