THRESHOLD FOR SUPERFLUID VORTEX LINES IN A ROTATING ANNULUS*

Philip J. Bendt† and Russell J. Donnelly
Department of Physics, University of Oregon, Eugene, Oregon
(Received 5 June 1967)

An experimental observation of the onset of quantized vortex lines in a rotating annulus of He II has been made. The results agree with the predictions of Donnelly and Fetter which were based on a free-energy approach, provided care is taken to ensure equilibrium.

Exact calculations of the equibrium array of vortices in a rotating annulus of He II have been recently carried out by Fetter.[1] The experimental significance of these results for superfluid gyroscopes has been examined by Donnelly and Fetter.[2] They also suggested that an experimental search for the initial angular velocity Ω_0 for the appearance of vortices in an annulus would be most valuable. This Letter reports experimental observation of Ω_0, using the attenuation of second sound as a probe.

Experiments on hydrodynamic stability are incomplete without a systematic study of changes in scale. Here the same anodized-aluminum outside cylinder was used with four interchangeable inside cylinders to form annuli with four different gap widths. The radius of the outside cylinder was 1.56 cm. The cylinders had opposite faces coated with Aquadag to serve as second-sound transmitters and receivers. Their surface roughness was about 100 μin. The Q of second sound in these cavities varied from 500 to 1300. The apparatus and procedure were similar to those used previously.[3]

The observation of the onset of second-sound attenuation required the detection of a change in the amplitude of the second-sound signal of 0.3 to 0.8%. The signal before amplification varied from 10 to 40 μV in the fundamental radial mode, and from 5 to 10 μV in the second harmonic (at double the frequency of the fundamental). An extremely quiet rotating system was necessary to keep the noise on top of the second-sound signal as small as the change in amplitude when attenuation comes in. The signal was brought out through mercury troughs, through a Hewlett-Packard model-302A wave analyzer, and was measured with a Hewlett-Packard 400E ac voltmeter, which drove a strip chart recorder.

The small changes in amplitude due to attenuation, which were often comparable with small drifts in the electronics, were determined by alternately comparing the amplitude when rotating with the amplitude when rotation was stopped. An observation at a single angular velocity was repeated one to three times, waiting 2 to 5 min for transients to die out after each start and stop of rotation.

The theory discussed by Donnelly and Fetter distinguishes two critical velocities,

$$\Omega_C = (\kappa/4\pi d R)\ln 2C \quad (1)$$

and

$$\Omega_0 = \frac{\kappa}{\pi d^2}\ln\frac{2d}{a}. \quad (2)$$

Here κ is the quantum of circulation, R is the mean radius of the cylinders, d is the width of the annulus, C is a constant of order unity, and a is the vortex core parameter which is of order 10^{-8} cm.

For $0<\Omega<\Omega_C$, the superfluid presumably remains at rest. When $\Omega=\Omega_C$, vortices can form near the inner wall and the superfluid starts to be involved in the rotation. Note that Ω_C is lower than Ω_0 by a factor $d/4R$. For $\Omega_C<\Omega<\Omega_0$ the equilibrium state is a set of quantized circulation states, without vortices, of the type first discussed by Bendt and Oliphant.[4] Just how this equilibrium comes about is not specified by free-energy calculations. If the equilibrium situation prevails, one does not expect any attenuation of second sound. At Ω_0 a single row of vortices form in the middle of the gap, and for $\Omega \gg \Omega_0$ a uniform array of vortices is produced. Thus the first attenuation is expected at $\Omega=\Omega_0$, and at higher speeds the attenuation should be governed by the mutual friction coefficient B.[5]

Our experiments showed no observable attenuation at low speeds of rotation followed by a definite onset of attenuation at a critical angular velocity, which we designate as Ω' for the fundamental and Ω'' for the second harmonic. On the first run after cooling through the λ point, the value of Ω' often exceeded Ω_0

Table I. Critical angular velocities for the appearance and disappearance of second-sound attenuation. Ω' refers to results for the fundamental and Ω'' to the second harmonic. Ω_0 is the theoretical value from Eq. (2), setting $a = 1.3\times10^{-8}$ cm. The radius of the outside cylinder was 1.56 cm. The temperature was 1.40°K.

