ASYMPTOTIC CHIRAL SYMMETRY

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In this note we point out a general consistency between the predictions of asymptotic chiral SU(3)⊗SU(3) invariance[1,2] and the extrapolations to large spacelike momentum transfer of the usual phenomenological fits to vector and axial-vector baryon form factors.

The point of view we take is the one often stressed by Nambu.[1] Chiral SU(3)⊗SU(3) invariance[3] is considered to be a fairly good symmetry of nature so the various chiral "charges" are (partially) conserved. However, because of its large mass the nucleon cannot ordinarily approximate to an eigenstate of these charges, and we may expect the usual group-theory approach to be misleading. On the other hand, for vertex functions involving extremely large momentum transfers even the nucleon mass may be negligible and there is a possibility that assigning the baryons to definite chiral representations will result in interesting predictions.

In the limit of SU(3) degeneracy we have the following expressions for the vector and pseudovector form factors of the octet baryons:

$$\langle N'(p')|V_{b\mu}{}^a(0)|N(p)\rangle = \frac{-iM}{(p_0p_0')^{1/2}}\bar{u}(p')\Big\{\gamma_\mu[d_1{}^V(q^2)D_b{}^a+f_1{}^V(q^2)F_b{}^a] + \frac{\sigma_{\mu\nu}q_\nu}{2M}[d_2{}^V(q^2)D_b{}^a+f_2{}^V(q^2)F_b{}^a]\Big\}u(p), \tag{1a}$$

$$\langle N'(p')|P_{b\mu}{}^a(0)|N(p)\rangle = \frac{-iM}{(p_0p_0')^{1/2}}\bar{u}(p')\Big\{\gamma_\mu\gamma_5[d_1(q^2)D_b{}^a+f_1(q^2)F_b{}^a] + \frac{iq_\mu}{2M}\gamma_5[d_2(q^2)D_b{}^a+f_2(q^2)F_b{}^a]\Big\}u(p). \tag{1b}$$

In Eqs. (1) $V_{b\mu}{}^a$ and $P_{b\mu}{}^a$ are the vector and pseudovector currents, M is the baryon mass, $D_b{}^a$ and $F_b{}^a$ are the symmetric and antisymmetric SU(3) matrices, and $q=p-p'$.

We shall assume that the eight baryons belong (asymptotically) to the eight-dimensional $[(\underline{8},\underline{1}),(\underline{1},\underline{8})]$ representation[4] of chiral SU(3)⊗SU(3). The other usual assignments bring in extra states which, besides being hard to explain away, lead to bad results in our scheme. Assuming the vector and axial-vector currents to transform as $(\underline{8},\underline{1})\pm(\underline{1},\underline{8})$ then leads to the predictions

$$d_1(q^2)=d_1{}^V(q^2),\quad f_1(q^2)=f_1{}^V(q^2), \tag{2}$$

$$d_2(q^2)=f_2(q^2)=d_2{}^V(q^2)=f_2{}^V(q^2)=0. \tag{3}$$

For completeness we note that the $[(\underline{3},\underline{3}^*),(\underline{3}^*,\underline{3})]$ and $[(\underline{6},\underline{3}),(\underline{3},\underline{6})]$ representations would also give Eq. 3; but instead of Eq. (2),

$$d_1(q^2)=f_1{}^V(q^2),\quad f_1(q^2)=d_1{}^V(q^2), \tag{4}$$

and

$$d_1(q^2)=f_1{}^V(q^2)-\tfrac{2}{3}d_1{}^V(q^2),\quad f_1(q^2)=\tfrac{2}{3}f_1{}^V(q^2)+\tfrac{5}{9}d_1{}^V(q^2), \tag{5}$$

respectively.

Equations (2) and (3) are expected to hold only at large spacelike q^2. For their mutual consistency it is necessary that the "induced" form factors of Eq. (3) fall away faster at large q^2 than those of Eq. (2). We now check to see that this is indeed the case if the usual form factor fit is extrapolated. The vector form factors are related to Sachs's form factors as follows:

$$d_1^V(q^2) = -\tfrac{3}{2}(1+\tau)^{-1}(G_E^n + \tau G_M^n),$$

$$d_2^V(q^2) = -\tfrac{3}{2}(1+\tau)^{-1}(G_M^n - G_E^n),$$

$$f_1^V(q^2) = (1+\tau)^{-1}[G_E^p + \tfrac{1}{2}G_E^n + \tau(G_M^p + \tfrac{1}{2}G_M^n)],$$

$$f_2^V(q^2) = (1+\tau)^{-1}(G_M^p + \tfrac{1}{2}G_M^n - G_E^p - \tfrac{1}{2}G_E^n), \tag{6}$$

where $\tau = q_2/4M^2$.

