All of the observations above refer to measurements made on the same sample of InSb and for one polarity of the applied electric field. When the polarity was reversed, the only significant change was in the over-all power level of the continuum radiation. Changes as high as 5 dB were noted. Different samples exhibited variations of power level by as much as 8 dB. The separation between spikes from sample to sample varied, however, less than 5%.

Threshold characteristics similar to those shown in Fig. 2 can be deduced[12,13] from linear instability theory based on the model that the observed emission comes from the excitation of longitudinal phonons by electrons drifting along $\vec{B}_0$. We assume that there is both deformation potential and piezoelectric coupling between the electron and phonon systems.[10] The associated electric field excites transverse electromagnetic waves at the surface of the sample. Since the magnitude of the phonon propagation constant $\vec{q}$ is much greater than that of the electromagnetic wave (ω/c), the observed fields must be mainly due to phonons that travel almost perpendicular to $\vec{B}_0$. In the range of experimental parameters E_0, B_0, and ω, the principal mechanism for generation of phonons propagating almost across $\vec{B}_0$ is found to be inverse Landau damping (Landau growth). We point out, however, that the resonant spikes observed at 4.2°K (Fig. 3) are at present unexplained.

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MEASUREMENT OF RECOMBINATION LIFETIMES IN SUPERCONDUCTORS

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It is shown that the experimentally measured quasiparticle recombination lifetime in a superconductor is not the same as the previously calculated theoretical lifetime. A simple expression relating the two is derived.

Over the past few years, several experiments have been carried out to measure the quasiparticle recombination lifetime in a superconductor,[1-3] i.e., the time τ_R required for a quasiparticle at the gap edge to recombine with a thermally excited quasiparticle, thereby forming a Cooper pair and becoming part of the superfluid.[4] In each of these experiments, a double-tunnel-junction structure was arranged so that one junction could be used to inject quasiparticles into a superconducting film and the second junction could be used to detect the resulting increase in the density of quasiparticles in the film (see Fig. 1). In calculating τ_R from the experimental data, it is assumed that the steady-state density of injected quasiparticles ΔN is small compared with N_T, the thermal number present, and that the phonons

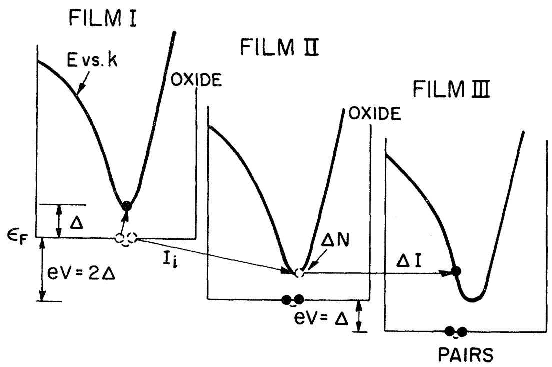

FIG. 1. Quasiparticle E-vs-k diagrams showing the double-junction structure used to determine τ_R experimentally. The extra tunnel current ΔI which flows from film II to III when a tunnel current I_i flows from film I to II is a measure of the increased number of quasiparticles ΔN in II and, in principle, τ_R in film II.

arising from the recombination of the injected quasiparticles can be ignored. Both of these assumptions are also implicit in the several theoretical calculations of τ_R.[5-7] In this Letter, we (1) point out that experimentally, the assumption of small ΔN can easily be invalid depending upon the injection levels and temperatures used, (2) show that even if the small-ΔN assumption is satisfied, the recombination phonons can in general never be ignored, and (3) derive a simple expression valid for both large and small ΔN which takes into account the effects of the phonons and gives the relationship between the lifetimes calculated theoretically and those measured experimentally. The temperature dependence expected for the experimentally measured lifetimes is also discussed.

Injection current densities used in such experiments may typically be on the order of 5×10^{-2} A/cm^2, corresponding to a quasiparticle injection rate of 3.1×10^{17}/sec cm^2. Since a typical middle-film thickness is about 3×10^{-6} cm, the density of injected carriers is $\sim10^{23}$/cm^3 sec. For aluminum, the primary superconductor used in the experiments, the theoretically calculated recombination lifetime at a reduced temperature $t=T/T_c$ of 0.3 is on the order of 10^{-7} sec.[8] As a result, the steady state density of injected quasiparticles is expected to be on the order of 10^{16}/cm^3. The thermal equilibrium number of quasiparticles N_T goes as $t^{1/2}e^{(-1.76/t)}$ at low reduced temperatures and for Al falls below 10^{16}/cm^3 at roughly $t=0.3$. Thus, for small t and/or large injection currents, the steady state density of injected quasiparticles can easily exceed the thermally generated density.

