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DECAY RATE AND SPECTRA IN $K_L^0 \to \pi^+ + \pi^- + \pi^0$ †

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In a study of 9629 decays of K_L^0, the branching ratio $[K_L^0 \to \pi^+ + \pi^- + \pi^0/K_L^0 \to$ all charged modes] has been measured as 0.161 ± 0.005. These decays have been selected from the data with a background from the K_L^0 leptonic modes of approximately 17%, and the energy spectrum of the π^0 meson fitted to a decay matrix element of the form $M = 1-(a_0/M_\pi^2)(S_3-S_0)$. The best fit value of the parameter a_0 was -0.249 ± 0.018.

The Brookhaven National Laboratory 14-in. liquid-hydrogen bubble chamber was exposed to a cleared neutral beam taken at 21° from the Brookhaven alternating-gradient synchrotron. Approximately 150 000 good quality photographs were obtained. From the number of K_L^0 decays observed in the fiducial volume and the known K_L^0 lifetime of 5.15×10^{-8} sec,[1] it was determined that approximately 18 long-lived neutral K mesons passed through the fiducial volume on each pulse, together with about six neutrons per K_L^0. Neutron interactions in hydrogen do not produce a background, and are not considered further.

All neutral decays (V's) observed in the chamber were measured on digitized microscopes and projectors, and reconstructed using the geometrical reconstruction program TRED.[2] All calculations were based on the measured variables, in that the beam momentum and the direction and momentum of the neutral decay product were not known, so that there were insufficient constraints to allow a kinematic fit, other than the zero-constraint variety. Approximately 18 000 such events were measured, of which 10 985 were inside a fiducial region chosen to reduce the background from such processes as $K_L^0 + p \to K_S^0 + p$ and $K_L^0 + p \to \Lambda^0 + \pi^+$. The final fiducial volume was 15.0 cm long, 4.5 cm wide, and 5.25 cm deep.

Three cuts were made on the data to purify the sample further. (1) The effective mass of each V interpreted as $\pi^- p$ was calculated, and those with 1108.86 MeV $< M_{\rm eff} \leq 1121.86$ MeV, consistent with the mass of the Λ^0, removed. (2) The effective mass of each V interpreted as $\pi^+\pi^-$ was calculated, and those with 469.2 MeV $< M_{\rm eff} \leq 529.2$ MeV, consistent with K_S^0 decay to two pions, removed. (3) All V's with opening angle less than 2° or greater than 165° were removed, to eliminate electron pairs and decay of charged background particles in flight.

For each event remaining after cuts (1) and (3), three kinematic variables were calculated: (1) the total visible momentum in the decay $\vec{p}_{\rm vis}$ [Fig. 1(b)]; (2) the Q value interpreting each charged track as a pion [Fig. 1(c)]; and (3) the variable

$$(p_0')^2 = \frac{(M_K^2 - M_0^2 - M_{12}^2)^2 - 4(M_0^2 M_{12}^2 + P_T^2 M_K^2)}{4(P_T^2 + M_{12}^2)},$$

where $M_K = K_L^0$ mass, $M_0 = \pi^0$ mass, M_{12} equals effective mass of the V interpreted as $\pi^+\pi^-$, and P_T equals transverse momentum of $\pi^+\pi^-$ [Fig. 1(d)].[3] Figure 1(c) clearly shows the peak of decays $K_S^0 \to \pi^+ + \pi^-$ with Q value 218.8 MeV; the shaded region has been removed and is not included in Figs. 1(b) or 1(d).

