## TEMPERATURE DEPENDENCE OF SUPERFLUID CRITICAL VELOCITIES NEAR $T_{\lambda}$ \*

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Persistent-current methods are used to obtain critical-velocity data for flow of superfluid helium through filter materials with uniform pore size. For the smallest pore size investigated,  $0.2 \mu$ , evidence is seen for a small depression  $(7.5 \times 10^{-4} \,^{\circ}\text{K})$  of the transition temperature. The critical velocity for this pore size is found to vary linearly with the superfluid density as the transition temperature is approached.

The frictionless flow of superfluid helium is limited by critical velocities. This phenomenon has proved to be complex. Persistentcurrent methods are well suited to the study of critical velocities in liquid helium, particularly near the transition temperature. The superfluid gyroscope was developed to make such measurements.<sup>1,2</sup>

In the present work, persistent currents are formed in an annular container filled with a fibrous filter material.<sup>3</sup> The outer radius of the container is 2.5 cm, the inner radius 1.6 cm, and the height 1.0 cm. The filter material used was selected for the wide range of pore sizes available and for its uniformity in pore size.

Persistent currents are created by rotating the container at some angular velocity  $\omega$  while cooling through the transition temperature  $T_{\lambda}$ . When some standard temperature  $T_S < T_{\lambda}$ is reached, the container is stopped, and the persistent-current angular momentum  $L_p$  is measured.<sup>4</sup> An example of the values of  $L_p$ at  $T_S$  obtained for different initial angular velocities is shown in Fig. 1. The pore size is



FIG. 1. Angular momentum  $L_p$  of persistent currents formed by rotating at an initial angular velocity  $\omega$  while cooling through  $T_{\lambda}$  to a temperature  $T_s$  100 mdeg below  $T_{\lambda}$ . At  $T_s$  the rotation is stopped and  $L_p$  measured.

10  $\mu$ . For values of  $\omega$  less than  $\omega_c$ ,  $L_p$  is proportional to  $\omega$ . For speeds greater than  $\omega_c$ , the angular momentum saturates and becomes independent of the initial angular velocity. Any  $L_p$  measured at this temperature can be converted to an effective angular velocity required to create the measured amount of angular momentum.

In principle, the measurements shown in Fig. 1 could be repeated at a number of temperatures to determine  $\omega_c$  as a function of temperature. However, we have used another method which takes advantage of the reversibility of persistent-current angular momentum. A current is formed at the standard temperature  $T_s$ . Then the temperature is raised to a value T near  $T_{\lambda}$ . If the critical velocity at T is less than the original persistent-current velocity, then, when the temperature is reduced and  $L_{D}$  is remeasured at the standard temperature  $T_s$ ,  $L_b$  will have a smaller value. The reduced value of  $L_{b}$  is converted to an equivalent critical angular velocity using the data of Fig. 1.

The critical angular velocities obtained by this method do not require knowledge of the superfluid density. The temperature dependence of the superfluid density,  $\rho_S$ , is determined independently by a method described in an earlier paper.<sup>5</sup> In Fig. 2 we have plotted the values of the effective critical angular velocity  $\omega_C$  obtained for several different pore sizes against the quantity  $T_{\lambda}-T$ , where  $T_{\lambda}$  is taken as that temperature where  $\rho_S$  goes to 0.

For the larger pore sizes, the value of  $T_{\lambda}$  determined in this way is consistent with the value obtained by observation of the discontinuity in the behavior of a resistance thermometer placed in the helium bath. In the 0.2- $\mu$  pore size, however,  $\rho_S$  is found to approach 0 at a temperature 0.75±0.20 mdeg below the  $T_{\lambda}$  obtained for bulk helium. If this correction is not applied to the 0.2- $\mu$  data, the values of  $\omega_C$  drop below the critical angular ve-



FIG. 2. The effective critical angular velocity  $\omega_c$  is shown as a function of  $T_{\lambda}-T$  for flow through 0.2-, 10-, and 150- $\mu$  pores. In the case of the 0.2- $\mu$  data  $T_{\lambda}$  is taken as a temperature  $7.5 \times 10^{-4}$  °K below the bulk transition temperature.

locity for the larger pore sizes.

The critical velocities shown in Fig. 2 display only a weak dependence on temperature for values of temperature away from  $T_{\lambda}$ . In this lower temperature region the usual geometry dependence is seen; i.e., the smaller pore sizes exhibit the larger critical velocities. The marked decrease in the critical velocity seen in the 10- and 0.2- $\mu$  data has appeared in other investigations.<sup>6</sup> In the present experiment, however, we are able to approach much closer to the transition temperature. The 0.2- $\mu$  data in Fig. 2 exhibit an interesting power law dependence in the critical region near  $T_{\lambda}$ :

$$\omega_c = \omega_0 (1 - T/T_{\lambda})^{\zeta},$$

where  $\omega_0 = 1.9 \times 10^2 \text{ rad/sec}$  and  $\zeta = 0.68 \pm 0.03$ .

