is necessary to pursue these points.

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NONUNIQUENESS OF MASS-FORMULAS FROM INFINITE-DIMENSIONAL GROUPS AND ALGEBRAS

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The possibility of getting any mass spectrum one desires in the framework of finitely-generated infinite-dimensional associative algebras and unitary groups is discussed. Inevitable consequences for the mass problem are then suggested.

Recently, some authors have tried once more to resolve some difficulties of interpretation concerning the fundamental problem of hadron mass splittings.[1] The authors of this note feel, however, that such an interpretation should not be achieved at any price and no matter how. The aim of this short Letter is to point out that once a suitable algebraic structure is supposed (the only goal and "raison d'être" of which is the justification of the observed mass splittings), any mass formula can be obtained. The connection of this with some no-go theorems is then discussed.

(1) Let H be a separable Hilbert space, and U the group of all unitary operators acting on H (U is an infinite-dimensional Lie group of the Banach type). Denote by $\mathcal{P}$ the universal covering of the connected Poincaré group; if V is any continuous unitary representation of $\mathcal{P}$ on H, we have evidently $V(\mathcal{P}) \subset U$. For simplicity, we shall restrict ourselves to the baryon octet $\frac{1}{2}^+$ (though the suggested construction is easily generalizable to all hadrons and all internal groups, and can easily include discrete symmetries). We thus choose a particular representation

$$V = \bigoplus_{i=1}^{8} V(\tfrac{1}{2}, m_i),$$

the direct sum of eight irreducible unitary continuous representations of $\mathcal{P}$ characterized by $J=\frac{1}{2}$, m_i being the set of the eight $\frac{1}{2}^+$ masses. We identify it with the unitary representation of a "physical Poincaré group" in U. This representation acts on the direct sum of eight (trivially isomorphic) Hilbert spaces $H = \oplus_i H_i$. Let φ_{in} ($n = 1, 2, 3, \cdots$) be a complete orthonormal basis of H_i ($i = 1, \cdots, 8$). For every fixed n, we can define on the vector space generated by the φ_{in} ($i = 1, \cdots, 8$) a representation Ad_n of SU(3) isomorphic to the adjoint representation. We thus get a subgroup

$$\bigoplus_{n=1} Ad_n\,(\mathrm{SU}(3))$$

of U, which is the image of SU(3) under a (very reducible) unitary representation on H.

We have therefore suceeded in formulating the octet $\frac{1}{2}^+$ classification and mass splitting (for example!) by mixing unitary reducible representations of SU(3) and $\mathcal{P}$, both contained in our infinite-dimensional Banach-Lie group U. Furthermore, it is of interest to notice that any mass formula can be obtained in the framework of associative algebras.

Indeed, consider the tensor algebra $T(P,S)$ over the vector space $P+S$, where P (S) the Poincaré (internal) Lie algebra. Divide it by the ideal generated (for instance) by the elements $pp'-p'p-[p,p']_P$, $ss'-s's-[s,s']_S$

(for all $p, p' \in P$; $s, s' \in S$), and $p_\mu p^\mu - f$, $w_\mu w^\mu - g p_\mu p^\mu$, where f is a polynomial with internal quantum numbers and the invariants Inv(S) of S, and g, a polynomial with Inv(S). [Even more general relations can be considered in the quotient field, or some closure, of $T(P, S)$]. In the quotient finitely generated infinite-dimensional associative algebra A, we thus get, at least formally since one has still to look for suitable representations of A (the existence of which is not trivial) in order to realize this, any mass formula and even spin assignments.

(2) From this, the following consequences are suggested:

(a) The case of "infinite algebraic structures" is much different from the case of finite-dimensional Lie groups or algebras. Any a priori wanted mass formula, spin relations with internal symmetry, etc. (including wanted predictions) can be justified. Therefore obtained relations of this type can by no means justify a posteriori such a use of these structures. Before using them, one should have a priori more convincing physical reasons (or at least ideas) to do so.

(b) In the framework of finite-dimensional Lie algebras containing P, the nonpossibility of having a discrete mass spectrum was conjectured.[2] The conjecture was criticized and a counter-example given.[3] This clarification was useful since it lead to the weaker but interesting no-go result[4] that several isolated mass eigenvalues cannot be obtained with continuous unitary irreducible representations of connected finite-dimensional Lie groups containing $\mathcal{P}$. But in the finite dimensional case the following objects can be considered, all of which are of interest: Lie algebras (and their representations), nonconnected Lie groups, more general representations, and nonisolated eigenvalues or more generally densities in continuous spectrum. The power of this case is that it is more limited with possibilities than "infinite structures," that in some cases more convincing physical reasons for its choice can be given, and that in general the structure of finite-dimensional Lie algebras (or groups) is suggested by known physical motivations. This kind of treatment may be quite difficult, physically and mathematically, but this is not a sufficient reason to disregard it. Of course, if one is ready to change[5] the traditional definition of mass of the relativistic particle, many other speculations are still possible in the framework of finite-dimensional Lie groups.

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SEARCH FOR $S = +1$ BARYON STATES IN PHOTOPRODUCTION*

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The photoproduction of negative K mesons on hydrogen has been studied in a search for positive-strangeness baryon states. A missing-mass spectrometer was used in which the production angle and momentum of the K^- meson were measured. The experiment was performed at the Cambridge Electron Accelerator using the bremsstrahlung beam from a 0.1-radiation-length tungsten target. Extensive data on $K^+ + Y^*$ two-body final states have also been taken, and new results will be reported separately.[1]

Some reactions which yield a negative K^- meson are as follows:

$$\gamma + p \to K^+ + K^- + p, \qquad (1)$$

$$\gamma + p \to p + \varphi \to p + K^+ + K^-, \qquad (2)$$

$$\gamma + p \to K^+ + Y^* \to K^+ + K^- + p, \qquad (3a)$$

$$\gamma + p \to K^{+*} + Y^* \to K^{+*} + K^- + p, \qquad (3b)$$

$$\gamma + p \to Z^{++} + K^{-*} \to Z^{++} + K^- + \pi^0, \qquad (4a)$$

$$\gamma + p \to Z^+ + K^{0*} \to Z^+ + K^- + \pi^+, \qquad (4b)$$

$$\gamma + p \to K^- + Z^{++}. \qquad (4c)$$

The final states may also include one or more