Empirically[5] the following relations appear to hold:

$$G_M^n/\mu_n = G_M^p/\mu_p = G_E^p \equiv G, \tag{7a}$$

$$G_E^n = 0, \tag{7b}$$

where $\mu_p \simeq 2.79$ and $\mu_n \simeq 1.91$. Combining Eqs. (6) and (7) gives the asymptotic behaviors ($q^2 \gg 4M^2$)

$$d_1^V \sim -\tfrac{3}{2}\mu_n G; \quad f_1^V \sim (\mu_p + \tfrac{1}{2}\mu_n)G; \tag{8}$$

$$d_2^V \sim -\tfrac{3}{2}\mu_n G(4M^2/q^2); \quad f_2^V \sim (\mu_p + \tfrac{1}{2}\mu_n - 1)G(4M^2/q^2). \tag{9}$$

Thus we see that Eqs. (2) and (3) are consistent at large q^2 if our extrapolation is allowable. Normally[5] G is taken to be the "double-pole" expression:

$$G = (1 + q^2/M_v^2)^{-2} \tag{10}$$

with $M_v^2 = 0.71$ (BeV)^2. The precise theoretical meaning of Eq. (10) is not clear, but it agrees with the experimental data so far obtained.

In the case of the axial-vector form factors the greater fall-off of the induced term is guaranteed if we use Nambu's partially conserved axial-vector current argument[6] which yields

$$d_2(q^2) = \frac{4M^2}{q^2+\mu^2}d_1(q^2); \quad f_2(q^2) = \frac{4M^2}{q^2+\mu^2}f_1(q^2), \tag{11}$$

where μ is the pseudoscalar meson mass.

It is interesting to observe how Eq. (11) is roughly maintained in the approximation where the axial-vector current is dominated by a pion pole plus an axial-vector meson pole. Here we have for the neutron β-decay axial current

$$\langle p(p')|P_{2\mu}'(0)|n(p)\rangle = \frac{-iM}{(p_0 p_0')^{1/2}}\bar{u}(p')\left[\frac{g_A M_a^2 \gamma_\nu\gamma_5}{q^2+M_a^2}\left(S_{\mu\nu} + \frac{q_\mu q_\nu}{M_a^2}\right) + \frac{iq_\mu\gamma_5}{2M}\frac{g_A 4M^2}{q^2+\mu^2}\right]u(p)$$

$$= \frac{-iM}{(p_0 p_0')^{1/2}}\frac{g_A M_a^2}{q^2+M_a^2}\bar{u}(p')\left[\gamma_\mu\gamma_5 + \frac{iq_\mu\gamma_5}{2M}\left(\frac{M_a^2-\mu^2}{M_a^2}\right)\frac{4M^2}{q^2+\mu^2}\right]u(p), \tag{12}$$

where M_a is the axial-vector mass and $g_A \simeq 1.18$. The presence of the term $q_\mu q_\nu$ in the axial-vector meson propagator enables us to write the expression in essentially Nambu's form.[6] With this reassurance, let us assume by analogy that the following double-pole expression for the axial-vector form factors is valid:

$$\frac{d_1(q^2)}{d_1(0)} = \frac{f_1(q^2)}{f_1(0)} = \left[1 + \frac{q^2}{M_a^2}\right]^{-2}. \qquad (13)$$

With this choice a double pole without extraneous kinematical factors appears also in the Breit frame. In Eq. (13) we have taken the parameter to be the axial-vector meson mass by the analogy that the corresponding parameter for the vector form factors is compatible with an average vector-meson mass. Actually we may even use a completely different high-energy form such as that of Wu and Yang.[7]

Using Eqs. (8) and (13) in the chiral SU(3) ⊗SU(3) relation, Eq. (2) gives the remarkable result

$$\frac{d_1(0)}{f_1(0)} = \frac{-3\mu_n}{2\mu_p + \mu_n} \simeq 1.56 \qquad (14)$$

which is to be compared with the experimental value[8] of 1.7. If we had used Eqs. (2) at small q^2 we would have instead found $d_1(0)/f_1(0) = 0$. This result led Gell-Mann[3] to reject the assignment of the $\frac{1}{2}^+$ baryons to the $[(\underline{8},\underline{1}), (\underline{1},\underline{8})]$ representation at low q^2. We stress that Eq. (14) requires only the assumption that Eqs. (7) hold asymptotically, not the detailed choice of a "double-pole" fit. In the same way Eqs. (4) and (5) would lead to the bad results $d_1(0)/f_1(0) = 0$ and $\frac{2}{3}$ for the cases of the $[(\underline{6},\underline{3}), (\underline{3},\underline{6})]$ and $[(\underline{3},\underline{3}^*), (\underline{3}^*,\underline{3})]$ baryon assignments, respectively.

