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[15]An equally good fit to the total cross sections using the same number of parameters is $\sigma_+ + \sigma_- = 44.20 + 18.4/p^{0.69}$ and $\sigma_- - \sigma_+ = 3.85/p^{0.306}$. This fit gave similar predictions for D^+ but gave results for D^- which disagreed with the data because of the slower convergence of the total cross section difference at high energies. However, a two-standard-deviation change in the exponent of the momentum dependence of this fit would give a reasonable agreement with D^-.

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PROTON AND PION SPECTRA FROM PROTON-PROTON INTERACTIONS AT 10, 20, AND 30 BeV/c*

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The secondary proton and pion spectra from proton-proton interactions have been measured over nearly the entire momentum range and over a wide range of laboratory angles. Most of the data are at an incident-proton momentum of 30 BeV/c.

The proton spectra were transformed into cross sections in terms of $p_\perp^*$ and $p_\parallel^*$ all in the c.m. system. These cross sections were found to be largely independent of $p_\parallel^*$ and an empirical fit was obtained quite similar to one derived by Hagedorn using a statistical model. The pion spectra were also expressed in terms of $p_\perp^*$ and $p_\parallel^*$ and found to be different from the proton spectra by showing a marked dependence on $p_\parallel^*$.

As part of a proton-proton scattering experiment reported earlier,[1,2] we obtained spectra of secondary protons and pions covering a wide range of angles and momenta. A description of the wire-plane spectrometer system and analysis procedure is found elsewhere.[3] Separation of protons and π^+ mesons was accomplished with a 7-ft-long, freon-filled, threshold Cherenkov counter operating at pressures between 15 and 35 psi. The efficiency of this

