are most serious near the edge of the zone. We expect that our results should be valid in the same temperature range as Dyson's, since both the thermodynamic quantities calculated by Dyson and the dynamical properties we calculate are influenced by long-wavelength thermal spin waves. On the other hand, one might argue that since the effects we find depend intimately on the influence of bound states, our results will be valid only in the (possibly smaller) temperature range where bound states are meaningful. However, it seems to us that these two regions are essentially the same, since when many spin-wave processes become important (and two-particle bound states lose their meaning), Dyson's formulas must also break down.

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STATIC QUADRUPOLE MOMENT OF THE FIRST 2^+ STATE IN ^{114}Cd MEASURED BY COULOMB EXCITATION*

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Recently, several experiments[1-4] have been reported which make use of higher order effects in Coulomb excitation in order to determine quadrupole moments of excited states of nuclei. One of the surprising results of these measurements was the systematic occurrence of large quadrupole moments for the first 2^+ states of nuclei which heretofore were believed to be good examples of the harmonic vibrator model.

In all but one of these experiments, excitation probabilities were obtained by measuring γ rays in coincidence with backscattered ions. The quadrupole moment can be obtained by comparing the excitation probabilities which result from using different types of projectiles, keeping the geometry of the experiment fixed. There are two difficulties associated with this type of experiment. First, it is hard to extract from a particle-γ coincidence experiment the excitation probability with the kind of accuracy needed (i.e., 1%). Secondly, it is not possible to differentiate between the effect due to the quadrupole moment of the 2^+ state and possible other higher order effects such as virtual excitation via the giant dipole resonance. Therefore, it seemed desirable to develop a second and at least partly independent method for measuring quadrupole moments.

In the present experiment the scattered ions are energy analyzed and the quantity measured is the ratio $R_{\mathrm{exp}}=d\sigma_{\mathrm{inel}}/d\sigma_{\mathrm{el}}$ of the inelastic to the elastic cross section as a function of the scattering angle. A similar experiment was performed by de Boer et al.,[1] who measured R_{exp} for different projectiles, keeping the scattering angle fixed.

The experiment was performed with 42-MeV O^{16} ions from the University of Pittsburgh tandem accelerator. The method of production of the oxygen beam is similar to the one employed in other laboratories, with the exception that we use a foil stripper. We typically obtain 0.3 to 1 μA of 6^+ beam on target. The scattered ions are energy analyzed in an Enge split-pole spectrograph and detected by means of position-sensitive detectors.

There are two effects which have discouraged previous attempts to determine excitation probabilities by resolving the inelastic and elastic groups. First, energy losses and straggling are very large, which necessitates the use of

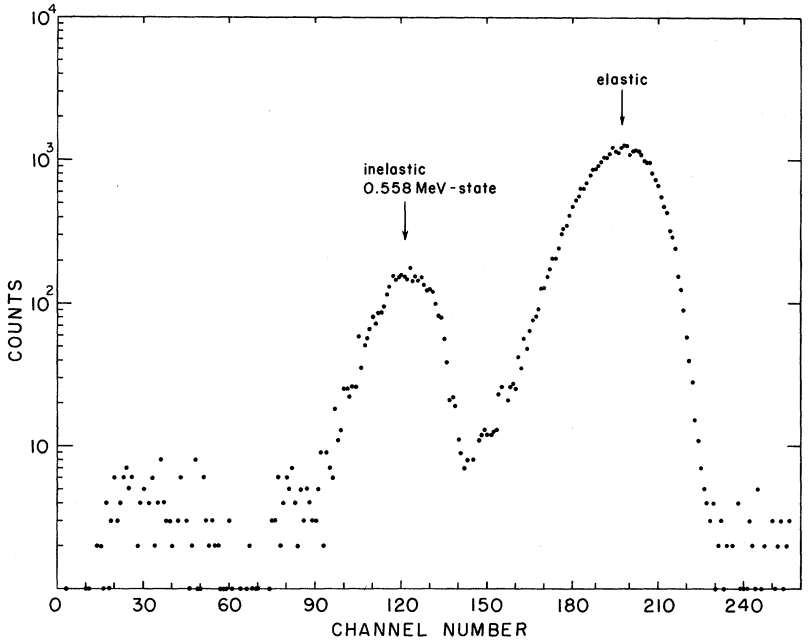

FIG. 1. Spectrum of O^{16} ions scattered from Cd^{114} at a scattering angle $\theta_{lab}=142.8°$ and $E_{O^{16}}=42$ MeV.

extremely thin targets. Secondly, the energy spread due to kinematics is large and severely limits, in conventional spectrographs, the usable solid angle. In the Enge spectrograph it is possible to compensate for kinematic energy spreading by proper positioning of the focal plane. Thus it was possible to use solid angles of 4 msr. Figure 1 shows a typical spectrum for a scattering angle of 142.8°. The target consisted of approximately 15 $\mu g/cm^2$ of Cd^{114} evaporated on a 10-$\mu g/cm^2$ carbon backing. Two detectors were used in the present experiment and measurements were made on either and the 6^+ and the 7^+ states or the 7^+ and 8^+ states. The measured ratios R_{exp} were corrected for the difference in charge-state distributions of the elastic and inelastic groups and are listed in Table I.

