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HIGH-PRECISION $\pi^{\pm}$-p TOTAL CROSS SECTIONS FROM 8 TO 29 BeV/c*†

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We have measured the total cross section from 8 to 29 BeV/c for π^--p and from 8 to 22 BeV/c for π^+-p, using a counter-hodoscope system to obtain an absolute accuracy of 0.3%. Previous measurements[1] extend only to 20 BeV/c with an accuracy ~1%. The improved accuracy was necessary for investigation of asymptotic behavior and to reduce systematic uncertainty in a recent test of the validity of the forward dispersion relations.[2]

Two x-y hodoscopes were used to measure the incident angle of each particle in a momentum-selected (~0.2% rms) beam with a resolution of ±0.15 mrad. After transmission through a 10-ft long liquid-hydrogen target, the angle of the outgoing particle was measured by a third x-y hodoscope with a resolution of ±0.20 mrad. Pions were selected by two threshold Cherenkov counters. The density of the liquid hydrogen was held constant to within 0.03% by pressure control. The beam and general features of the data-handling and on-line computer system were similar to those previously described.[2]

At each momentum, we used about 20 points in 1-mrad steps to extrapolate the observed transmission to zero angle. The extrapolation form used was

$$\sigma_{\text{Tot}}(t) = \sigma_{\text{Tot}}(0) + Ae^{bt}.$$

It gave good fits down to $|t| = 0.005$ (BeV/c)^2 at which point π-e scattering and pion decay caused a departure from the smooth curve. A cubic polynomial extrapolation was also tried and gave the same result but with a somewhat worse χ^2. The entire contribution of the extrapolation was only ~2% of the total cross section and the total cross section was insensitive to a wide variation in minimum-$|t|$ cutoff. Thus this experimental procedure virtually eliminated the extrapolation error which was the dominant error in previous investigations.

Corrections were made for Coulomb-nuclear interference, multiple scattering, and muon contamination. The muon contamination of the beam (3-5%) was measured before and after each total cross-section measurement by the transmission through 8 ft of iron raised into the beam just downstream of the target. The uncertainty in the muon contamination was smaller than 0.1%. The electron contamination, as measured by a 40-ft Cherenkov counter set just below the muon threshold, was less than 0.1% and made a negligible contribution to the error.

In order to reduce accidental effects, events in which more than one counter in any incident hodoscope screen was struck were vetoed electronically. An increase in rate by a factor of 25 was made at 18 BeV/c π^--p and the resulting total cross section agreed within 0.1%. Total cross sections were also measured with a 2-ft long hydrogen target and the cross sec-

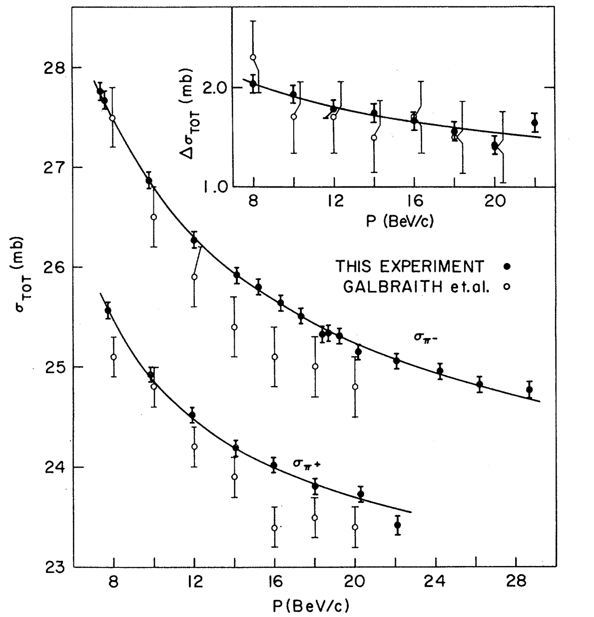

FIG. 1. The measured total cross sections versus laboratory momentum. Also shown are the data of Ref. 1. The lines are fit I of Table II. The inset shows the difference data $\sigma_{\pi^-}-\sigma_{\pi^+}$ with fit II. The fits are made to the data of this experiment only.

tion agreed with the 10-ft target value within the statistical error of 0.2%. Different areas of the hodoscope were used to show that possible bias effects due to efficiency variations were negligible.

The resulting cross sections are shown in Table I and Fig. 1. The errors shown are mostly systematic and were obtained by compounding the uncertainties in the corrections for π-e scattering, efficiency variations, muon contamination, accidentals, Coulomb effects, the effect of uncertainty in the t values of the measurements, and counting statistics (~0.1%). The compounding was justified since the effects were of comparable magnitude and uncorrelated. Although, as can be seen in Fig. 1, the present results are systematically higher than previous values,[1] we do not consider the difference to be significant since the errors are mainly systematic and in all cases nearly overlap. In considering the differences $\sigma(\pi^- \text{-} p)-\sigma(\pi^+ \text{-} p)$, some of the error cancels and an error of 90 μb is appropriate.

In order to examine possible asymptotic behaviors of the total cross sections, we have fitted the data to the functional form shown in Table II. For the three fits, we have assumed that the difference $\sigma_{\pi^-}-\sigma_{\pi^+}$ vanishes asymptotically as the momentum approaches infinity and that the momentum dependence can be expressed by two cross sections, each involving a single power of the momentum. The first (I) fits the measured cross sections themselves. The second (II) is a fit to the sum and difference of the measured cross sections, and fit III used

Table I. The measured total cross sections. Measurements within 100 MeV/c of the same momentum have been combined. The sums and differences shown have been produced by interpolating the measured points to even-numbered momenta by moving the points parallel to fit I.

