TOTAL MUON-CAPTURE RATE AND THE CONSISTENCY OF MIGDAL'S THEORY*

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Migdal's theory of nuclear structure is recast in the shell-model language, enabling us to show that the excellent agreement with experiment of the theory applied to the total muon-capture rate is obtained by an appreciable symmetry breaking of the Wigner's SU(4) supermultiplet structure contrary to other models where the symmetry seems to be well preserved. Thus, the μ-capture result is inconsistent with the dipole photoabsorption and inelastic electron scattering processes reflecting the possible breakdown of the quasiparticle hypothesis.

Recent successful calculation of the total μ-capture rates in complex nuclei by Bunatyan[1] using Migdal's theory[2] has an important implication in that none of current nuclear models has succeeded in reducing the theoretical capture rates to the experimental values. Because of the close analogy between the muon capture, the dipole photoabsorption, and inelastic electron scattering, success in one might naturally lead to better understanding in others. The trouble with Bunatyan's calculation is, however, that it is not clear how and why Migdal's theory works. The situation is quite different from the case of partial-capture rates involving low-lying states,[3] where the transition matrix elements can be directly related to static magnetic and quadrupole moments and β-decay transition rates, etc.

We demonstrate in this Letter a mechanism by which Migdal's approach can obtain the fit. For this we consider for simplicity the reaction $\mu^- + O^{16}(0^+;\text{ground state}) \to \nu + N^{16}$ (all final states). If we take the standard Fujii-Primakoff Hamiltonian[4] assuming the validity of the universal Fermi interaction (UFI) hypothesis,[5] then the capture rate for the process $0^+ \to$ all final states in terms of the coupling constants $G_{V,A,P}$ is proportional to

$$G_V^2\mathfrak{M}_V + 3G_A^2\mathfrak{M}_A + (G_P^2 - 2G_PG_A)\mathfrak{M}_P, \tag{1}$$

where

$$\mathfrak{M}_{V,A,P} \equiv \sum_f \left(\frac{\nu_f}{m_\mu}\right)^2 |\langle\tilde{f}|t_{V,A,P}|0^+\rangle|^2$$
$$= \sum_f \left(\frac{\nu_f}{m_\mu}\right)^2 \int \frac{d\hat{\nu}}{4\pi} |\langle\tilde{f}| \sum_{\tau=1}^{A} Q_{V,A,P}(i)\exp[-i\vec{\nu}_f\cdot\vec{r}(i)]|0^+\rangle|^2, \tag{2}$$

where $Q_V = \tau_-$, $Q_A = (1/\sqrt{3})\tau_-\vec{\sigma}$, $Q_P = \tau_-\hat{\nu}\cdot\vec{\sigma}$, ν_f the neutrino energy, m_μ the muon mass, and $|\tilde{f}\rangle$ an exact final state. We neglect in our discussion recoil corrections which amount to about 10%. Migdal's theory provides a means of calculating Eq. (2) in terms of a renormalized single-particle operator t^R (the renormalization is caused by the presence of multi-quasiparticle configurations), a vertex function $\mathcal{T}(\omega)$, and the pole part of the Green's function A:

$$|\langle\tilde{f}|t|0^+\rangle|^2 = \mathrm{Res}\{\textstyle\sum_n \langle 0|t^R|n\rangle A_n \langle n|\mathcal{T}(\omega)|0\rangle\}_{\omega=\omega_f}, \tag{3}$$

where ω_f is the transition energy. In matrix notation $\mathcal{T}$ satisfies the equation $\mathcal{T} = t^R + \Gamma A \mathcal{T}$, where Γ is the interaction amplitude to be defined below.