The calculations and experimental determinations of the lifetime of the injected quasiparticles[9] are based essentially on the existence of a differential equation for the excess quasiparticle density of the form[10]

$$d\Delta N/dt=I_0-\Delta N/\tau_R, \tag{1}$$

with the steady state solution

$$\Delta N=I_0\tau_R, \tag{2}$$

where I_0 is the number of quasiparticles injected per cm^3 per sec and τ_R is the recombination lifetime (or equivalently, τ_R^{-1} is the transition probability that a quasiparticle injected at the gap edge will combine with a thermally excited quasiparticle). Equations (1) and (2) would be appropriate if the number of injected quasiparticles were much smaller than the thermal number and if the lifetime of the emitted phonons were zero. As indicated above, the first of these conditions may not be satisfied over a wide range of temperatures and injection currents, while the second is never satisfied.

The differential equations which more correctly describe the experimental situation are

$$dN/dt=I_0+\beta N_\omega-RN^2, \tag{3}$$

and

$$\frac{dN_\omega}{dt}=\frac{RN^2}{2}-\beta\frac{N_\omega}{2}-(N_\omega-N_{\omega T})\tau_\gamma^{-1}, \tag{4}$$

where N is the total number of quasiparticles, R is the recombination coefficient, N_ω is the total number of phonons with energy $\hbar\omega>2\Delta$, β is the transition probability for the breaking of pairs by such phonons, and τ_γ^{-1} is the net transition probability for phonons to be lost out of the energy range $\hbar\omega>2\Delta$ by processes other than pair excitation. The factors of $\frac{1}{2}$ in Eq. (4) arise because one phonon creates two quasiparticles. The term $(N_\omega-N_{\omega T})\tau_\gamma^{-1}$ takes into account the fact that when $N_\omega=N_{\omega T}$, the equilibrium number of phonons, there is no net loss or gain of phonons to the energy range in question. In obtaining Eqs. (3) and

(4), we have neglected the quasiparticle tunneling current which leaves the film since it is orders of magnitude smaller than the injection current. We have also assumed that the number of pairs is much greater than the number of quasiparticles and remains essentially unchanged by the injection process. Near T_c, this assumption will break down. Additionally, we have ignored possible spatial variations in the density of the injected quasiparticles since the junction geometries and film thicknesses used in the experiments of Refs. 2 and 3 seem to preclude any such effects.

The steady state solution is obtained by setting Eqs. (3) and (4) equal to zero. The result is

$$N^2=\frac{\beta N_{\omega T}}{R}+\frac{I_0}{R}\left(1+\frac{\beta}{2}\tau_\gamma\right). \tag{5}$$

From the equilibrium condition with no injection current ($I_0=dN/dt=0$) and the reasonable assumption that R and β are independent of the number of injected quasiparticles, we have, from Eq. (3), $N_T^2=\beta N_{\omega T}/R$. When this is substituted into Eq. (5) with $\Delta N=N-N_T$, we obtain

$$\Delta N=I_0\left[\left\{\left(\frac{N_T}{I_0}\right)^2+2\frac{N_T}{I_0}\left(\frac{1}{2RN_T}+\frac{N_T}{4N_{\omega T}}\tau_\gamma\right)\right\}^{1/2}-\frac{N_T}{I_0}\right]. \tag{6}$$

In the limit of $\tau_\gamma\to\infty$ and $I_0\to 0$, Eq. (6) reduces to Eq. (2) with $\tau_R^{-1}=2RN_T$. Therefore we can write

$$\tau_{\text{exp}}=\left\{\left(\frac{N_T}{I_0}\right)^2+2\frac{N_T}{I_0}\left[\tau_R+\tau_\gamma\frac{N_T}{4N_{\omega T}}\right]\right\}^{1/2}-\frac{N_T}{I_0}, \tag{7}$$

where τ_{exp} is the lifetime that one would calculate from the experimental data assuming $\Delta N=I_0\tau_{\text{exp}}$. Solving Eq. (7) for τ_R in terms of τ_{exp} gives

$$\tau_R=\tau_{\text{exp}}\left(1+\frac{\tau_{\text{exp}}}{2}\frac{I_0}{N_T}\right)-\frac{N_T}{4N_{\omega T}}\tau_\gamma. \tag{8}$$

Equation (8) is the one which should be used to compare theory with experiment.

