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CONVERGENT CALCULATION OF NONLEPTONIC K DECAY IN THE INTERMEDIATE-BOSON MODEL*

S. L. Glashow
Harvard University, Cambridge, Massachusetts

and

Howard J. Schnitzer
Brandeis University, Waltham, Massachusetts

and

Steven Weinberg†
University of California, Berkeley, California

The nonleptonic decay amplitudes of the K mesons are calculated using current algebra and the spectral-function sum rules. The result is convergent in the intermediate-boson model, but not for a direct current-current interaction. Comparison with the observed rate for K_1^0 decay shows that the intermediate-boson mass is about 8 BeV.

Do the weak interactions arise from a local four-fermion interaction, or are they mediated by an intermediate boson? It has not previously been possible to decide between these two models, because they give nearly equivalent descriptions of the leptonic and semileptonic decays, and because the calculation of nonleptonic decay rates in either model has been prevented by ultraviolet divergences and by our general inability to handle strong interaction effects. In this Letter we shall describe a calculation of the matrix element for nonleptonic K decay from the known properties of the semileptonic weak interactions, using a technique recently employed[1] to calculate the $\pi^+-\pi^0$ mass difference. Our calculation diverges for the local model, but gives a finite result, in terms of measurable parameters, in the intermediate-boson theory. Moreover, our result depends upon the vector boson mass M_B in an essentially different way than do the matrix elements for leptonic or semileptonic decays; so it is possible to determine M_B from the observed K_1^0 lifetime. We obtain for the intermediate-boson mass the value[2]

$$M_B \simeq 8 \text{ BeV}. \qquad (1)$$

Our primary assumption is that the weak interaction has the form

$$\mathcal{H}_w = g(L_\mu + J_\mu)B^\mu + H.C., \quad (2)$$

where B^μ is an intermediate-boson field of mass M_B, L_μ is the usual lepton current, and J_μ is the Cabibbo current[3]

$$J^\mu = \tfrac{1}{2}(V_1^\mu + iV_2^\mu - A_1^\mu - iA_2^\mu)\cos\theta + \tfrac{1}{2}(V_4^\mu + iV_5^\mu - A_4^\mu - iA_5^\mu)\sin\theta. \quad (3)$$

The coupling constant observed in leptonic and semileptonic decays is then

$$G \equiv 2^{1/2}g^2/M_B^2. \quad (4)$$

We have also separately considered the case of a local current-current interaction; the result coincides with the limit $M_B \to \infty$ (g^2/M_B^2 being fixed) of the intermediate-boson theory. Our calculation will be divided into four stages, and the assumed properties of the V_i^μ and A_i^μ will be stated as they are needed in each stage.

(I) We shall assume the validity of the "soft-pion" calculations,[4] which give the matrix element for $K \to n + \pi$ in terms of the matrix element for $K \to$ vacuum. In particular, we shall use the result[5] that the matrix element for $K_1^0 \to \pi^+ + \pi^-$ is

$$\mathfrak{M} = \tfrac{1}{2}F_\pi^{-2}\langle 0|[Q_+,[Q_-,\mathcal{H}_{\rm eff}]] + [Q_-,[Q_+,\mathcal{H}_{\rm eff}]]|K_1^0\rangle, \quad (5)$$

where F_π is the usual pion decay amplitude, and

$$Q_\pm \equiv 2^{-1/2}\int d^3x\,(A_1^0 \pm iA_2^0), \quad (6)$$

$$\mathcal{H}_{\rm eff} \equiv g^2\int d^4y\, T\{J_\mu(y), J_\nu^+(0)\}\Delta_B^{\mu\nu}(y), \quad (7)$$

$$\Delta_B^{\mu\nu}(y) \equiv -i(2\pi)^{-4}\int d^4p\, e^{ip\cdot y} \times [g^{\mu\nu} + p^\mu p^\nu/M_B^2][p^2 + M_B^2]^{-1}. \quad (8)$$

All matrix elements are defined here with the usual factors $(2\pi)^{-3/2}(2E)^{-1/2}$ omitted; the rate for $K_1^0 \to \pi^+ + \pi^-$ is thus

$$\Gamma(K_1^0 \to \pi^+ + \pi^-) = (16\pi m_K)^{-1}[1 - 4m_\pi^2/m_K^2]^{1/2}|\mathfrak{M}|^2. \quad (9)$$

Performing the double commutation in (5) gives

$$\mathfrak{M} = F_\pi^{-2}\langle 0|\mathcal{H}_{\rm eff}|K_1^0\rangle. \quad (10)$$

It should perhaps be stressed that we are concentrating on the decay process $K_1^0 \to \pi^+ + \pi^-$ only for the sake of definiteness and convenience; in fact, the soft-pion approach[4] gives all nonleptonic K-meson decay rates in term of $\mathfrak{M}$, and in accord with the $\Delta I = \frac{1}{2}$ rule.

(II) In order to calculate the matrix element $\langle 0|\mathcal{H}_{\rm eff}|K_1^0\rangle$, we shall work in an ideal world in which the currents V_i^μ, A_i^μ satisfy the SU(3) $\otimes$ SU(3) commutation relations[6] and are exactly conserved. The breaking of SU(3) and chirality then enforces the presence of massless "Goldstone bosons," which we identify[7] as the π, K, κ, and η. Taking the K_1^0 at zero four-momentum, we have then[8]

$$\mathfrak{M} = F_K^{-1}F_\pi^{-2}\int d^4x\,\partial_\mu\langle T\{A_4^\mu(x), \mathcal{H}_{\rm eff}\}\rangle_0 = F_K^{-1}F_\pi^{-2}\int d^4x\,\langle[A_4^0(x), \mathcal{H}_{\rm eff}]\rangle_0\delta(x^0)$$
$$= ig^2F_K^{-1}F_\pi^{-2}\cos\theta\sin\theta\int d^4y\,\Delta_B^{\mu\nu}(y)[\Delta_{\mu\nu}^A(y) + \Delta_{\mu\nu}^V(y) - \Delta_{\mu\nu}^{A'}(y) - \Delta_{\mu\nu}^{V'}(y)], \quad (11)$$