The crucial point of our discussion is the theorem[3] relating Migdal's method to the more familiar shell-model (SM) approach. If the quasiparticle hypothesis[6] holds for all excitations and if the discussion is limited to closed shell nuclei, the following rules[3] give Migdal's result:

(A) Diagonalize in one-particle, one-hole (1p-1h) space the nuclear Hamiltonian with the matrix element of the residual interaction given by the direct term of Γ, where Γ is the Fourier transform of Γ_p:

$$\Gamma_p = V_0(\vec{\tau}_1\cdot\vec{\tau}_2)\textstyle\sum_\kappa (f_\kappa' + g_\kappa'\vec{\sigma}_1\cdot\vec{\sigma}_2)P_\kappa(\vec{p}_1\cdot\vec{p}_2/p_F^2), \tag{4}$$

where P stands for Legendre polynomial. This prescription gives the wave function $|f\rangle$ and eigenvalue ω_f. It is easy to show that $\mathcal{T}(\omega)$ in Eq. (3) has a pole at $\omega=\omega_f$.

(B) Evaluate the matrix element of the operator t^R between $|f\rangle$ and $|0\rangle$. Invoking the quasiparticle hypothesis once more leads to the simple result[2]

$$t^R = e(t)t, \tag{5}$$

where $e(t)$ is the state-independent "effective charge" of an operator t. The "effective charges" in Eq. (2) are $e(\tau A)$, $e(\tau_-\vec{\sigma})$, and $e(\tau_-\vec{\sigma}\cdot\hat{\nu})$. It follows from these prescriptions that any difference between Migdal's and other SM results should be found in Eqs. (4) and (5).

Table I contains the results of such calculations using the following values of Migdal's constants[7] (with $V_0/4\pi = 35$ MeV F^3):

$$f_0' = 0.35, \quad g_0' = 0.50, \quad \zeta = 0.05. \tag{6}$$

Here ζ measures the renormalization of the spin operator, $e(\tau_-\vec{\sigma}) = e(\tau_3\vec{\sigma}) = 1-2\zeta$. Equation (5) and the nonrenormalizability of scalar vertices[2] imply that $e(\tau_-)=1$. The values given in (6) correspond to those used by Bunatyan,[1] and describe correctly most of the properties of low-lying states[8] as was discussed in Ref. 3. Unlike Bunatyan, however, we have computed only the first forbidden (dipole) transition rates, leaving out the other multipoles 0^+, 2^+, 3^-, 4^+, 5^-, etc. The sum over the first forbidden transitions yields

$$\Lambda_D = 8.96\times10^4 \text{ sec}^{-1}. \tag{7}$$

In order to compare with experiment, one should add the contributions from other multipoles (denoted by Λ_{OM}). Various models[9] predict Λ_{OM} to lie between 10 and 20 % of the total capture rate.[10] If we take this range, we obtain for $\Lambda_T \equiv \Lambda_D + \Lambda_{OM}$

$$9.95\times10^4 \text{ sec}^{-1} \lesssim \Lambda_T \lesssim 11.2\times10^4 \text{ sec}^{-1}. \tag{8}$$

This is essentially Bunatyan's result,[11] to be compared with the corresponding pure shell model (i.e., $V_0=\zeta=0$) value 17×10^4 sec^{-1}.

Table I. Capture rates[a] for $\mu^- + O^{16}(0^+, \text{g.s.}) \to \nu_\mu + N^{16}(T=1,\ 0^-, 1^-, 2^-)$ in 10^4 sec^{-1}.

J^π	ω_f [b]	Λ_f
0^-	12.15	0.038
	26.35	0.258
	Sum	0.296
1^-	11.88	0.282
	15.44	0.304
	17.82	0.208
	19.52	1.964
	24.92	2.027
	Sum	4.785
2^-	10.85	0.666
	16.00	0.138
	17.08	0.005
	19.62	1.918
	22.23	1.156
	Sum	3.883
3^-	11.22	0.014
	16.24	0.007
	23.47	0.009
	Sum	0.030

[a]Calculated with $V_0 = 4\pi\times35$ MeV F^3, $f_0'=0.35$, $g_0'=0.50$ and the pseudoscalar to axial-vector weak coupling constant ratio $C_P=8$. Radial wave function is taken to be harmonic oscillator with length parameter $b=(\hbar/m\omega)^{1/2}=1.75$ F.