MOM (BeV/c)	σ (π^-p) (mb)	MOM (BeV/c)	σ (π^+p) (mb)
7.38	27.755 ± 0.089	7.73	25.564 ± 0.084
7.60	27.671 ± 0.088	9.84	24.921 ± 0.079
9.78	26.871 ± 0.084	11.90	24.517 ± 0.078
12.01	26.273 ± 0.083	14.07	24.187 ± 0.081
14.13	25.915 ± 0.081	15.96	24.025 ± 0.076
15.21	25.799 ± 0.082	18.02	23.805 ± 0.081
16.31	25.642 ± 0.081	20.29	23.731 ± 0.079
17.32	25.509 ± 0.081	22.10	23.422 ± 0.098
18.36	25.327 ± 0.084		
18.68	25.344 ± 0.081		
19.22	25.308 ± 0.081		
20.17	25.150 ± 0.082		
22.09	25.064 ± 0.079		
24.27	24.955 ± 0.082		
26.19	24.822 ± 0.079		
28.68	24.774 ± 0.083		

MOM (BeV/c)	$\sigma_{\pi^-} - \sigma_{\pi^+}$ (mb)	MOM (BeV/c)	$\sigma_{\pi^-} + \sigma_{\pi^+}$ (mb)
8	2.04 ± 0.09	8	52.98 ± 0.12
10	1.93 ± 0.09	10	51.71 ± 0.12
12	1.78 ± 0.09	12	50.79 ± 0.12
14	1.74 ± 0.09	14	50.14 ± 0.12
16	1.66 ± 0.09	16	49.71 ± 0.12
18	1.56 ± 0.09	18	49.18 ± 0.12
20	1.42 ± 0.09	20	48.92 ± 0.12
22	1.64 ± 0.10	22	48.50 ± 0.13

Table II. The results of three least-squares fits to the data. Fit II used the sums and differences given in Table I.

Fit	$\sigma = A + B/p^C$	A	B	C
I	σ_{π^+}	22.60 ± 0.40	25.9 ± 9.6	1.06 ± 0.24
	σ_{π^-}	22.60 ± 0.40	19.6 ± 1.8	0.67 ± 0.08
II	$\sigma_{\pi^-} + \sigma_{\pi^+}$	44.24 ± 1.88	37.0 ± 8.5	0.69 ± 0.21
	$\sigma_{\pi^-} - \sigma_{\pi^+}$	0	3.85 ± 0.56	0.31 ± 0.06
III	σ_3	22.52 ± 0.51	24.8 ± 10.7	1.02 ± 0.28
	σ_1	22.52 ± 0.51	19.86 ± 1.92	0.58 ± 0.08

the isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ states. Each fit involved five arbitrary parameters and gave a good χ^2. Since the cross-section errors are mostly of a systematic nature, it is difficult to ascribe an exact meaning to the errors in the parameters derived from the least-squares fit. However, our opinion is that allowing a range of uncertainty of ± 2 standard deviations in these parameters is reasonable.

If fit I is used to evaluate the asymptotic contributions to the pion-nucleon forward dispersion relations, good agreement is obtained with recent experimental values[2] of the real part of the pion-nucleon scattering amplitude. Fit III also gives good agreement within 1 standard deviation. However, because of the slow convergence of the π^+ and π^- total cross section obtained from fit II, a good fit to the real part is obtained only by going to the limits of the results of the two experiments. Even if we take the most convergent fit to the difference (fit I) and estimate where the Pomeranchuk theorem will be valid to ~0.1%, we obtain $E \gtrsim 20\,000$ Gev.

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UPPER BOUND ON POSSIBLE MASS SPECTRA FROM SINGLE-PARTICLE SATURATION OF CURRENT ALGEBRA SUM RULES

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Since the recent success of the Adler-Weisberger relation[1] there has been considerable interest in attempts to saturate current algebra and superconvergence sum rules with a set of single-particle intermediate states.[1-5] The superconvergence relations reflect a pure Regge asymptotic behavior which is a multiparticle effect and it is very difficult to see how one could obtain a solution for a range of t with only single-particle states.[4,5] But with current-algebra sum rules things are different; here we are dealing with a perturbative amplitude whose high-energy behavior is dominated by a fixed pole in the j plane[4,6]:

$$A(s,t) \underset{s\to\infty}{\longrightarrow} F(t)/s + \text{superconverging Regge terms}, \quad (1)$$

and it is quite possible that a saturation with single-particle intermediate states could generate rather a lot of the structure of the fixed-pole term. Of course we need an infinite number of particles if $F(t)$ is to have any singularities in the finite t plane.

On the one hand, we could be very optimistic and seek an exact single-particle representation of current algebra in the $P_z \to \infty$ frame,[2] but the only solution found so far is the degenerate-mass case where vector current conservation can be satisfied locally and the completeness properties of the functions describing the particles plays a very prominent role.[3,4] Faced with the lack of nontrivial solutions, Gell-Mann and Zachariasen have tried to find any general restrictions on the possible mass spectra of any solution (assuming one exists), and have failed to do so up to order $1/m^2$.[2]

On the other hand, whether one believes in the existence of exact solutions or not, it is clearly interesting to treat the approximation of saturating the sum rules with arbitrarily narrow resonances.

In this Letter we demonstrate that the saturation of current-algebra sum rules by single-particle intermediate states indeed implies a strong limitation on the mass spectrum: $m(j)$ cannot grow asymptotically faster than j, the spin of the particle.

In order to avoid as much complication due to external spin as possible, we treat a sum rule obtained by sandwiching the commutator of a vector and scalar density between spinless single-particle states.[7] Writing as usual

$$T_\mu = i\int d^4x\, e^{iq\cdot x}\,\theta(x_0)\langle P'|[j_\mu^{\,i}(x), s^j(0)]|P\rangle, \quad (2)$$