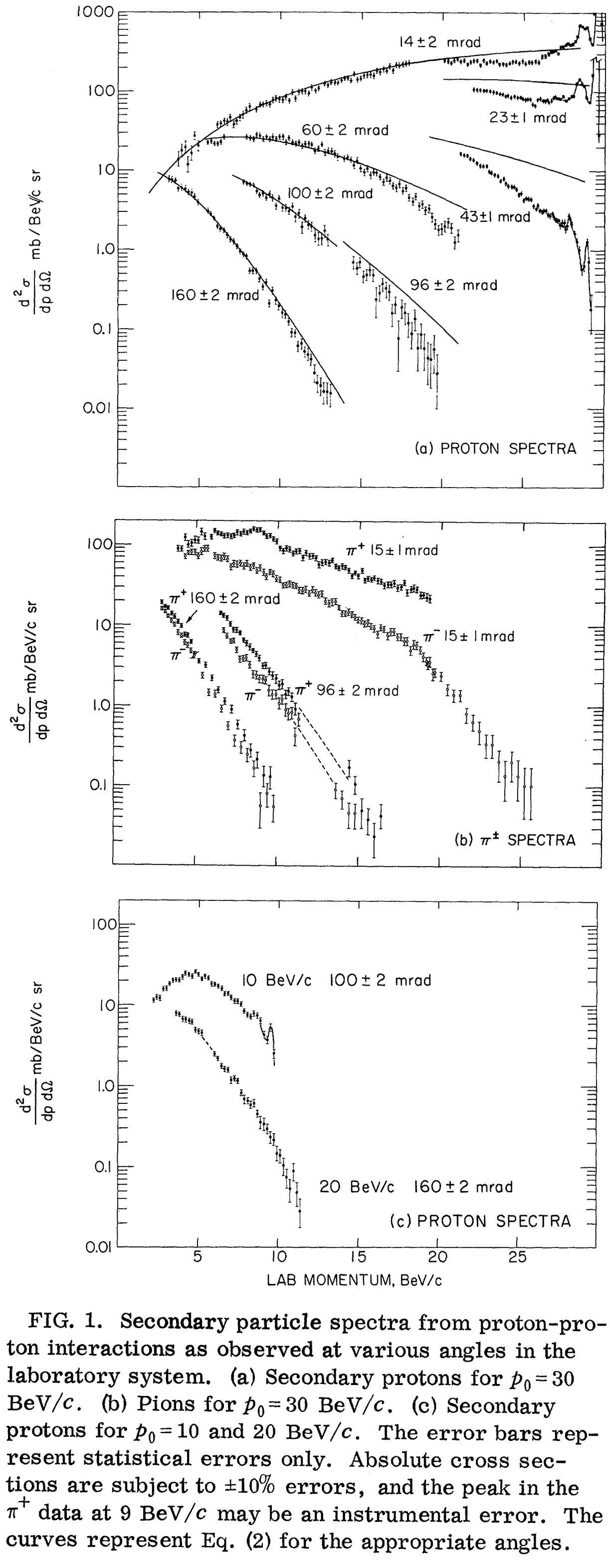

FIG. 1. **Secondary particle** spectra from proton-proton interactions as observed at various angles in the laboratory system. (a) Secondary protons for $p_0=30$ BeV/c. (b) Pions for $p_0=30$ BeV/c. (c) Secondary protons for $p_0=10$ and 20 BeV/c. The error bars represent statistical errors only. Absolute cross sections are subject to ±10% errors, and the peak in the π^+ data at 9 BeV/c may be an instrumental error. The curves represent Eq. (2) for the appropriate angles.

counter was determined by measurements to be greater than 99% over the entire range of β covered in this experiment.

Figures 1(a) and 1(b) show the measured differential cross section, in the laboratory system, for secondary protons and pions at various scattering angles for 30-BeV/c protons incident on a liquid-hydrogen target. Figure 1(c) shows the proton spectra for 10- and 20-BeV/c incident protons. Dekkers et al.[4] have also measured secondary spectra in this energy range.

The meaningful role played by $p_\perp^*$, the component of the secondary-particle momentum perpendicular to the incoming momentum, has been pointed out by several authors.[5-8] These considerations lead us to express our proton and pion cross sections in terms of $p_\perp^*$ and $p_\parallel^*$ (c.m. momentum perpendicular and parallel to incoming momenta) instead of the usual parameters p^* and Ω^* (c.m. momentum and solid angle). Figure 2(a) shows the result of transforming in this way all the data shown in Fig. 1(a) for $p<20$ BeV/c, where the products of resonant interactions are not expected to predominate. Here the differential cross section $d^2\sigma/dp_\perp^* dp_\parallel^*$ is plotted against $p_\perp^*$. We expected to obtain a series of distributions corresponding to different values of $p_\parallel^*$. Instead, the rather limited distribution shown resulted, which includes values of $p_\parallel^*$ from 0.2 to 3.0 BeV/c. This suggests that the cross section for producing these secondary protons is rather independent of $p_\parallel^*$, although some dependence is noted for points corresponding to high values of laboratory momenta. The

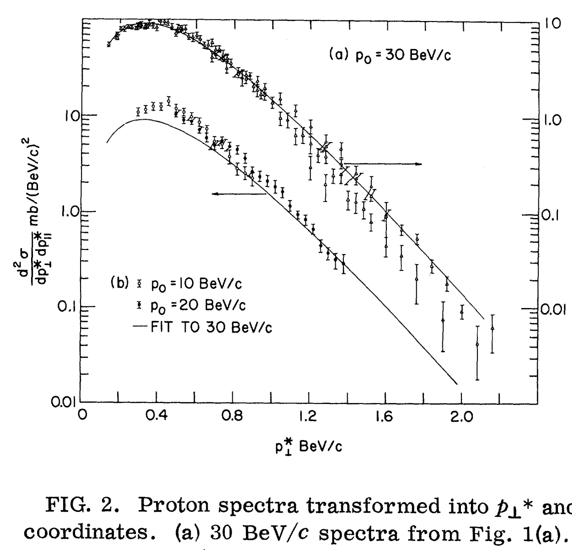

FIG. 2. Proton spectra transformed into $p_\perp^*$ and $p_\parallel^*$ coordinates. (a) 30 BeV/c spectra from Fig. 1(a). (b) 20- and 10-BeV/c data from Fig. 1(c). The curves in (a) and (b) are the same maximum-likelihood fits to 30-BeV/c data.

