PLASMA WAVE ECHO*

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It is shown that if a longitudinal wave is excited in a collision-free plasma and Landau-damps away, and a second wave is excited and also damps away, then a third wave (i.e., the echo) will spontaneously appear in the plasma.

It has long been recognized that electron plasma waves can be damped, even in the absence of collisions.[1] Collisionless damping (Landau damping) has been the subject of extensive theoretical treatments in recent years and is now believed to play an important role in many related, but more complicated, oscillation and instability phenomena. Only recently has Landau damping been demonstrated experimentally.[2] Landau's treatment shows that macroscopic quantities such as the electric field and the charge density are damped exponentially, but the perturbations in the electron phase-space distribution $f(x,v,t)$ oscillate indefinitely. Since the electron density is given by $n_e = \int f(x,v,t)dv$, one may think of the damping as arising out of the phase mixing of various parts of the distribution function. In this paper, we will show how the direction of the phase evolution of the perturbed distribution function can be reversed by the application of a second electric field. This results in the subsequent reappearance of a macroscopic field (i.e., the echo), many Landau-damping periods after the application of the second pulse. The plasma echo is related to other known echo phenomena[3] in that the decay of a macroscopic physical quantity of the system, caused by phase mixing of rapidly oscillating microscopic elements in the system, is reversed by reversing the direction of phase evolution of the microscopic elements.

The basic mechanism behind the plasma echo can easily be understood. When an electric field of spatial dependence e^{-ik_1x} is excited in a plasma and then Landau-damps away, it modulates the distribution function leaving a perturbation of the form[1] $f_1(v)\exp[-ik_1x+ik_1vt]$. For large t, there is no electric field associated with this perturbation, since a velocity integral over it will phase mix to 0. If after a time τ a wave of spatial dependence e^{ik_2x} is excited and then damps away, it will modulate the unperturbed part of the distribution leaving a first-order term of the form $f_2(v)\exp[ik_2x-ik_2v(t-\tau)]$, but it will also modulate the perturbation due to the first wave leaving a second-order term of the form

$$f_1(v)f_2(v)\exp[i(k_2-k_1)x+ik_2v\tau-i(k_2-k_1)vt].$$

The coefficient of v in this exponential will vanish when $t=\tau[k_2/(k_2-k_1)]$; so at this time a velocity integral over this term will not phase mix to 0, and an electric field will reappear in the plasma. If τ is long compared with a collisionless damping period and $[k_2/(k_2-k_1)]$ is of order unity, then this third electric field will appear long after the first two waves have damped away (i.e., it will be an echo).

This echo phenomenon can be rigorously derived from the collisionless Boltzmann equation and Poisson's equation. For the sake of simplicity, we limit the presentation to one dimension and treat the ions as a uniform positive background charge. If we assume that the electron distribution is initially spatially homogeneous, $f(x,v,t=0)=f_0(v)$, and that the two externally applied pulses are given by[4]

$$\varphi_{\text{ext}} = \Phi_{k_1}\cos(k_1x)\delta[\omega_pt] + \Phi_{k_2}\cos(k_2x)\delta[\omega_p(t-\tau)],$$

then the Fourier transform of the spatial dependence and Laplace transform of the time dependence of the Boltzmann equation and Poisson's equation can be written as

$$(p+ikv)\tilde{f}_k(v,p)=\frac{e}{m}ik\tilde{\varphi}_k(p)\frac{\partial f_0}{\partial v}+\frac{e}{m}\sum_q{}'\int\frac{dp'}{2\pi}i(k-q)\tilde{\varphi}_{k-q}(p-p')\frac{\partial f_q}{\partial v}(p'), \quad (1)$$

$$k^2\tilde{\varphi}_k(p)=4\pi ne\int dv\tilde{f}_k(v,p)+\frac{k_1{}^2\Phi_{k_1}}{2\omega_p}[\delta_{k,k_1}+\delta_{k,-k_1}]+\frac{k_2{}^2\Phi_{k_2}}{2\omega_p}[\delta_{k,k_2}+\delta_{k,-k_2}]e^{-p\tau}, \quad (2)$$

where $\tilde{\varphi}_k(p)$ and $\tilde{f}_k(v,p)$ are the transformed electric potential and distribution function, and the prime on the sigma in Eq. (1) indicates that the $q=0$ term is being treated separately in the manner usual for mode coupling calculations.[5]