Figure 3 shows the critical velocity obtained in the 0.2- $\mu$  material plotted against  $\rho_S/\rho_0$ , where  $\rho_S$  is the superfluid density measured in the same system, and  $\rho_0$  is the value of  $\rho_S$  at a temperature 25 mdeg below the bulk transition temperature. Figure 3 demonstrates a linear dependence between  $\omega_c$  and  $\rho_s$ . The small value of  $\omega_c$  remaining when  $\rho_s$  is 0 is attributed to currents flowing in the small percentage of larger pores as well as in the channels which inevitably remain around the edges when the container is packed. This residual flow contributes a small angular momentum (approximately a 3% effect) which has been



FIG. 3. The critical angular velocity  $\omega_c$  obtained for flow through the 0.2- $\mu$  pores is plotted against  $\rho_s/\rho_0$ , where the superfluid density  $\rho_s$  is measured in the same material and  $\rho_0$  is the value of  $\rho_s$  at a temperature 25 mdeg below the bulk transition temperature.

subtracted to obtain the 0.2- $\mu$  data shown in Fig. 2. The need for this type of correction emphasizes the need for uniform materials if sharp determination of superfluid density, critical velocities, or  $\lambda$ -point depressions are desired when studying helium in porous materials.

The existence of a superfluid phase coherence length as discussed by Josephson<sup>7</sup> implies a limit on the gradient of the superfluid phase.<sup>8</sup> Tyson and Douglass<sup>9</sup> have noted that this length is inversely proportional to  $\rho_S$ . Since the superfluid velocity  $v_S$  is proportional to the gradient of the superfluid phase, one may expect a limit to  $v_S$  proportional to  $\rho_S$  for temperatures near the transition.

Langer and Fisher<sup>10</sup> have considered a critical-velocity mechanism based on fluctuations of the order parameter in the critical region. They find that this type of critical velocity will be proportional to the superfluid density. In order to make a comparison between our data and the theory of Langer and Fisher it is necessary to multiply the effective angular velocity  $\omega_c$  by a mean radius of about 2.0 cm. When this is done, we have, for the flow in the 0.2- $\mu$  material,

$$v_{s,c} = v_c (1 - T/T_{\lambda})^{0.68 \pm 0.03}$$

where  $v_c = 3.8 \times 10^2$  cm/sec and  $T_{\lambda} - T < 25 \times 10^{-3}$  °K. Langer and Fisher have obtained a value of  $v_c$  approximately four times great-

er. However, it should be noted that the flow through a complex material such as these filters is not uniform. A range of velocities will be present depending on the details of the internal geometry.

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<sup>8</sup>The authors are indebted to Professor David Mermin for making this point.

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## **NEGATIVE ELECTRON CYCLOTRON RESONANCE ABSORPTION DUE TO COLLISIONS\***

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Negative absorption or amplification due to cyclotron resonance interaction at microwave frequencies has previously been reported for a system of weakly relativistic electrons in vacuo.<sup>1,2</sup> Measurements of amplification of radiation near cyclotron resonance by electrons in a xenon discharge<sup>3</sup> and observations of anomalously high radiation temperatures near cyclotron resonance in non-Maxwellian plasmas<sup>4</sup> have also been reported. The physical process underlying the latter results has been attributed to the rapid variation with energy of the elastic-electron-collision cross sections of the Ramsauer gases argon, krypton, and xenon. In this note we describe an experiment in which the detailed nature of the negative cyclotron resonance absorption due to the elastic collisions of electrons with such atoms is revealed by using nearly monoenergetic electrons. The results are compared with the predictions of well-known kinetic theory of waves in plasmas.<sup>5</sup>

The phenomenon has generally been described by considering the small-amplitude transverse electromagnetic plane wave  $\vec{E} \exp i(\vec{k} \cdot \vec{r} - \omega t)$  with  $\vec{k} \perp \vec{B} \perp \vec{E}$  in a tenuous, infinite, uniform, and isotropic plasma immersed in a large constant magnetic field  $\vec{B}$ . When plasma collective effects are ignored and when, in the absence of wave fields, the electrons are distributed in velocity according to  $f_0(v)$ , standard methods utilizing the Boltzmann equation and Maxwell equations lead to the dispersion relation

$$\omega^2 - c^2 k^2 + \frac{2\pi \omega \omega_p^2}{3} \int_0^\infty dv \, v^3 \frac{\partial f_0 / \partial v}{\omega + \Omega + i \nu_c(v)} = 0.$$
(1)

The electrons which gyrate with angular frequency  $\Omega = eB/mc < 0$  also experience occasional collisions with heavy, stationary neutral atoms. The collisions are short ranged and are therefore phenomenologically described by the velocity-dependent collision frequency  $\nu_c(v)$ . For very low densities Eq. (1) is solved approximately for the small growth or damping rate Im $\omega$  by expanding it about  $\omega = ck$  to obtain

$$\mathrm{Im}\omega \simeq -\frac{\pi\omega_{p}^{2}}{3} \int_{0}^{\infty} dv f_{0}(v) \frac{\partial}{\partial v} \left[ \frac{v^{3} v_{c}(v)}{(ck+\Omega)^{2} + v_{c}^{2}} \right].$$
(2)

When the distribution of electron velocities  $f_0(v) = (4\pi V^2)^{-1}\delta(v-V)$ , representing a narrow spread of electron energy about the value  $\frac{1}{2}mV^2$ , is inserted into Eq. (2), the cyclotron resonance absorption spectrum is given by

$$-\frac{4\nu_{c}(V)}{\omega_{p}^{2}}\operatorname{Im}\omega \simeq \frac{1+\frac{1}{3}\alpha}{1+\delta^{2}} - \frac{\frac{2}{3}\alpha}{(1+\delta^{2})^{2}}.$$
(3)