In order to understand the effects of the experimental resolution on the data, and to determine the beam momentum spectrum, a Monte Carlo calculation simulating the experiment was carried out. In this calculation the incoming K_2^0 beam momentum spectrum was taken as a variable to be determined by comparison with the data, as were the relative rates $K_{e3}:K_{\mu 3}:K_{+-0}$. The K_{e3} and $K_{\mu 3}$ decay interactions were assumed to be pure vector with no form factor variation,[4] and the ratio of the form factors[5] ξ was taken to be 0.9. A linear variation with T_0 in the squared matrix element in the K_{+-0} decay was assumed, with parameter $a_0 = -0.24$. The Monte Carlo program generated decays in the bubble chamber, from

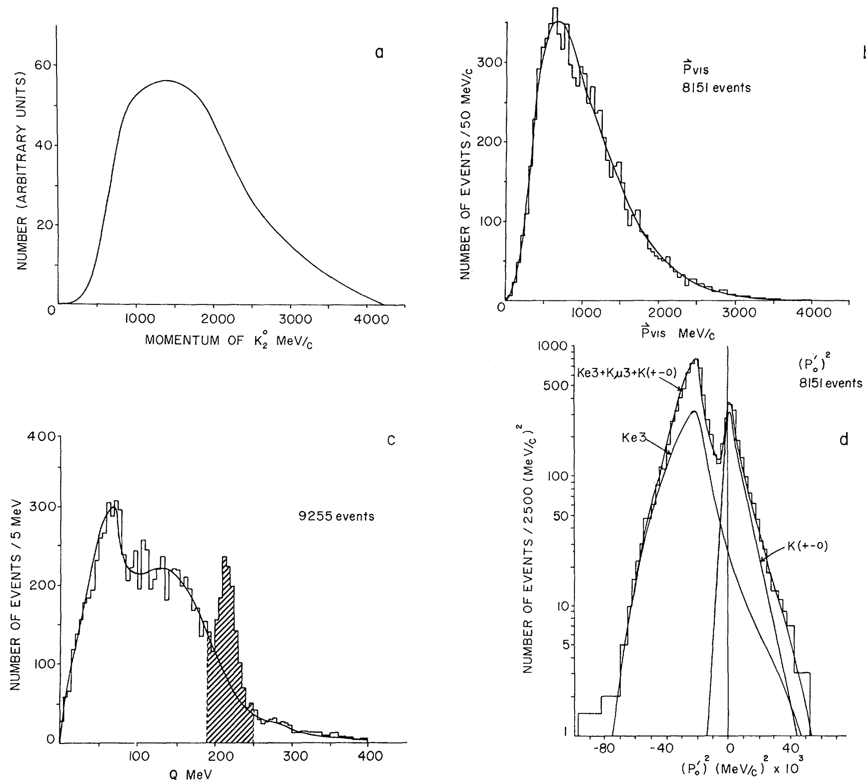

FIG. 1. (a) K_L^0 beam momentum spectrum. (b) Visible momentum in K_L^0 decays $\vec{p}_{vis}$. (c) Q value of decay interpreted as $\pi^+\pi^-$. Events which have 1108.86 MeV $< M_{eff}(\pi^-p) <$ 1121.86 MeV, or opening angle less than 2° or greater than 165°, are not included on this histogram. Events in the shaded region have been removed from the data used in the remainder of the analysis. (d) Distribution in $(p_0')^2$ (see text). Events which have 1108.86 MeV $< M_{eff}(\pi^-p) <$ 1121.86 MeV, or 469.2 MeV $< M_{eff}(\pi^-\pi^+) <$ 529.2 MeV, or opening angle less than 2° or greater than 165°, are not included on this histogram. The smooth curves on Figs. 1(a)-(d) are obtained from the Monte Carlo calculation.

the curvature and direction of the charged tracks and the position of the vertex calculated the errors in the same way as the geometry program,[6] and then swept the variables in a Gaussianly distributed way about the central calculated values.