To solve Eqs. (1) and (2), we expand in terms of the applied potentials Φ_{k_1} and Φ_{k_2}. The first-order (or linear) solution just describes a Landau-damped plasma wave following each pulse.[1] The second-order solution associated with wave number[6] $k_3\equiv k_2-k_1$ can be written as

$$\varphi_{k_3}{}^{(2)}(t)=\frac{e}{m}\frac{\Phi_{k_1}\Phi_{k_2}k_1k_2}{4k_3{}^2}\int_{-\infty}^{+\infty}dv\int_{-i\infty+\sigma}^{i\infty+\sigma}\frac{dp}{2\pi i}\int_{-i\infty+\sigma'}^{i\infty+\sigma'}\frac{dp'}{2\pi i}\frac{ik_3}{\epsilon(k_3p)(p+ik_3v)^2}\frac{\partial f_0}{\partial v}$$
$$\times\left[\frac{e^{pt}e^{-p\tau'}}{\epsilon(k_2,p')\epsilon(-k_1,p-p')(p'+ik_2v)}+\frac{e^{p(t-\tau)}e^{p'\tau}}{\epsilon(k_2,p-p')\epsilon(-k_1,p')(p'-ik_1v)}\right], \quad (3)$$

where $\epsilon(k,p)\equiv 1-\omega_p{}^2/k^2\int dv(\partial f_0/\partial v)[v+p/ik]^{-1}$ is the Landau dielectric function[1] and the p and p' contours are defined by requiring that $0<\sigma'<\sigma$. To carry out the p and p' integrations, we use the Cauchy residue method, closing the contours on the side which produces vanishingly small exponentials in the numerator. If we assume that τ is long compared with a collisionless damping period and that the time between the second pulse and the echo is same order as τ {i.e., that $|\gamma(k_1)\tau|$, $|\gamma(k_2)\tau|$, $|\gamma(k_3)\tau|\gg 1$ and that $[k_1/k_3]\simeq 1$}, then the residues at the poles arising from the roots of the dielectric constants will all be exponentially small and we may neglect them. Picking up the poles at $p'=ik_1v$ and $p=-ik_3v$ yields the result

$$\varphi_{k_3}{}^{(2)}(t)=\frac{e}{m}\frac{\Phi_{k_1}\Phi_{k_2}k_1k_2ik_1\tau}{4k_3{}^2}\int_{-\infty}^{+\infty}dv\frac{\partial f_0}{\partial v}\left\{\frac{e^{iv[k_1\tau-k_3(t-\tau)]}}{\epsilon(-k_1,ik_1v)\epsilon(k_2,-ik_2v)\epsilon(k_3,-ik_3v)}\right\}. \quad (4)$$

This integral does not phase mix to 0 when $k_3(t-\tau)\simeq k_1\tau$ [i.e., when $t\simeq\tau'\equiv\tau(k_2/k_3)$], and this results in the echo. One recognizes the various dielectric functions in the denominator as resulting from the effect of the self-consistent fields associated with first and second pulses and the echo. By setting these dielectric functions equal to unity, one recovers the result for weakly interacting or free streaming particles [an easy limit in which to check Eq. (4)].

When $t<\tau'$, we can evaluate the integral in Eq. (4) by closing the contour in the upper half v plane.[7] In this region of the v plane, we pick up poles from the Landau roots of $\epsilon(-k_1,ik_1v)$. On the other hand, when $t>\tau'$, we must close the contour in the lower half v plane and we pick up poles from the Landau roots of $\epsilon(k_3,-ik_3v)$ and $\epsilon(k_2,-ik_2v)$. However, we may neglect the latter compared with the former, since our assumption that $k_1/k_3\simeq 1$ implies that $k_2\simeq 2k_3$ and that $|\gamma(k_2)|\gg|\gamma(k_3)|$. Carrying out the integrations for these two cases and using a Maxwellian of mean thermal energy T to evaluate the dielectric functions yields the following time-asymptotic solution (i.e., $|t-\tau'|\omega_p>1$):

