Our measurements of Ω' are analogous to the observation of H_{c1} in a type-II superconductor. Supercurrents are confined to a penetration length λ which acts as a natural long-distance cutoff in a bulk type-II superconductor. The corresponding length for the superfluid helium appears to be infinite. The presence of the inner cylinder in our apparatus introduces the length d which now plays a role similar to λ. Thus H_{c1} is of order $(\varphi_0/4\pi\lambda^2)\times\ln(\lambda/\xi)$, where $\varphi_0=hc/2e$ is the quantum of magnetic flux and ξ is the coherence length. This expression is similar to the for Ω_0.

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MEASUREMENTS OF ANGULAR MOMENTUM IN SUPERFLUID HELIUM†

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We demonstrate for the first time a rotational Meissner effect for liquid helium, analogous to the magnetic Meissner effect in superconductors. Liquid helium in a rotating cylinder is cooled through the lambda temperature. At sufficiently slow rotational speeds, the superfluid forms in a state of 0 total angular momentum, causing the container to rotate faster.

Landau[1] proposed that an essential characteristic of liquid helium II is the contraint of an irrotational superfluid velocity field. To determine if the equilibrium state of the superfluid is indeed irrotatational, London[2] suggested cooling through the lambda temperature a cylindrical sample of liquid helium I which is rotating uniformly with its container. One should then observe the transfer of a fraction ρ_s/ρ of the angular momentum of the helium to the container, as the superfluid stops rotating. (ρ_s/ρ is the fraction of the density associated with the superfluid in the final state.) London anticipated that this effect would be observed only below a critical angular velocity of the order of $\hbar/mR^2$, where m is the mass of the helium atom and R is the inside radius of the container. A number of experimenters[3] have since looked for nonrotation of superfluid helium in a rotating vessel; in every case in which the helium was plausibly in equilibrium with the container, it was found that the whole fluid rotated with the angular velocity of the container. We have recently carried out London's experiment at sufficiently low angular velocities and have observed for the first time the formation of stationary superfluid on cooling rotating helium I in a rotating vessel.[4]

The nature of the critical angular velocity is now understood on the basis of the Onsager-Feynman quantized-vortex model,[5] which has recently received considerable experimental support.[6] Onsager and Feynman suggested that the superfluid velocity field may contain vortex-line singularities. The superfluid circulation is not necessarily 0 around every closed path, as supposed by Landau, but is equal to h/m times the net number of vortices threading the path. For helium contained in a circular cylinder rotating about its axis, the nonrotating, vortex-free superfluid state has the lowest free energy only at angular velocities less than $\omega_{c1} = (\hbar/mR^2)\ln(R/a)$, where $a \approx 10^{-8}$ cm is the effective radius of the vortex core. At slightly larger angular velocities, the state of lowest free energy has one vortex on the cylinder axis, and the superfluid has angular momentum $L_0 = N\hbar\rho_s/\rho$, where N is the total number of helium atoms. At considerably larger angular velocities, the equilibrium superfluid contains an approximately uniform array of many vortices, and the angular momentum asymptotically approaches that of a classical, rigidly rotating fluid. ω_{c1} is analogous to the lower critical field H_{c1} in a type-II superconductor. This model predicts that at least a few hundred vortices would have been present in equilibrium in every previous experiment[3] on the rotation of superfluid helium in a simply connected container.

We have achieved a sensitivity to angular-momentum transfer on the order of L_0 at angular velocities in the neighborhood of ω_{c1} by supporting a container of small radius with a Beams-type magnetic bearing. An electromagnet at 4°K, controlled by feedback from a height sensor, attracts a ferrite slug attached to the end of an 0.089-cm-i.d. tube containing 7 mg of helium. This rotor is thereby suspended in free space inside a 1°K vacuum chamber just below the magnet pole piece and may rotate with extremely small dissipative torque. The rotor is cooled by temporarily admitting exchange gas to the vacuum chamber, and is heated above the lambda temperature in about 10 sec by shining light through a window. A set of small induction coils are used to set the rotor into rotation. The period of rotation is monitored with an autocollimator, which observes a small mirror attached to the rotor.

In practice, the angular momentum of the superfluid is determined by reheating the rotor above the lambda temperature and measuring the small change $\Delta\omega$ in the angular velocity ω of the rotor as the helium I comes into rigid rotation. The angular momentum L of the superfluid in the initial state is given by

$$\frac{L}{L_0} = \frac{1}{2}\frac{\omega}{\omega_0}\left[1 + \frac{I_R}{I_{SC}}\frac{\Delta\omega}{\omega}\right],$$