where $\Delta^A, \Delta^V, \Delta^{A'}$, and $\Delta^{V'}$ are, respectively, the propagators of A_i $(i = 1,2,3)$, V_i $(i = 1,2,3)$, A_i $(i = 4,5,6,7)$, and V_i $(i = 4,5,6,7)$. [Note that this matrix element vanishes in the limit of exact SU(3), as it must.[9]] We may evaluate (11) by using the well-known spectral representation for a conserved current, e.g.,

$$\Delta_{\mu\nu}^A(y) = -i(2\pi)^{-4}\int d^4p\, e^{ip\cdot y}\{\int d\mu^2\,\rho_A(\mu^2)[g_{\mu\nu} + p_\mu p_\nu/\mu^2][p^2 + \mu^2]^{-1} + F_\pi^2(p_\mu p_\nu/p^2)\}. \quad (12)$$

Similar formulas hold for Δ^V, $\Delta^{A'}$ and $\Delta^{V'}$, with F_π replaced, respectively, with 0, F_K, and F_K. Using these formulas in Eq. (11) then gives

$$\mathfrak{M} = -ig^2F_K^{-1}F_\pi^{-2}\cos\theta\sin\theta\int d^4p\,\{A + B(p^2 + M_B^2)^{-2} + 3\int d\mu^2\,\sigma(\mu^2)(M_B^2 - \mu^2)(p^2 + \mu^2)^{-2}(p^2 + M^2)^{-1}\}, \quad (13)$$

where

$$\sigma(\mu^2) \equiv \rho^A(\mu^2) + \rho^V(\mu^2) - \rho^{A'}(\mu^2) - \rho^{V'}(\mu^2), \tag{14}$$

$$A \equiv M_B^{-2}\{\int d\mu^2\, \mu^{-2}\sigma(\mu^2) + F_\pi^2 - F_K^2 - F_\kappa^2\}, \tag{15}$$

$$B = \int d\mu^2\, \sigma(\mu^2). \tag{16}$$

(III) We now assume the validity of the two spectral-function sum rules[10] for SU(3). It follows immediately that $A = B = 0$; so the quartic and logarithmic divergences drop out of (13).[11] The remaining finite integral can be easily calculated, and we find

$$\mathfrak{M} = -\left(\frac{3g^2\cos\theta\sin\theta}{16\pi^2 F_K F_\pi^2}\right)\int d\mu^2\, \mu^2(M_B^2-\mu^2)^{-1}\sigma(\mu^2)\ln(M_B^2/\mu^2). \tag{17}$$

(IV) In order to evaluate the integral in Eq. (17), we shall assume the spectral functions ρ^A, ρ^V, $\rho^{A'}$, $\rho^{V'}$ to be saturated[12] by the observed[13] axial vector and vector mesons $A1(1080)$, $\rho(770)$, $K_A(1320)$, and $K^*(890)$. For the coefficient of the δ functions in these spectral functions, we shall use the current-algebra estimate[14] $2F_\pi^2 m_\rho^2$. With Eq. (4) this now gives

$$\mathfrak{M} = -\left(\frac{3Gm_\rho^2\cos\theta\sin\theta}{8\pi^2 F_K 2^{1/2}}\right)\{f(m_{KA}) + f(m_{K^*}) - f(m_{A1}) - f(m_\rho)\}, \tag{18}$$

where

$$f(m) \equiv M_B^2 m^2 (M_B^2 - m^2)^{-1}\ln M_B^2/m^2 \simeq m^2 \ln M_B^2/m^2. \tag{19}$$

Taking $G = 1.02\times10^{-5} m_p^2$, $\cos\theta\sin\theta = 0.22$, and $F_K = 1.28F_\pi = 220$ MeV, we find that

$$|\mathfrak{M}| \simeq 10^{-7} m_K [5.46\ln(M_B/m_\rho) - 4.68]. \tag{20}$$

This is to be compared with the experimental value of $\mathfrak{M}$, determined from Eq. (9), and the observed value 0.77×10^{10} sec^{-1} of $\Gamma(K_1^0 \to \pi^+ + \pi^-)$:

$$|\mathfrak{M}|_{\text{exp}} = 7.85\times10^{-7} m_K. \tag{21}$$

Equating (20 and (21), we find that the intermediate-boson mass M_B should be roughly 8 BeV.

It is to be noted that Eq. (17) gives a logarithmically divergent result for $\mathfrak{M}$ in the limit $M_B \to \infty$, g^2/M_B^2 fixed, unless the spectral functions obey one additional sum rule:

$$\int \sigma(\mu^2)\mu^2 d\mu^2 = 0. \tag{22}$$

This is not satisfied in the meson-dominance approximation, as shown by the nonvanishing coefficient of $\ln M$ in Eq. (20). If Eq. (22) is indeed false, we may conclude that a local current-current interaction does not yield finite matrix elements for nonleptonic K decay.

We are well aware that our calculation is based on questionable approximations, particularly at stage II. However, other applications of the spectral-function sum rules have worked better than might have been expected, and we may hope for the same good fortune here. Of course, we will lose most of our scruples about this calculation if an 8-BeV intermediate boson is found at Serpukhov or Weston. In this event, we might reasonably infer that the weak currents are linear combinations of hadron gauge fields, since these are the only currents which are known[15] to satisfy the spectral-function sum rules.

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†Presently Morris Loeb Lecturer at Harvard University.

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