We note that the significance of the $[(\underline{8},\underline{1}), (\underline{1},\underline{8})]$ representation for the baryons becoming asymptotically suitable may reflect nothing more than the fact that the term

$$-\bar{N}_b^{\ a} \gamma_\mu \partial_\mu N_a^{\ b}$$

($N_a^{\ b}$ = baryon octet field) in the effective strong-interaction Lagrangian becomes dominant in the high-energy region. This term is easily seen to be chiral SU(3)⊗ SU(3) invariant with the octet baryons belonging to $[(\underline{8},\underline{1}), (\underline{1},\underline{8})]$. At more usual energies where other terms contribute substantially, it is not clear that this irreducible representation assignment has any meaning. However, one may hope that at low energies the unit baryon-number states (baryons and pseudoscalar mesons) can be assigned to a reducible representation of SU(3) ⊗ SU(3) in such a way that in the high-energy limit we recover our results.

Now if we become more speculative and assume that the "double-pole" fit actually holds for q^2 well beyond the present experimental region, we find the following amusing relation:

$$g_A \equiv d_1(0) + f_1(0) = (M_v/M_a)^4(\mu_p - \mu_n). \qquad (15)$$

Putting experimental numbers into Eq. (15) leads to

$$(M_a/M_v)^4 \simeq 3.99.$$

We note that this value agrees almost exactly with Weinberg's result[9] $M_a/M_v = \sqrt{2}$, although we have no rigorous reason to associate M_v and M_a with the ρ and A_1 resonances. The use of a different high-energy form such as that of Wu and Yang[7] would lead to a similar relation between the vector and axial-vector charge radii.

To summarize, we point out that the use of asymptotic chiral SU(3)⊗ SU(3) invariance leads (in order of increasing speculation) to the following three consistencies with the above extrapolations of vector and axial-vector form factors: (a) For large q^2 the induced form factors fall off faster than the direct ones. (b) A good value for axial d/f ratio at zero-momentum transfer emerges. (c) A plausible expression for the axial-vector renormalization constant is found.

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DETERMINATION OF THE SCATTERING AMPLITUDES FROM POLARIZATION MEASUREMENTS

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A necessity condition is proved for the set of polarization measurements which have to be performed for a full determination of the scattering amplitudes at fixed angles. The special cases of resonance scattering, Regge poles, and potential scattering are discussed.

Introduction and main result.—Scattering processes are completely described by $N = \sum_\nu (2j_\nu + 1)$ scattering amplitudes $R_{m_\nu} = R_{m_1 \cdots m_n}$ which are functions of the dynamical variables. Each m_ν labels the magnetic quantum number of the intrinsic spin j_ν of the νth particle in some suitable coordinate system and n is the number of particles with spin $j \neq 0$ in the reaction. In some cases the number N of independent amplitudes can be reduced by parity conservation, etc.

By measurements, however, linear combinations of bilinear products $R_{m_\nu} R_{m_\nu'}^*$ are determined. There are N^2 linearly independent measurements at given dynamical variables. These include measurements of the polarization of all incoming and outgoing particles in coincidence, which is in general outside the reach of experimental possibilities.

Because $(R_{m_\nu} R_{m_\nu'}^*)^* = (R_{m_\nu'} R_{m_\nu}^*)$, the N^2 possible measurements give N^2 real parameters from which $2N-1$ real parameters of the amplitudes can be determined while one overall phase is undeterminable. Conversely these $2N-1$ parameters uniquely determine the N^2 measurements.

This leads to the following question: what measurements are necessary and which sets of measurements are sufficient to determine all the scattering amplitudes R_{m_ν} up to a common phase?

In the present paper the following necessity statement will be proved[1]: If no phase analysis is performed, the polarization of each particle (with $j \neq 0$) must be measured in coincidence with at least one other polarization, and this in such a way that it is impossible to divide the particles into two sets with no polarization correlation measured between a particle from one and a particle from the other set, unless the missing information according to this criterion can be obtained from the performed measurements by the use of symmetry operations which interchange the role of the particles in the reaction.[2] A continuous family of possible amplitudes will be found if this condition is not met.[3]

A mathematical exception to this statement will be given at the end of the proof and the physical meaning of the exception will be discussed in the last paragraph. The statement holds with or without parity conservation. The proof, however, will be given only without consideration of parity.[1]

Proof.—To prove the above statement we divide the particles with $j \neq 0$ into two sets S_1 and S_2 and show that even if all possible correlation measurements within the two sets are performed there is still a continuous family of solutions for the scattering amplitudes.

Let α and β denote all possible combinations of the magnetic quantum numbers m_i, for $i \in S_1$ and $i \in S_2$, respectively. Then the set of amplitudes R_{m_ν} can be written in form of a (nonsquare) matrix $R = (R_{\alpha\beta})$ which can be interpreted as a mapping R: $E_{S_2} \to E_{S_1}$ of the product space E_{S_2} of the particles in S_2 into the product space E_{S_1} of the particles in S_1.

Now measurements where all particles of S_2 are unpolarized give bilinear products which