Annulus width (mm)	Ω_0 (rad/sec)	Ω' (rad/sec)	Ω'' (rad/sec)	Ω''/Ω'
0.62	1.22	0.83 ± 0.06	0.84 ± 0.06	1.0 ± 0.1
1.04	0.46	0.49 ± 0.05	0.91 ± 0.05	1.9 ± 0.3
1.40	0.25	0.23 ± 0.04	0.46 ± 0.04	2.0 ± 0.4
1.90	0.14	0.14 ± 0.04	0.26 ± 0.04	1.8 ± 0.5

by a factor of ~3. After this first sequence Ω' was observed to appear near Ω_0 for both increasing and decreasing sequences of measurements. Despite the precaution of starting and stopping rotation for each measurement, some hysteresis was observed in the Ω' transition: Sequences with increasing Ω had a somewhat larger Ω' than sequences with decreasing Ω. These values have been averaged to give the numbers quoted in Table I, and they fall within the stated limits of uncertainty.

These results are compared with theory in Fig. 1, and agreement within experimental uncertainty is obtained for all but the narrowest annulus. All the measurements were made at 1.40°K, where the superfluid density is about 0.93 of the total density.[6]

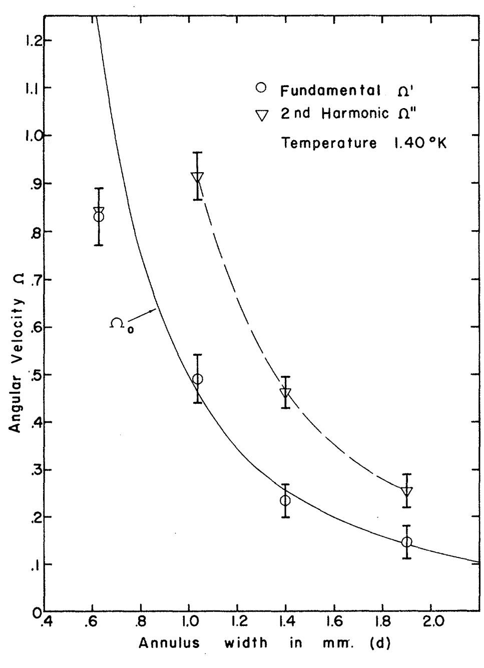

FIG. 1. Plot of Ω' and Ω'' as a function of d. The theoretical value of Ω_0 is given by the solid line.

A further test of the theory was obtained by measurements of the second harmonic. The vortices are predicted to lie in the middle of the gap, and since the second harmonic has a node in the middle (in $v_n - v_s$), one would not expect Ω'' to coincide with Ω_0. The data here were taken with a weaker signal, but the indication was that Ω'' occurred at a definite and reproducible angular velocity which was higher than Ω' (except for the narrowest gap, where it coincided with the results for the fundamental). One possible interpretation of these results is that Ω'' indicates the formation of a second row of vortices at a significantly greater speed than Ω_0. The theoretical calculation of the onset of a double row is formidable. For $\Omega \gg \Omega_0$ the attenuation [defined as $(A_0/A-1)$, where A is the amplitude of the signal in rotation and A_0 at rest] was reasonably consistent with values of B determined in an earlier experiment.[3]

The experiments indicate that for the 1.04-, 1.40-, and 1.90-mm gaps, Fetter's calculations are confirmed. One concludes that (at least after some initial stirring) the hydrodynamic flow coincides with the state of lowest free energy. The fact that relatively small hysteresis occurs in increasing and decreasing sequences of measurements suggests that this equilibrium is nearly independent of the path by which it is reached. The relative independence is similar to that reported for Couette flow by Caldwell and Donnelly[7] and suggests that a dynamic theory of quantum stability could be based on the usual linear approximations.