Table I. The experimental ratio $R_{exp}=d\sigma_{inel}/d\sigma_{el}$ as a function of laboratory angle. The beam energy of the oxygen ions was 42 MeV.

Laboratory angle (deg)	R_{exp}
45.0	0.009 04 ± 0.000 06
89.4	0.0543 ± 0.0004
119.4	0.0855 ± 0.0007
133.8	0.0962 ± 0.0011
134.2	0.0979 ± 0.0012
142.8	0.1050 ± 0.0010

The extraction of the quadrupole moment from the experimental data was accomplished by means of the Coulomb-excitation program of Winther and de Boer,[5] which solves the time-dependent Schrödinger equation for the transition amplitudes. Figure 2 shows the levels included in the calculation. They are indexed in order of their excitation energy, and those quadrupole transitions which were taken into account in the calculation are indicated by arrows.

The reduced matrix elements of the electric quadrupole operator, $M_{if}=-\langle I_i \| \mathfrak{M}(E2) \| I_f\rangle$, for each of the transitions shown are part of

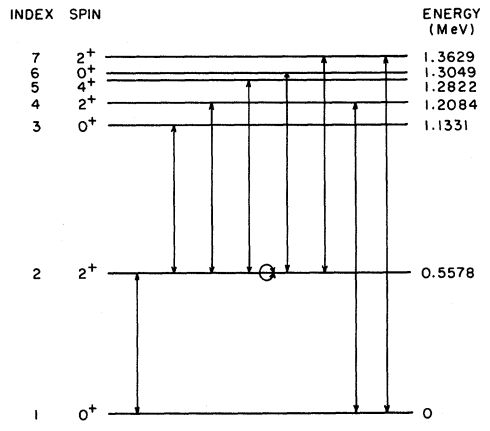

FIG. 2. Low-lying states of Cd^{114}. The arrows indicate the matrix elements used in the calculation.

Table II. The matrix M_{fi} (in units of $e \times 10^{-24}$ cm^2) used as input to the computer. M_{12} and M_{22} are fitted to the experimental data, and the higher matrix elements are from Ref. 6. No measured value for M_{26} was available so $M_{26}=M_{23}$ was used. $B(E2, i \to f) \times (2I_i + 1) = (M_{fi})^2$.

f \ i	1	2	3	4	5	6	7
1	0	M_{12}	0	±0.09	0	0	±0.09
2	M_{21}	M_{22}	±0.31	±0.84	±1.38	±0.31	±0.34
3	0	±0.31	0	0	0	0	0
4	±0.09	±0.84	0	0	0	0	0
5	0	±1.38	0	0	0	0	0
6	0	±0.31	0	0	0	0	0
7	±0.09	±0.34	0	0	0	0	0

the computer input. The matrix elements are subscripted with the level indices of Fig. 2. All matrix elements except M_{12} and M_{22} have been taken from results of Stelson and McGowan[6] and are listed in Table II. The matrix elements M_{12} and M_{22} are treated as variable parameters to fit the data. The quadrupole moment Q of level 2 and the $B(E2)$ for the transition between levels 1 and 2 are related to M_{12} and M_{22} by

$$B(E2) = |M_{12}|^2$$

and

$$Q = -0.758\, M_{22}.$$

A least-squares fit of $R_{\mathrm{comp}}(Q) = (d\sigma_{\mathrm{inel}}/d\sigma_{\mathrm{el}})_{\mathrm{comp}}$ to R_{exp} was made to determine the best values of M_{12} and M_{22}. The effect of the finite angular acceptance on $R_{\mathrm{comp}}(Q)$ was taken into account. In order to display the effects of M_{22} and the transitions to higher states, both $R_{\mathrm{comp}}(Q)$ and R_{exp} are divided by $R_{\mathrm{comp}}(0)$ in which all matrix elements except M_{12} have been set to 0. In Fig. 3 the quantities $1-R_{\mathrm{comp}}(Q)/R_{\mathrm{comp}}(0)$ and $1-R_{\mathrm{exp}}/R_{\mathrm{comp}}(0)$, which represent the deviations due to higher order effects, are plotted as a function of the scattering angle. The value of the quadrupole moment Q is sensitive to the relative signs of M_{14} and M_{24} and also of M_{17} and M_{27}. Unfortunately, the relative phases of these matrix elements are not known. In Fig. 3(a) all matrix elements except M_{22} were assumed to be negative whereas in Fig. 3(b) M_{14} and M_{17} were assumed to be positive. The quality of the fits is equally good for both assumptions. Since the largest uncertainty in Q comes from the unknown signs, the