[b]Energy in MeV relative to the ground state of O^{16}.

Although recoil corrections would raise Λ_T somewhat, it is in essential agreement with the experimental value $\Lambda_{\text{expt}} = (9.8\pm0.3)\times10^4$ sec^{-1}.

In order to understand how the result (8) comes out, it is necessary to examine what modification to the supermultiplet relation

$$\mathfrak{M}_A = \mathfrak{M}_V = \mathfrak{M}_P \tag{9}$$

is implied by the rules (A) and (B). Equation (9) holds exactly in the absence of spin-dependent forces[12] and was assumed in all other calculations.[9] We now assert that the success of Migdal's theory in getting the correct results is based on a large deviation from these equalities. According to (B),

$$\mathfrak{M}_A/\mathfrak{M}_V = (1-2\zeta)^2\left\{\sum_f\left(\frac{\nu_f}{m_\mu}\right)^2|\langle f|t_A|0\rangle|^2\Big/\sum_f\left(\frac{\nu_f}{m_\mu}\right)^2|\langle f|t_V|0\rangle|^2\right\}. \tag{10}$$

The ratio in the curly bracket is usually about 1.13 if one were to take Γ to be the shell-model effective force.[12] But Migdal's amplitudes [Eq. (6)] contain much stronger spin dependence than the ordinary SM forces. As a comparison, using the relationship between the SM constants $a_{0,\sigma,\tau,\sigma\tau}$ and the Migdal constants f_0', g_0', etc., i.e., $f_0' = -\frac{1}{4}(-a_0 - 3a_\sigma + 5a_\tau + 3a_{\sigma\tau})$ and $g_0' = -\frac{1}{4}(-a_0 - a_\sigma + a_\tau + 3a_{\sigma\tau})$, one finds that the Soper mixture[13] is equivalent to $f_0' = 0.49$ and $g_0' = 0.28$ for the given strength constant V_0. It is known that the effect of increasing g_0' is to rob the axial-vector strength of the low-lying states of $J = 2^-$ leading to a large suppression of $\mathfrak{M}_A$. This is exhibited in Table II, where the percentage deviation from the supermultiplet symmetry $\mathfrak{M}_A/\mathfrak{M}_V = 1$ is given as g_0' varies for a fixed f_0'. For the constants of (6), the ratio is

$$\mathfrak{M}_A/\mathfrak{M}_V \lesssim 0.76, \qquad (11)$$

which is an appreciable deviation from the usual particle-hole model value of 1.13. The inequality is put in cognizance of the possibility that $\xi > 0.05$.[14] As for the relationship between $\mathfrak{M}_A$ and $\mathfrak{M}_P$, we find that the equality holds to within 1%, because both $\mathfrak{M}_A$ and $\mathfrak{M}_P$ contain the operator σ and are influenced by the same spin-dependent amplitude g_0'. Assuming the UFI together with one-pion-pole dominance for the weak coupling constants,[6] one gets from the above considerations

$$\Lambda_D^{\ M}/\Lambda_D^{\ P} \lesssim 0.61,$$

where the superscripts M and P stand for the Migdal and the pure shell-model values. This is just the amount of reduction needed in O^{16} to obtain the result given by (7). The situation is similar in Ca^{40}.

If the suppression of the axial-vector term relative to the vector term is the only mechanism, as our theorem implies, it then follows that the wave functions which yield the capture rates for the 1^- states in Table I will exhaust the dipole photoabsorption sum rule below 30 MeV contrary to the experimental observation, where only a part of the classical sum-rule strength is found.[11] This is a well-known difficulty with all 1p-1h shell-model descriptions. In Migdal's theory it appears as an evidence for the breakdown of the quasiparticle hypothesis either in deriving Eq. (3) or in writing down an energy-independent form Eq. (5) or both.