The fact that the measured lifetime depends critically upon τ_γ persists for all cases. In the limit $\Delta N/N_T\ll 1$ (small injection current) one has simply

$$\tau_{\text{exp}}=\tau_R(1+\tfrac{1}{2}\tau_\gamma\beta)=\tau_R+(N_T/4N_{\omega T})\tau_\gamma, \tag{9}$$

while for the opposite extreme, $\Delta N/N_T\gg 1$ (large injection current), one has

$$\tau_{\text{exp}}=\left[2\frac{N_T}{I_0}\left(\tau_R+\frac{N_T}{4N_{\omega T}}\tau_\gamma\right)\right]^{1/2}. \tag{10}$$

To see the relative magnitudes of τ_R and the phonon contribution, consider Al. At low reduced temperatures the theoretical value[8] for τ_R is $1.25\times10^{-10}\,t^{-1/2}e^{1.76/t}$ while from statistics, $N_T/4N_{\omega T}=1.47\times10t^{-1/2}e^{1.76/t}$. Thus, for $\tau_\gamma=8.5\times10^{-12}$ sec, τ_R and the phonon contribution would be equal. The smallest value of τ_γ is expected to arise from the phonons simply leaving the film. For this decay mode, τ_γ is given by $d/2s\approx5\times10^{-12}$ sec, where d is the film thickness (~300 Å) and s is the velocity of sound (~3×10^5 cm/sec). This value is already comparable with that required for the two contributions to be equal. More realistically, a significant fraction of the phonons will be reflected because of the acoustic mismatch between the film and substrate (or bath); a crude estimate indicates that the effective transmission coefficient is less than one half. It is thus likely that the phonon effects will completely dominate, and the experiment will determine τ_γ rather than τ_R.[11] This dominance will hold over nearly the entire temperature range since τ_R and the phonon contribution to τ_{exp} have the same temperature dependence; the low t expressions for τ_R and $N_T/4N_{\omega T}$ given above show this explicitly, while Eq. (9) shows it more generally, since β depends only weakly on temperature through its dependence on the number of Cooper pairs.[12] However, since τ_γ is expected to depend on the thickness of the center film, measurements on films of different thicknesses could perhaps be used to separate the two contributions.

The temperature dependence observed for τ_{exp} will be markedly different for the two limiting cases [Eqs. (9) and (10)]. From Eq. (9) one finds a strong dependence, $\tau_{\text{exp}}\sim\tau_R\sim t^{-1/2}e^{1.76/t}$, while from Eq. (10), one finds that τ_{exp} is independent of t since $N_T\sim t^{1/2}e^{-1.76/t}$. It is also evident from Eq. (10) that τ_{exp} will depend on the magnitude of the injection current, varying as $I_0^{-1/2}$. (A decrease in τ_{exp} as I_0 was increased has been noted in

Ref. 3.)

In closing, we note that the important role played by the recombination phonons stems from the fact that they are generated at a great rate, i.e., of order I_0, and that β is relatively large and comparable with τ_γ^{-1}. As a result, the phonons produced by the recombination of the injected quasiparticles will themselves create quasiparticles at a rate comparable with I_0. On the other hand, it may be possible to minimize the effects of the phonons by carrying out a pulse experiment. For times small compared with τ_R, the number of recombination phonons will be negligible and thus the initial decay rate of a pulse of injected quasiparticles will be determined solely by τ_R.

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[12]It is unlikely that any temperature dependence in τ_γ would be sufficiently strong to invalidate these conclusions.

EFFECT OF BOUND STATES ON THE EXCITATION SPECTRUM OF A HEISENBERG FERROMAGNET AT LOW TEMPERATURE*

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A compact expression for the energy shift and inverse lifetime (energy width) of spin waves in a Heisenberg ferromagnet at low temperatures is given. The two-particle bound states are observable via the resonance they cause in the self-energy of spin waves.

The purpose of this brief note is to report calculations of the transverse component of the generalized wave-vector and frequency-dependent susceptibility $\chi_{+-}(k,\omega)$ for the Heisenberg ferromagnet at low temperatures. These calculations show that at short wavelengths the two-particle bound states[1] influence the single-particle spin-wave excitations in a dominant and nonperturbative way.

We use the Dyson-Maleev transformation[2]

$$S_+ = (2S)^{1/2}(1-a^\dagger a/2S)a; \quad S_- = (2S)^{1/2}a^\dagger;$$

$$S_z = S - a^\dagger a \tag{1}$$

to write the Heisenberg Hamiltonian in terms of boson operators as

$$H = E_0 + \sum_k \epsilon_k a_k^\dagger a_k + \frac{1}{2N}\sum_{k\lambda\lambda'} V_k(\lambda,\lambda') \times a^\dagger_{\frac12 k+\lambda} a^\dagger_{\frac12 k-\lambda} a_{\frac12 k+\lambda'} a_{\frac12 k-\lambda'}, \tag{2}$$

where

$$\epsilon_k = J_z S(1-\gamma_k), \tag{3a}$$

$$V_k(\lambda,\lambda') = -\tfrac12 J_z[\gamma_{\lambda-\lambda'} + \gamma_{\lambda+\lambda'} - \gamma_{\frac12 k+\lambda} - \gamma_{\frac12 k-\lambda}] \tag{3b}$$

in the usual notation.[3] The susceptibility is