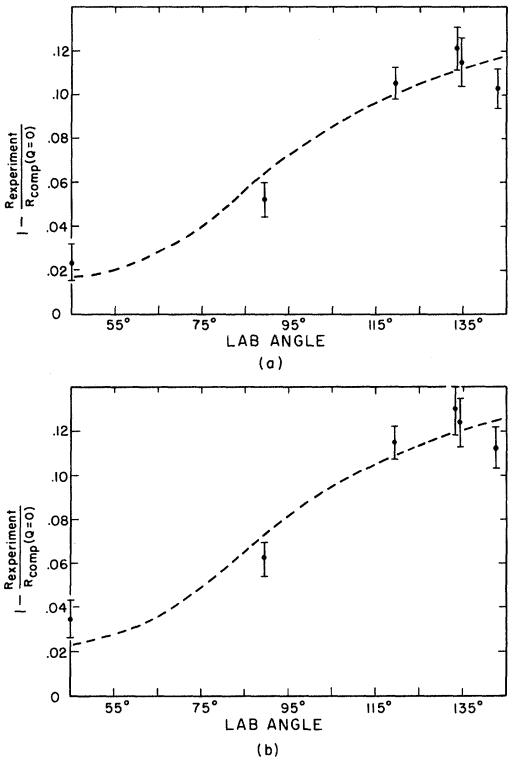

FIG. 3. Least-squares fit of $1-R_{\mathrm{comp}}(Q)/R_{\mathrm{comp}}(0)$ to $1-R_{\mathrm{exp}}/R_{\mathrm{comp}}(0)$. The dashed curve is $1-R_{\mathrm{comp}}(Q)/R_{\mathrm{comp}}(0)$. The error bars represent statistical errors only. (a) All matrix elements except M_{22} negative. (b) All matrix elements except M_{22}, M_{14}, and M_{17}, negative.

data were analyzed making all possible assumptions about the relative signs of M_{14} and M_{17}. Table III shows the results of this analysis. Table IV gives a comparison with the results of other groups. The agreement is excellent within the uncertainties quoted.

Eichler[7] has shown that large contributions from virtual excitation via the giant dipole resonance would considerably change the shape of the angular distributions of Fig. 3. Our re-

Table III. Results of a least-squares fit of M_{12} and M_{22} to the measured quantities R_{exp} shown in Table I. $(M_{12})^2 = B(E2, 1 \to 2)$ and $Q = -0.758 M_{22}$.

Signs of M_{14}	Signs of M_{17}	$B(E2)$ (e^2 b^2)	Q (e b)
−	−	0.566 ± 0.018	−0.67 ± 0.15
−	+	0.566 ± 0.018	−0.58 ± 0.15
+	−	0.567 ± 0.018	−0.48 ± 0.15
+	+	0.572 ± 0.018	−0.42 ± 0.15

Table IV. Comparison of our results with those previously reported in Refs. 2-4. See Table III for our results.

Institution	Q (e b)
Rutgers-Cal. Tech.	$-0.54 < Q < -0.90$
Aldermaston	-0.49 ± 0.25
Oak Ridge	-0.60 ± 0.14

sults indicate that contributions from this effect must be very small. This is in agreement with theoretical predictions by MacDonald.[8]

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DIRECT-COMPOUND INTERFERENCE AT AN ISOBARIC ANALOG RESONANCE IN DEUTERON STRIPPING

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Isobaric analog resonances are identified in the reaction $Pb^{207}(d,p)Pb^{208}$. A calculation of the cross section based on adding the scattering matrix elements for direct stripping (obtained from distorted-wave Born approximation) and for compound-nucleus formation (from R-matrix theory) fits the data near the $d_{5/2}$ resonance at 11.43 MeV well.

The interference of compound-nucleus formation and direct interactions in rearrangement reactions is not well understood. One of the problems is that the available experimental data have been difficult to analyze: The spins, parities, and widths of the resonances are not known and often there are several overlapping resonances.[1] In this Letter we present data and a successful analysis for a single resonance of known spin, parity, and width which occurs in a predominantly direct reaction. Furthermore, the reaction is on a heavy target nucleus at a much higher bombarding energy than previous work[1]; the direct reaction should therefore be well described by distorted-wave Born approximation (DWBA).

The reaction $Pb^{207}(d,p)Pb^{208}$g.s. was studied as a function of deuteron energy in the range 9.2 to 13 MeV. The yield curves below 10.3 MeV are smooth functions of the energy[2]; the data above 10.3 MeV are shown in Fig. 1.[3] Resonances are apparent near 11.4, 11.9, and 12.4 MeV, and it is clear that there is interference between the resonances and the nonresonant background. These deuteron energies correspond closely to the analog states, in the compound nucleus Bi^{209}, of the following single-particle states of Pb^{209}: $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$-$g_{7/2}$ (unresolved). The same analog states have been studied by means of the reaction $Pb^{208}(p,p)$, and the positions, total widths, and partial elastic-proton widths have been determined.[4]

The presence of the resonances in the (d,p) reaction is of interest also from other points of view. The formation of the analog resonances in the deuteron channel is isospin forbidden because deuterons have isospin $T=0$ and the analog states have T one unit larger than that of the target. Their appearance is then eith-