Table II. Supermultiplet symmetry breaking $K \equiv 100 \times (1 - \mathfrak{M}_A/\mathfrak{M}_V)$ as a function of g_0' for $f_0' = 0.35$, $e(\tau_-\vec{\sigma}) = 0.90$.

g_0'	K (%)
0	5-14[a]
0.30	17
0.50	24
0.70	31
0.90	38

[a]Calculated with $e(\tau_-\vec{\sigma}) = 1$.

It is probably not feasible to do any practical calculation without invoking the hypothesis to obtain Eq. (3). Even if we take Eq. (3), the energy independence of t^R is not likely to hold if one is concerned with higher excitations such as the giant resonance region. This is because the separability of the Green's function is highly doubtful at such excitation energy. In order to incorporate into Migdal's method the fact that a large fraction of strength is located above the giant resonance region due to the ground-state correlation and the coupling to more complex configurations than the 1p-1h ones,[15] it seems necessary (for such a region) to have $e(\tau_-) = e(\tau_3)$ effectively reduced from unity. The same applies to the renormalization of vector operators. As we have learned in Ref. 3, the constants given in (6) are consistent with the experimental values of the transverse form factors $F^2(q)$ for the process $e^- + O^{16}(0^+; T=0) \rightarrow e^- + O^{16}(2^-; T=1; E^* = 13\text{ MeV})$. But for the transition to the giant magnetic quadrupole resonance[16] state ($T = 1$, $J^\pi = 2^-$, $E^* = 20.2$ MeV), the theoretical $F^2(q)$ for increasing momentum transfer q overestimates the experiment by more than 40%. Since the large q selects out the σ-dependent term, this implies that the 20.2-MeV state also requires $e(\tau_-\vec{\sigma}) = e(\tau_3\vec{\sigma})$ effectively reduced from the quasiparticle value $(1-2\xi)$. Thus, if Migdal's approach is to be consistent with these experiments while keeping the simple form of Eq. (3), it seems necessary to take into account the energy dependence of the operator t^R. Details will be reported elsewhere.

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[6]This hypothesis corresponds to the separability of a pole part from a nonpole part of the Green's function $G_{\lambda\lambda'}(\omega)$ as $G_{\lambda\lambda'}(\omega)=a_\lambda\delta_{\lambda\lambda'}/(\omega-\epsilon_\lambda+i\delta\ \mathrm{sgn}\epsilon_\lambda)+G_{\lambda\lambda'}{}^{NP}(\omega)$, where a_λ is the Green's-function renormalization constant which we can set to unity without loss of generality and $G_{\lambda\lambda'}{}^{NP}(\omega)$ is a slowly varying function in ω.

[7]The possible density dependence of Γ is ignored here. For $T=1$ states, the external (vacuum) and internal (nuclear matter) values of f_0' coincide approximately, and consequently the density-dependent factor vanishes approximately. There is yet no evidence that g_0' needs to be density dependent.

[8]All the partial μ-capture transition rates $O^{16}(0^+)\rightarrow N^{16}$ (J^π; bound states) come out correctly. For example, the μ-capture rate $\Lambda(0^+\rightarrow 2^-)=6.4\times10^3\ \mathrm{sec}^{-1}$ in agreement with experiment $(6.3\pm0.7)\times10^3\ \mathrm{sec}^{-1}$, the β-decay matrix element $4\pi|\langle 0^+\|\sum_i\tau_+(i)r(i)[Y_1\times\sigma]_2\|2^-\rangle|^2 = 18.0\ \mathrm{F}^2$ to be compared with experiment $(20.0\pm4.6)\ \mathrm{F}^2$.

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[10]Foldy and Walecka (Ref. 9) assume that Λ_{OM} does not undergo reduction in the presence of residual interaction, and take the pure shell-model value. This does not seem to be justified. An estimate with the 3^- states shows that the reduction can be 40% or more.

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