erated through a potential of 3.7 V; the current traversing the cavity was 0.13 mA. The xenon gas pressure was approximately 12 mTorr. At such low electron-beam currents the cavity fields are only slightly perturbed by the electrons, even at cyclotron resonance, and after proper alignment of the microwave spectrometer, the rate of absorption of field energy by the electrons in the cavity was the quantity measured. The center horizontal line of the oscilloscope trace represents zero absorption. Negative absorption near $\delta = 0$ is apparent. Comparison with Fig. 1 shows that the experimental value of the parameter α is approximately 4. This compares favorably with the data published by Frost and Phelps[8] which gives 3.8 as the value of α for 3.7-eV electrons in xenon.

Upon variation of the electron energy, the dip was observed only at those energies which correspond to a large positive slope of the collision cross section of the background gas used. In helium at energies of a few eV no dip whatsoever was observed. The spectrum corresponded to that labeled $\alpha = 0$ in Fig. 2, a result to be expected since the collision frequency for electrons in helium is about constant in that range.

It is suggested that the method employed in these experiments may be applied fruitfully to the development of accurate elastic-collision cross-section data.

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POSSIBLE RELATION BETWEEN PHASE SEPARATION AND THE λ TRANSITION IN ³He-⁴He MIXTURES*

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Along the λ line in ³He-⁴He mixtures, the specific heat at constant chemical potential presumably becomes infinite. It is suggested that the λ transition changes over to a first-order transition at a point at which the specific heat at constant composition becomes infinite. Possible reasons for such a change are discussed. The thermodynamic relationships are introduced through analogy with a system in which the variables are pressure and volume rather than chemical potential and composition.

In any order-disorder phenomenon there is a pair of complementary thermodynamic variables which are directly connected with the establishment or destruction of order, for example, the magnetic field and magnetization of an Ising lattice. Sometimes these variables are "hidden" and cannot be controlled by the experimenter, as in the λ transition of liquid helium. It is also often necessary to consider other "parametric" variables, which affect the process indirectly. In the liquid-vapor equilibrium, pressure P and (molal) volume V are primary variables, but in the Ising lattice they are parametric variables. The energy necessary to turn over a spin, which controls the transition in the Ising lattice, depends upon P or V, or, possibly, upon a parameter Y, which is itself a function of V and P. If $(\partial Y/\partial V)_T$ and $(\partial Y/\partial T)_V$ are everywhere finite and $(\partial Y/\partial V)_T$ is everywhere nonvanishing, Y may be said to be a volumelike variable, and C_v, then, tends to become infinite along a λ line. On the other

hand, if $(\partial Y/\partial P)_T$ and $(\partial Y/\partial T)_P$ are finite and $(\partial Y/\partial P)_T$ nonvanishing, Y is pressurelike and C_p becomes infinite along a λ line.

If C_v tends to become infinite along a λ line in P-T or V-T space, it can be shown[1] that $(\partial P/\partial T)_V$ parallels C_v as the latter becomes infinite, that along the λ line $dV_\lambda/dT_\lambda = (\partial V/\partial T)_P$, and that $(\partial P/\partial V)_T$ also eventually tends toward infinity as the λ line is approached. However, it becomes <u>positive</u>, which causes the isotherm to take the form of a van der Waals loop. Then the system becomes unstable and the transition changes over to a first-order one before the infinite value of C_v can be reached.

On the other hand, if C_p becomes infinite, then $(\partial V/\partial T)_P$ parallels C_p; while along the λ line $dP_\lambda/dT_\lambda = (\partial P/\partial T)_V$, and eventually, close to the λ line, $(\partial P/\partial V)_T$ tends to vanish, but is never positive, so that no instability appears.

It is instructive to consider the thermodynamic relation between C_p and C_v:

$$C_p = C_v + T(\partial P/\partial T)_V (\partial V/\partial T)_P.$$

This may be written

$$C_v = C_p + T(\partial P/\partial T)_V{}^2(\partial V/\partial P)_T$$

$$\xrightarrow[\lambda \text{ line}]{} C_p + T(dP_\lambda/dT_\lambda)^2(\partial V/\partial P)_T. \quad (1)$$

If C_p becomes infinite, $(\partial V/\partial P)_T$ becomes infinite and remains negative, and the second term in Eq. (1) cancels the infinity in C_p.

If C_v tends to become infinite, we write

$$C_p = C_v - T(\partial V/\partial T)_P{}^2(\partial P/\partial V)_T$$

$$\xrightarrow[\lambda \text{ line}]{} C_v - T(dV_\lambda/dT_\lambda)^2(\partial P/\partial V)_T. \quad (2)$$

With $(\partial P/\partial V)_T$ tending to become positively infinite, the second term in Eq. (2) would cancel the infinity in C_v if this condition could be reached.

The question now arises, what would happen to a λ line if Y changed from being a pressurelike parameter to a volumelike parameter? Since $(\partial Y/\partial V)_T = (\partial Y/\partial P)_T (\partial P/\partial V)_T$, we see that if Y is a pressurelike parameter, then $(\partial Y/\partial V)_T = 0$ along the λ line, because $(\partial P/\partial V)_T$ vanishes and $(\partial Y/\partial P)_T$ is finite. Similarly, if Y is volumelike, $(\partial Y/\partial P)_T$ vanishes along the λ line.