$$\varphi_{k_3}{}^{(2)}(t)=\Phi_{k_1}(\omega_p\tau)\frac{e\Phi_{k_2}}{T}\frac{k_1{}^4k_2L_D{}^2}{k_3(k_1+k_3)^2}$$
$$\times\frac{-(k_3/k_1)\gamma(k_1)e^{\gamma(k_1)k_3/k_1(\tau'-t)}\cos[\omega(k_1)(k_3/k_1)(\tau'-t)+\delta]}{\{\omega_p[(k_3-k_1)/(k_3+k_1)]^2+\gamma(k_1)^2\}^{1/2}},\quad\text{for } t<\tau'$$
$$\times\frac{\gamma(k_3)e^{\gamma(k_3)(t-\tau')}\cos[\omega(k_3)(t-\tau')+\delta']}{\{\omega_p{}^2[(k_3-k_1)/(k_3+k_1)]^2+\gamma(k_3)^2\}^{1/2}},\quad\text{for } t>\tau', \quad (5)$$

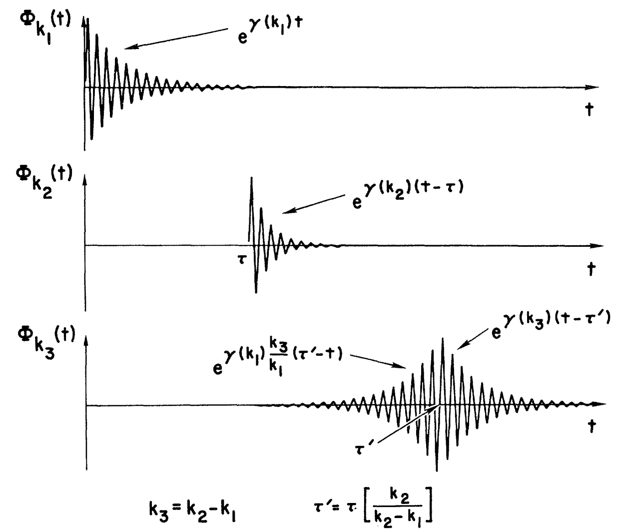

FIG. 1. Approximate variation of the principal Fourier coefficients of the self-consistent field for the case $k_3 \cong k_1 \cong \frac{1}{2}k_2$. Upper line: response to the first pulse; middle line: response to the second pulse; lower line: echo.

where

$$\tan\delta = \gamma(k_1)(k_3 - k_1)/\omega_p(k_3 + k_1)$$

and

$$\tan\delta' = \gamma(k_3)(k_1 - k_3)/\omega_p(k_1 + k_3).$$

It is interesting to note that the echo is not symmetric in that it grows up at the rate $\exp[\gamma(k_1)k_3/k_1(\tau' - t)]$ and damps away at the rate $\exp[\gamma(k_3)\times(t - \tau')]$.

The results of both the first- and second-order calculations are summarized in Fig. 1. The exponentials written in this figure indicate the general dependence of the envelopes of the oscillating curves, which have actually been drawn for the case where $k_1 \simeq k_3$.

The above calculation was based on the collisonless Boltzmann equation and is invalidated if collisions are strong enough to destroy the phase information before the echo can appear. Small angle Coulomb collisions are particularly effective in this regard, since the Fokker-Planck operator representing these collisions enhances the collision rate by a factor $(k\bar{v}\tau)^2 \simeq (\omega_p\tau)^2$ when operating on a perturbation of the form $e^{ikv\tau}$. By working in a marginal range, one might be able to use this effect as a tool to measure the Coulomb collision rate, even though the neutral collision rate is somewhat higher.

We have considered several variations on the above calculation. Although in this paper we have discussed explicitly only second-order echoes, higher order echoes are also possible. For example, a third-order echo is produced when the velocity space perturbation from the first pulse is modulated by a spatial harmonic of the electric field from the second pulse. The echo then occurs at $t = \tau 2k_2/(2k_2 - k_1)$ or $t = 2\tau$ when $k_2 = k_1$. This result is more closely related to echoes of other types[3] which are also third order for small amplitudes.