data which were obtained at 20 and 10 BeV/c [Fig. 1(c)] are plotted in Fig. 2(b) together with a duplicate of the best-fit curve for the $p_0 = 30$ BeV/c data of Fig. 2(a) to show the degree of energy independence. The curve shown in Fig. 2(a) is a graph of the expression

$$d^2\sigma/dp_\perp{}^* dp_\|{}^* = 610 p_\perp{}^{*2} \exp(-p_\perp{}^*/0.166)\ \text{mb}/(\text{BeV}/c)^2. \quad (1)$$

The three constants in this equation were obtained from a maximum likelihood fit and their errors are estimated to be ±10%. When transformed into the usual laboratory coordinates, Eq. (1) takes the form

$$\frac{d^2\sigma}{dp\,d\Omega} = \frac{p^2}{2\pi p_\perp} \frac{\gamma_c(E-\beta_c p\cos\theta)}{E} 610 p_\perp{}^2 \times \exp(-p_\perp/0.166)\ \text{mb}/(\text{BeV}/c\ \text{sr}), \quad (2)$$

where β_c and γ_c refer to the transformation to the over-all c.m. system. The curves shown in Figs. 1(a) and 1(c) are Eq. (2) for the appropriate angles and energies.

Our pion data do not extend over a sufficient range of $p_\|{}^*$ to make a presentation like Fig. 2(a) possible. Instead, we have selected $p_\perp{}^* = 0.18 \pm 0.02$ BeV/c for which a range of $p_\|{}^* = 1.0$ to $p_\|{}^* = 2.7$ BeV/c is available and plotted $d^2\sigma/dp_\perp{}^* dp_\|{}^*$ vs $p_\|{}^*$ in Fig. 3. Clearly the production cross section for pions is quite dependent on $p_\|{}^*$. For comparison the corresponding plots for protons with $p_\perp{}^* = 0.18 \pm 0.04$ BeV/c and $p_\perp{}^* = 1.0 \pm 0.04$ BeV/c are included to show again the high degree of independence of these secondary particles on $p_\|{}^*$.

We wish to point out that our empirical Eq. (2) is quite similar to one derived by Hagedorn[9] using a statistical model for central p-p interactions. Like our empirical expressions, Hagedorn's also becomes inexact at high momenta due to the absence of kinematical contraints in his theory. In addition this statistical theory does not account for the difference between secondary pions and protons evident in Fig. 3.

It seems useful to view these spectra as originating from central interactions, as distinct from the now-well-known peripheral interactions.[10,11] Peripheral interactions are responsible for the structure due to resonant states seen at high momenta and small angles. Central interactions are believed responsible for the smooth nonresonant spectra seen at large angles and lower momentum and showing a fast, approximately exponential falloff with momentum. At larger angles the pion spectra also show an exponential falloff, suggesting a nonresonant interaction. It should be noted as well that the π^+/π^- ratio approaches unity at large angles, indicating a high production multiplicity and a statistical, nonresonant interaction. With this distinction in mind we did not attempt to include all the data in the empirical fit, but limited the range to what appeared to be mainly central spectra; that is, to protons with laboratory momenta less than 20 BeV/c.

