NONPARABOLICITY OF THE L_1 CONDUCTION BAND IN GERMANIUM FROM MAGNETOPIEZOTRANSMISSION EXPERIMENTS

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Sharp oscillations have been observed in the magnetopiezotransmission spectrum of the indirect transition in germanium at $T \sim 20°K$. From the energy spacing of the observed transitions, we have deduced the nonparabolicity of the L_1 conduction band.

One of the most important contributions of the stress modulation technique[1,2] is the capability of probing the energy bands by magneto-optical experiments to higher energies deep into the Brillouin zone than heretofore.[3-5] We wish to report the observation of the oscillatory magnetopiezotransmission in germanium at low temperatures up to photon energies at which the direct transition dominates. The quantitative analysis of the results shows large nonparabolic behavior of the electron mass for the L_1 conduction band. We believe this is the first experimental determination of the variation in the transverse effective mass as a function of the electron energy relative to the bottom of the conduction band.

The experimental apparatus employed for magnetopiezotransmission experiments is very similar to that used for magnetopiezoreflection previously.[6] The optical system was modified to a transmission configuration in which the same Dewar used for reflection was employed. This was achieved by the inclusion of two diagonal mirrors on either side of the sample allowing a Voigt configuration of the crystal in respect to the incident light and the magnetic field.

The inset in Fig. 1 shows the sample-transducer assembly as used in the present experiments. The sample is attached to the transducer with vacuum grease applied at the ends only. There is a rectangular hole cut in the transducer under the sample for the light to be transmitted through it. The above sample-transducer package is then mounted on the cold finger of the optical Dewar. With liquid helium as the coolant, the sample temperature was estimated to be ~20°K. When the grease freezes during cooling from room temperature to low temperatures, it provides a rigid bond between the sample and the transducer. A periodic uniaxial stress was impressed upon the sample by driving the transducer sinusoidally at 500 cps. A peak-to-peak strain $\sim 7 \times 10^{-5}$ was used in the present experiments.

Figure 1 illustrates the piezotransmittance $\Delta T/T$ as a function of photon energy. The general increase in the amplitude of oscillations with photon energy is attributed to the thickness of the sample (~150 μ), which is better matched to the larger absorption at higher energies. The remarkable feature of the magnetopiezotransmission spectrum is the sharp lines as contrasted to the less distinct step pattern obtained from ordinary magnetoabsorption experiments for the indirect transition.[7,8] There are two sets of oscillations observed with the magnetic field $\vec{H}$ along a [110] direction, one identified as the light-electron transitions indicated by the letter "L" with appropriate subscript denoting the Landau quantum number n; the second, observed only at lower energies, is the heavy-electron transitions indicated by the letter "H." These apparently are all from the same level at the top of the valence band, since the selection rules Δn are

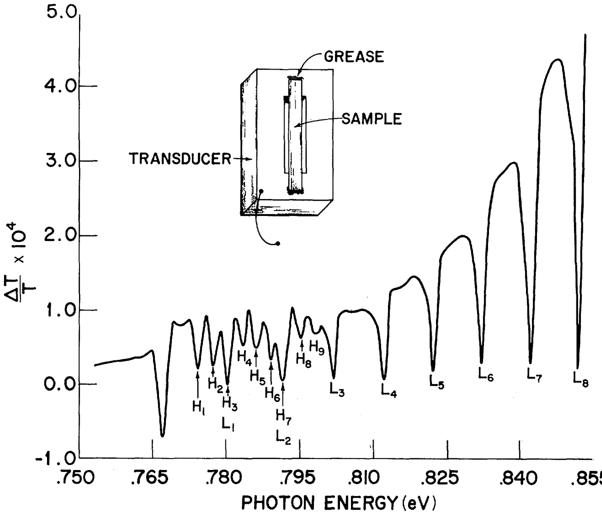

FIG. 1. Magnetopiezotransmission spectrum of the incirect transition in germanium at $T \sim 20°K$ for a magnetic field $H = 88.9$ kG along a [110] direction. The direction of light propagation $\vec{q}$ is parallel to [001] with $\vec{E} \parallel \vec{H} \parallel$ stress.

not conserved for the indirect transition. The first sharp line which persists down to zero field is the transition to the lowest exciton state which is strongly bound by the Coulomb potential to the $n=0$ Landau level in the high-field limit.[9] The higher levels are less effected by the Coulomb interaction since their wave functions are less concentrated at the center of the orbit. As the higher levels are less perturbed by the Coulomb field, we have used these to calculate the effective mass value as a function of energy. When the corresponding effective mass is extrapolated to the bottom of the conduction band, we obtain $m_1^* = (0.0963 \pm 0.001)m_0$ for the light-electron mass. For the heavy electron we deduce an average effective mass $m_2^* = (0.344 \pm 0.008)m_0$. Using the expressions

$$m_1^* = \sqrt{3}m_t(1+2m_t/m_l)^{-1/2},$$

$$m_2^* = (m_t m_l)^{1/2},$$

where m_t and m_l are the transverse and longitudinal effective masses, respectively, we find $m_t = (0.0796 \pm 0.001)m_0$, and $m_l = (1.49 \pm 0.06)m_0$ for the bottom of the conduction band. These are somewhat lower than the cyclotron-resonance values at liquid-helium temperature.[10,11] A comparison of the results is summarized in Table I.