It is possible also to have spatial echoes, and these will probably be easier to observe experimentally than the temporal echoes described above. If an electric field of frequency ω_1 is continuously excited at one point in a plasma and an electric field of frequency $\omega_2 > \omega_1$ is continuously excited at a distance l from this point, then a spatial echo of frequency $\omega_2 - \omega_1$ will appear at a distance $l\omega_1/(\omega_2 - \omega_1)$ from the point where the second field is excited.

Finally, although our discussion has been entirely in terms of electron wave echoes, it is clear that the above treatment can be extended in a straightforward manner to include ion dynamics, and this leads to temporal as well as spatial ion wave echoes.

An observation of plasma echoes would be of fundamental interest, since it would experimentally verify the reversible nature of collisionless damping. The analogy with spin echoes[3] strongly suggests the possible use of the ehco technique as a means for studying collisional relaxation phenoma in plasmas.

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[6]Of course, $\Phi_{k_1+k_2}$, $\Phi_{k_1+k_1}$, and $\Phi_{k_2+k_2}$ all have second-order terms, but there is no echo associated with these terms.

[7]The $\partial f_0/\partial v$ term in Eq. (4) looks like it makes the integrand diverge for large imaginary v, but this term is actually canceled by a similar term hidden in $\epsilon(-k_1, ik_1v)$.

NONDIRECT PROCESSES AND OPTICAL PROPERTIES OF METALS

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Recent photoemission studies of noble and transition metals show a predominance of "nondirect" electronic excitations and indicate an anomalous density-of-states peak below the expected d-band. We attribute these observations to second-order (and higher) processes involving secondary phonon- and electron-pair excitations.

Photoemission studies of the noble and transition metals by Spicer and collaborators[1] imply, contrary to the expected selection rule for photon absorption, that the dominant absorption mechanism involves electron excitations that do not conserve wave vector $\vec{k}$. In addition to this "nondirect" character of the excitation process, an anomalous peak in the valence density of states is deduced to lie several volts below observed d-band structure. The d-band structure itself appears to be in good agreement with theoretical computations. It has been shown,[1] particularly for Cu and Ni, that the photoemission results, including the anomalous peaks, are in agreement with optical-reflectance data. X-ray emission data on Ni have been interpreted by Phillips[2] to provide confirmation for the photoemission results, but Cuthill, McAlister, and Williams[3] dispute this interpretation. They associate the observed large x-ray emission peak with the d-band density of states, while Phillips identifies this peak with the anomalous peak observed several electron volts below the d-band structure in optical and photoemission data. Ion-neutralization spectra[4] of Cu and Ni indicate prominent d-band density-of-states peaks, with an anomalous peak in Cu at the same energy as the anomalous optical peak, but with no second peak in Ni.

We shall not attempt to resolve the questions raised by existing inconsistencies among experimental data. However, we shall point out additional evidence for the importance of nondirect optical transitions in metals, and we propose a general mechanism for the appearance of anomalous shifted peaks in effective densities of states deduced from optical data.

If optical experiments on metals really measure nonlocal bulk properties and not surface effects, then $\vec{k}$ conservation must be assumed for the over-all absorption process. Observation of nondirect processes implies that second-order (or higher) processes are involved, so that nondirect electronic excitations are coupled to secondary excitations that make up the necessary $\vec{k}$ vector. If nondirect processes are observed in any part of the spectrum, the dipole sum rule implies that the observed intensity is borrowed from first-order direct electronic excitations. It follows from this that any nondirect process has the effect of decreasing the relative intensity of direct processes, thus increasing the relative strength of other nondirect transitions.

Any intrinsic absorption below the threshold for direct excitation must be due to a nondirect process. In particular, the well-known classical Drude theory of free electron absorption corresponds in the quantum theory to a second-order indirect process in which $\vec{k}$ is conserved by phonon emission or absorption. The quantum theory of such indirect processes has been applied by Nettel[5] to describe not only the Drude absorption but also the anomalous structure observed in Na below the threshold for direct one-electron excitation.[6] Similar structure observed in other alkali metals shows temperature dependence that is not yet accounted for,[6] but, as pointed out above, such structure must be attributed to a nondirect excitation process because it lies below the direct excitation threshold. It has been shown by Ferrell[7] that an indirect electronic excitation from the Fermi surface to the Brillouin zone boundary would