where $\omega_0 = \hbar/mR^2$, I_R is the moment of inertia of the rotor, and $I_{SC} = \frac{1}{2}(Nm\rho_s/\rho)R^2$. If the superfluid behaved classically, $\Delta\omega/\omega$ would be zero and $L_{\text{classical}} = \omega L_0/2\omega_0 = \omega I_{SC}$. Our reduced experimental data are shown in Fig. 1. The superfluid angular momentum, in units of $L_0 = 9.4 \times 10^{-7}$ dyn cm sec (at 1.6°K), is plotted against the angular velocity of the rotor, in units of $\omega_0 = 0.0808$ rad/sec. Note that both the angular velocity and the angular momentum involved are rather small. An attempt to increase one by changing the container radius requires the other to decrease. The equilibrium angular momentum predicted[7] by the vortex model is indicated by the heavy line segments. It is 0 up to $\omega/\omega_0 = \ln(R/a) \approx 15.3$ and then increases in steps as the number of vortices increases. The equilibrium angular momentum for a classical fluid would follow the diagonal broken line. Each point on the figure is the average angular momentum for a number of experimental runs at the same angular velocity. The error bars represent the scatter of the runs in the group, presented as one standard deviation for the mean, and they do not include estimated systematic errors. No significant difference was found between data for clockwise and for counterclockwise rotation. The principal systematic uncertainty is in the determination of the ratio I_R/I_{SC}

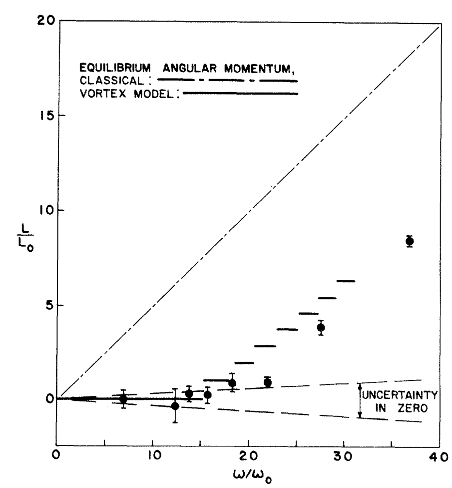

FIG. 1. Angular momentum L of the superfluid versus angular velocity ω of the rotor, after helium is cooled in rotation. $L_0 = N\hbar\rho_s/\rho$ and $\omega_0 = \hbar/mR^2$. Each point is the average for all runs at that angular velocity; the number of runs averaged is, from left to right, respectively, 10, 4, 22, 23, 10, 17, 13, and 18. The solid line segments are the equilibrium state predicted by the vortex model.

$=654 \pm 50$ and is due primarily to uncertainty of the amount of helium in the rotor. The rotor is permanently sealed, and its helium content was determined by measurements of its heat capacity between 1.6 and 5°K. This uncertainty results, in Fig. 1, in an uncertainty proportional to ω in the location of the 0 of superfluid angular momentum. If we attempt to fit the theory to the data by adjusting the constant $\hbar/m$, we obtain a value about 1.2 times the accepted value. The discrepancy seen in Fig. 1 can probably be attributed instead to a departure from equilibrium.

For the data included in Fig. 1, the temperature of the rotor just before heating ranged from 1.6 to 1.73°K, corresponding to values of ρ_s/ρ from 0.82 to 0.73. We have made some additional runs at $\omega = 14\omega_0$ over a wider range of helium-II temperatures. At this speed the superfluid in equilibrium should have no vortices. The fractional change in the angular velocity of the rotor on heating is plotted in Fig. 2 against the temperature just before heating. The same heat input was used at each temperature. It is clear that within experimental error the angular-momentum transfer is proportional to ρ_s/ρ, as expected.

In a few additional experiments, the superfluid was formed at rest and the vessel then accelerated into rotation. There is evidence that the superfluid may acquire some angular momentum by mechanical generation of vortices when the angular velocity is somewhat larger than ω_{c1}. Further work on these lines is in progress.

We conclude that superfluid formed in a container of liquid helium rotating at a sufficiently low angular velocity does not participate in the rotation, and that this is an equilibrium rather than a metastable phenomenon. The experimental precision is sufficient to distinguish clearly between the angular momentum of the actual superfluid flow and that for classical rotation, but is not quite sufficient to resolve the angular momentum of an individual quantized vortex. At larger angular velocities, the superfluid is formed with angular momentum of approximately the magnitude predicted by the vortex model for the equilibrium state.

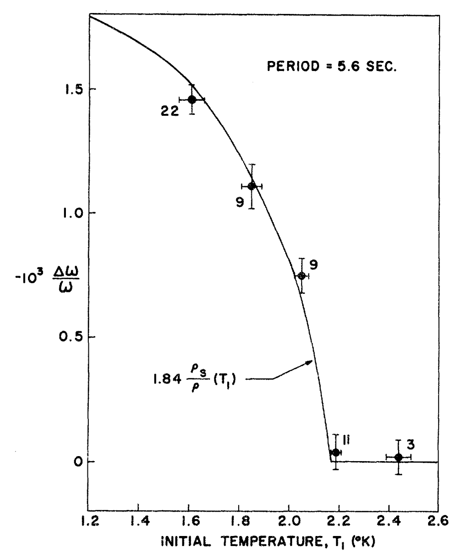

FIG. 2. The fractional change in rotor angular velocity on heating through the lambda temperature, as a function of the initial temperature T_1. At the angular velocity used, the superfluid is expected to have no angular momentum. The solid line is the predicted effect, which is proportional to ρ_s/ρ at T_1. The number of runs averaged is indicated by each point. The leftmost point represents the same data as the point for $\omega = 14\omega_0$ in Fig. 1.

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