The most interesting feature of our results is the variation of the effective mass with energy as obtained from the spacing of the light-electron levels. This is shown in Fig. 2 as a function of $\Delta\epsilon/\epsilon_i$, where $\Delta\epsilon$ is the electron energy above the bottom of the conduction band and ϵ_i is the photon energy for the zero-field indirect transition. We ascribe this variation as due to the nonparabolic character of the energy bands for increased momenta into the Brillouin zone. From the $\vec{k}\cdot\vec{p}$ theory, it can be shown that the electron energy relative to

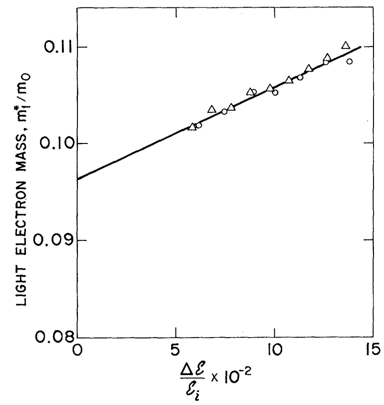

FIG. 2. Plot of the light electron mass m^* as a function of $\Delta\epsilon/\epsilon_i$, where $\Delta\epsilon$ is the electron energy above the bottom of the conduction band and ϵ_i is the photon energy for the zero-field indirect transition. ○, data for $H=88.9$ kG; △, data for $H=68.0$ kG; –, least-squares fit.

the bottom of the conduction band is given by[12]

$$\Delta\epsilon = \frac{p_\perp^2}{2m_0} + \frac{p_\perp^2}{2}\left(\frac{1}{m_t(0)} - \frac{1}{m_0}\right)\frac{\epsilon_g(L_1, L_3')}{\epsilon_g(L_1, L_3')+\Delta\epsilon} + \frac{p_\parallel^2}{2m_l}, \quad (1)$$

where $p_\parallel$ and $p_\perp$ denote the momenta parallel and perpendicular to the major axis of the L_1 ellipsoid; $\epsilon_g(L_1, L_3')$ is the energy separation between the L_1 and L_3' conduction-band minima; $m_t(0)$ is the transverse effective mass at the bottom of the conduction band.

In order to derive a simple expression for the effective mass as a function of energy, we may neglect all the small terms in the above equation for $\Delta\epsilon$. This yields the following approximate expression for the energy:

$$\Delta\epsilon = \frac{p_\perp^2}{2m_t(0)}\frac{\epsilon_g}{\epsilon_g+\Delta\epsilon}. \quad (2)$$

Table I. Effective masses for the L_1 conduction band in germanium.

Effective mass	Cyclotron resonance	Magnetoabsorption[c]	Present work
m_t/m_0	0.0819 ± 0.0003[a] 0.082 ± 0.001[b]	0.079	0.080 ± 0.001
m_l/m_0	1.64 ± 0.03[a] 1.58 ± 0.04[b]	1.74	1.49 ± 0.06

[a]Ref. 11. [b]Ref. 12. [c]Ref. 8.

From Eq. (2), one can deduce the eigenvalues for the Landau levels.

Case I.– For magnetic field $\vec{H}$ along the major axis of the ellipsoid, the energy of the nth Landau level is given by

$$\Delta\epsilon = -\tfrac{1}{2}\epsilon_g + \tfrac{1}{2}[\epsilon_g^2 + 4(n+\tfrac{1}{2})\hbar\omega_c(0)\epsilon_g]^{1/2} \qquad (3)$$

when the electron spin is neglected; $\omega_c(0)$ is the cyclotron frequency equal to $eH/m_\perp(0)c$, corresponding to the bottom of the band.

From the energy spacing between the consecutive Landau levels, we obtain the cyclotron resonance energy:

$$\hbar\omega_c = \hbar\omega_c(0)(1+2\langle\Delta\epsilon\rangle_{\mathrm{Av}}/\epsilon_g)^{-1}, \qquad (4)$$

where $\langle\Delta\epsilon\rangle_{\mathrm{Av}}$ is the mean energy of the Landau levels n and $n-1$.

Thus the cyclotron effective mass should increase with energy as

$$m_\perp = m_\perp(0)(1+2\Delta\epsilon/\epsilon_g). \qquad (5)$$

Case II.– For the magnetic field $\vec{H}\parallel[110]$, the four $\{111\}$ conduction-band minima will be split into two groups. The group consisting of two valleys with their major axes normal to $\vec{H}$ exhibits smaller Landau-level splittings which are responsible for the set of heavy electron transitions as observed. For the other group where the valleys have their major axes in a plane containing $\vec{H}$, we can show as in Case I that the effective mass follows the relation

$$m_1^* = m_1^*(0)(1+2\Delta\epsilon/\epsilon_g), \qquad (6)$$

where θ is the angle between the direction of the magnetic field and the major axis of a given valley; $m_1^*(0) = m_\perp(0)/\cos\theta$.

Rewriting the above expression so as to correspond to the abscissa of Fig. 2, we obtain

$$m_1^* = m_1^*(0)\left(1+2\frac{\epsilon_i}{\epsilon_g}\frac{\Delta\epsilon}{\epsilon_i}\right). \qquad (7)$$

When this is reduced to the appropriate numerical value using $\epsilon_g = 2.2$ eV, $\epsilon_i = 0.77$ eV, we have

$$m_1^* = m_1^*(0)(1+0.7\Delta\epsilon/\epsilon_i) \quad \text{(theory)}.$$

The experiment, however, shows that the mass variation is actually larger than that predicted by theory, since we obtain from Fig. 2

$$m_1^* = m_1^*(0)(1+1.0\Delta\epsilon/\epsilon_i) \quad \text{(experiment)}.$$

Although there remains a discrepancy between experiment and the results of the $\vec{k}\cdot\vec{p}$ theory, we have been able to account largely for the observed nonparabolicity of the L_1 conduction band.

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