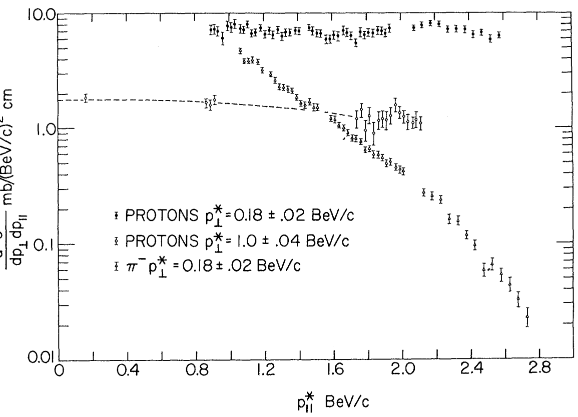

FIG. 3. $d^2\sigma/dp_\perp{}^* dp_\|{}^*$ vs $p_\|{}^*$ for pions and protons over the limited range of $p_\perp{}^*$ shown in the figure.

An analysis of the proton spectra is presented in the following Letter.

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INTERPRETATION OF HIGHLY INELASTIC PROTON-PROTON INTERACTIONS AT 10, 20, AND 30 BeV/c*

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A phenomenological analysis of the proton spectra resulting from inelastic proton-proton interactions is presented which suggests the existence of a bremsstrahlung-type process of pion production for highly inelastic collisions. A relation $\bar{n} = p_0{}^* \sin\theta^*/0.4$ is found between the implied average number of produced pions $\bar{n}$, the c.m. angle of scatter of the proton θ^*, and the c.m. incident-proton momentum $p_0{}^*$. A simple expression for the differential cross section for secondary protons yields excellent agreement with data over a wide range of angle, momenta, and incident-proton momenta.

The preceding article[1] (herein referred to as I) presents data on highly inelastic proton-proton interactions at incident proton momenta up to 30 BeV/c. We present a phenomenological analysis of these data in terms of a modified-statistical model. This analysis resulted from the observation that the c.m. momentum spectrum for secondary protons scattered at various angles showed marked similarity to phase-space momentum distributions calculated for systems containing two protons and $\bar{n}$ pions—i.e., secondary proton spectra at large (small) c.m. angles resembled phase-space distributions for systems with large (small) $\bar{n}$. In effect, the angle θ^* at which a proton inelastically scatters in the c.m. system (we use an asterisk to denote c.m. quantities) seems to be a sensitive and determining index of the collision process for nonperipheral interactions. Since the inferred pion multiplicity $\bar{n}$ can be related to θ^* as we shall see below, then having specified θ^*, the details of the proton and pion-momentum distributions, and perhaps even the angular correlations, appear to be described in their dominant features by phase space.

The multiple-pion production may, for example, be a "bremsstrahlung-type" process as proposed by Lewis, Oppenheimer, and Wouthuysen[2], and more recently modified by Kastrup,[3] to explain elastic proton-proton scattering. It was, in fact, this bremsstrahlung model that first suggested to us a relationship between the proton angle and the pion multiplicity.

Specifically, we take as the differential cross section for observing a proton scattered into $d\Omega_1{}^*$ with momentum $p_1{}^*$

$$\frac{d^2\sigma}{dp_1{}^*d\Omega_1{}^*} = \int \sigma_c(\theta_1{}^*\theta_2{}^*p_0{}^*n)\frac{1}{(K)^2n-1}\frac{dp_2{}^*}{E_1{}^*}\frac{d^3q_1{}^*}{\mathcal{E}_1{}^*}\cdots\frac{d^3q_n{}^*}{\mathcal{E}_n{}^*}\delta\Big(p_1{}^*+p_2{}^*+\sum_{i=1}^{n}q_1{}^*\Big)$$
$$\times\delta\Big(2[(p_0{}^*)^2+m_p{}^2]^{1/2}-E_1{}^*-E_2{}^*-\sum_{i=1}^{n}\mathcal{E}_i{}^*\Big), \qquad (1)$$

where $(p_2{}^*, E_2{}^*)$ and $(q_i{}^*, \mathcal{E}_i{}^*)$ are the four momenta of the unobserved proton and the ith pion, re-