If Y changes from a pressurelike to a volumelike parameter at a certain point on the λ line, this means that $(\partial Y/\partial V)_T$ ceases to be zero,

and rises in value. At the same time, $(\partial Y/\partial P)_T$ becomes zero. Presumably at the point of change they are both zero. While $(\partial Y/\partial V)_T$ is zero, $(\partial P/\partial V)_T$ remains zero; when, however, $(\partial Y/\partial P)_T$ becomes zero, $(\partial P/\partial V)_T$ becomes infinite and changes sign, and the λ transition must change to a first-order one. Though $(\partial P/\partial V)_T$ vanishes all along the λ line where Y is pressurelike, we expect the region in which $(\partial P/\partial V)_T$ is near zero to become smaller and smaller as the point of change is approached. When $(\partial P/\partial V)_T$ first has an infinity, the associated van der Waals loop will at first be very small, but its prominence will increase as one recedes from the point of change.

In the λ transition in liquid ^{4}He it has been observed[2] that $(\partial P/\partial T)_V$ and $(\partial V/\partial T)_P$ both increase in the neighborhood of the λ line.

However, $(\partial P/\partial T)_V$ cannot become infinite —it eventually rounds off to a finite value—and since $(\partial P/\partial V)_T = -(\partial P/\partial T)_V/(\partial V/\partial T)_P$, it is only in this region that $(\partial P/\partial V)_T$ becomes very small. If this behavior is typical, then we may say in general that as Y approaches the end of the region in which it is a pressurelike parameter, the region in which $(\partial P/\partial T)_V$ rounds off will also become small. Finally, when the point is reached where Y becomes volumelike, $(\partial V/\partial T)_P$ becomes the coefficient which rounds off, and $(\partial P/\partial T)_V$ would become infinite if this unstable situation could actually be realized.

In solutions of ^{3}He in ^{4}He it has recently been found[3] that the coexistence or solubility curve for the phase separation, which occurs at low temperatures, does not have a rounded top, but consists of two more or less straight lines (one being the ^{3}He-rich branch, the other the ^{4}He-rich branch), which meet the λ curve at their intersection; thus the two transitions seem to be intimately related. This suggests, but of course does not prove, that the upper part of the first-order phase transition is related to the λ transition in the way described above, except that the parametric variables are the chemical potential μ and the number n of moles of ^{4}He in a fixed quantity of ^{3}He.

Let us consider the total energy E of the mixture to be a function of the total entropy S and of n. We may then write (neglecting PV terms, or at constant P we could substitute enthalpy for E)

$$dE = TdS + (\partial E/\partial n)_S dn,$$

$$= TdS + \mu dn,$$

which replaces the usual relation for dE. All the thermodynamic calculations will go through as before[1] with $-n$ replacing V and μ replacing P. No instability occurs if C_μ becomes infinite along a λ line, but instability will occur if C_n tends to become infinite.

We speculate now that at the end of the λ line, where the phase separation starts to take place, an infinity in C_μ tends to go over to an infinity in C_n. This seems a fairly reasonable hypothesis. As long as ^{3}He is in low concentration, the interaction between rotons, which is responsible for the λ transition, would be expected to depend upon an intrinsic property of the ^{4}He part, such as its chemical potential. But as the concentration of ^{3}He increases and ^{3}He itself becomes more intimately concerned in the transition, the dependence might well go over into one on concentration.

Some measurements on the specific heat at constant composition C_n have been made on ^{3}He-^{4}He mixtures.[4] In general, C_n reaches a peak at the λ line, but does not appear at become infinite, as, of course, it should not. It would be much more difficult to obtain measurements of C_μ, which should become infinite. Relationships analogous to those with pressure and volume as the parametric variables should hold.

The peak of C_n along the λ line becomes lower as the temperature λ is lowered. This presumably occurs because the decrease in the temperature is correlated with a decrease in the concentration of ^{4}He. The λ transition thus becomes "diluted." Also, at a composition very close to that of the top of the phase-separation curve, the order-disorder changes in the phase separation and in the λ transition tend to overlap in the curve of C_n vs T, which gives an appearance of rounding off. These phenomena are probably not connected particularly with the change in the character of the transition. But it would be very interesting to have complete thermodynamic data in this region, in order to be able to test and analyze the thermodynamic relationships.

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NONLINEAR EVOLUTION OF A TWO-STREAM INSTABILITY*

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Calculations of a two-stream instability have been made by following the motion of the phase-space boundaries of an incompressible and constant-density phase-space fluid. Because of the condensation of holes, which to a good approximation act as gravitational particles, large-scale nonlinear pulses develop.

This Letter presents a picture of the nonlinear development of a two-stream instability that has resulted from numerical experiments with the one-dimensional Vlasov-Poisson equations. The motion of the electrons is described by Vlasov's equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - \frac{\partial \varphi}{\partial x}\frac{\partial f}{\partial v} = 0. \qquad (1)$$

There is a uniform neutralizing background,

and the potential φ is determined from Poisson's equation

$$\frac{\partial^2 \varphi}{\partial x^2} = \omega_p{}^2\left[\int\!\! f\frac{dv}{v_0} - 1\right], \qquad (2)$$

where ω_p is the plasma frequency and v_0 is a constant chosen so that in the mean, $\int\!f dv/v_0 = 1$.

We picture the distribution function $f(x, v, t)$ as the density of an incompressible "phase