The occurrence of attenuation at angular velocities smaller than the theoretical value in the narrowest annulus is apparently due to a "critical velocity," which lies above the experimental points for the wider annuli. This critical velocity will be discussed in a later publication.[8]

Our measurements of Ω' are analogous to the observation of H_{c1} in a type-II superconductor. Supercurrents are confined to a penetration length λ which acts as a natural long-distance cutoff in a bulk type-II superconductor. The corresponding length for the superfluid helium appears to be infinite. The presence of the inner cylinder in our apparatus introduces the length d which now plays a role similar to λ. Thus H_{c1} is of order $(\varphi_0/4\pi\lambda^2)\times\ln(\lambda/\xi)$, where $\varphi_0=hc/2e$ is the quantum of magnetic flux and ξ is the coherence length. This expression is similar to the for Ω_0.

The authors are greatly indebted to Professor A. L. Fetter for undertaking the detailed calculations, and for active interest in the progress of the experiments. One of us (PJB) thanks the U. S. Atomic Energy Commission for partial support, and the University of Oregon Physics Department for their hospitality during this investigation.

*Research supported by the National Science Foundation under Grants Nos. NSF-GP-6473 and NSF-GP-6482, and by the U. S. Air Force Office of Scientific Research under Grant No. AF-AFOSR 785-65.

†On leave from the Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

[1]A. L. Fetter, Phys. Rev. 153, 285 (1967).

[2]R. J. Donnelly and A. L. Fetter, Phys. Rev. Letters 17, 747 (1966).

[3]P. J. Bendt, Phys. Rev. 153, 280 (1967).

[4]P. J. Bendt and T. A. Oliphant, Phys. Rev. Letters 6, 213 (1961).

[5]R. J. Donnelly, Experimental Superfluidity (University of Chicago Press, Chicago, Illinois, 1967).

[6]In the Donnelly-Fetter theory, the angular velocity Ω_0 at which a row of superfluid vortex lines first becomes energetically favorable does not depend on ρ_s/ρ; so the results reported here are not expected to be temperature dependent, except possibly close to T_λ.

[7]D. R. Caldwell and R. J. Donnelly, Proc. Roy. Soc. (London) A267, 197 (1962).

[8]The experimental Ω'' for $d=1.04$ mm also may be determined by the critical velocity to which we refer.

MEASUREMENTS OF ANGULAR MOMENTUM IN SUPERFLUID HELIUM†

G. B. Hess and W. M. Fairbank
Stanford University, Stanford, California
(Received 5 June 1967)

We demonstrate for the first time a rotational Meissner effect for liquid helium, analogous to the magnetic Meissner effect in superconductors. Liquid helium in a rotating cylinder is cooled through the lambda temperature. At sufficiently slow rotational speeds, the superfluid forms in a state of 0 total angular momentum, causing the container to rotate faster.

Landau[1] proposed that an essential characteristic of liquid helium II is the contraint of an irrotational superfluid velocity field. To determine if the equilibrium state of the superfluid is indeed irrotatational, London[2] suggested cooling through the lambda temperature a cylindrical sample of liquid helium I which is rotating uniformly with its container. One should then observe the transfer of a fraction ρ_s/ρ of the angular momentum of the helium to the container, as the superfluid stops rotating. (ρ_s/ρ is the fraction of the density associated with the superfluid in the final state.) London anticipated that this effect would be observed only below a critical angular velocity of the order of $\hbar/mR^2$, where m is the mass of the helium atom and R is the inside radius of the container. A number of experimenters[3] have since looked for nonrotation of superfluid helium in a rotating vessel; in every case in which the helium was plausibly in equilibrium with the container, it was found that the whole fluid rotated with the angular velocity of the container. We have recently carried out London's experiment at sufficiently low angular velocities and have observed for the first time the formation of stationary superfluid on cooling rotating helium I in a rotating vessel.[4]

The nature of the critical angular velocity is now understood on the basis of the Onsager-Feynman quantized-vortex model,[5] which has recently received considerable experimental support.[6] Onsager and Feynman suggested that the superfluid velocity field may contain vortex-line singularities. The superfluid circulation is not necessarily 0 around every closed path, as supposed by Landau, but is equal to h/m times the net number of vortices thread-