The shape of the K_2^0 beam momentum spectrum was obtained by iterating the input spectrum to the Monte Carlo calculation, keeping the ratios $K_{e3}:K_{\mu3}:K_{+-0}$ fixed at 0.486:0.359:0.155,[7] until satisfactory agreement between the predicted $\vec{p}_{vis}$ spectrum and that observed was obtained [Fig. 1(b)]. The shape of the beam spectrum so derived is shown in Fig. 1(a). The total number of events generated by the calculation, after the appropriate cuts, was normalized to the number of experimental decays. The resulting distributions in $Q_{\pi\pi}$ and $(p_0')^2$ are shown as the smooth curves in Figs. 1(c) and 1(d). The agreement between the data and calculation is quite satisfactory.

To obtain the ratio $K_L^0(+-0)$/(all charged), Monte Carlo-generated distributions in $(p_0')^2$ for a range of ratios $K_{e3}:K_{\mu3}:K_{+-0}$ were fitted to the experimental distributions by the least-squares method. The best fit, with χ^2 probability 63 %, was with the relative branching ratio $K_{\mu3}/K_{e3} = 0.81 \pm 0.08$, and $K_L^0(+-0)/K_L^0(\text{all charged}) = 0.161 \pm 0.005$.

From the distribution in $(p_0')^2$ [Fig. 1(d)] it is clear that a good separation may be obtained between the $K_L^0(+-0)$ mode and the leptonic modes by making a $(p_0')^2$ cut in the data. The shape of the π^0 kinetic-energy spectrum (T_0) is quite sensitive to the point at which the cut is made, particularly the low-T_0 region which is strongly depleted as the cut moves to more positive $(p_0')^2$. Taking events with $(p_0')^2 \geq -5000$ (MeV/c)^2 included 98 % of the $K_L^0(+-0)$ decays, with the leptonic background contributing 24 % of the data in the region. In the calculation of T_0, events were rejected which had a beam momentum for both zero-constraint solutions outside the range 400 MeV/c-4 GeV/c. Approximately 17 % of the events with $(p_0')^2 \geq -5000$ (MeV/c)^2 were rejected for this reason or gave solutions which lay outside the kinematically allowed limits of the T_0 plot. The Monte Carlo calculation indicated that the final leptonic contribution to the T_0 spectrum was less than 17 %. The solid histogram of Fig. 2(a) shows the T_0 spectrum for a cut at -5000 (MeV/c)^2 after subtraction of the predicted leptonic background from the dashed histogram. Figure 2(b) is the spectrum of Fig. 2(a) divided by the phase space predicted by the Monte Carlo calculation for the same cuts as in the data. A least-squares fit of the squared matrix element

$$|M|^2 = 1 + 2a_0 \frac{M_k}{M_\pi^2} T_{\max} \times\left(\frac{2T_0}{T_{\max}} - 1\right) + a_0^2 \frac{M_K^2}{M_\pi^4} T^2_{\max}\left(\frac{2T_0}{T_{\max}} - 1\right)^2$$

to the points of Fig. 2(b) gives a value of $a_0 = -0.294 \pm 0.018$ if the quadratic term in $2T_0/T_{\max}-1$ is ignored (linear spectrum approximation), or $a_0 = -0.334 \pm 0.023$ if the quadratic term is included (linear matrix element).[8]

Since there is a correlation between the $(p_0')^2$ spectrum and the T_0 spectrum, in that a $(p_0')^2$ spectrum corresponding to uniform phase-space population is not identical to that spectrum coming from a population with a strong dependence on T_0, the value of a_0 obtained by the above method was checked by repeating the Monte Carlo calculation for $K_L^0(+-0)$ decays, with varying values of a_0. The best fits to the observed spectra for cuts in $(p_0')^2$ at both -5000 and 0 (MeV/c)^2 were obtained with a value of $a_0 = -0.30$.

The energy spectra of the charged pions in $K_L^0(+-0)$ decay has also been studied. In this case the ambiguity in the transformation from the laboratory to the K_L^0 rest system, which arises because the K_L^0 momentum is unknown, gives two solutions for the energy of the charged pions (the π^0 energy is unique). Events in which the discriminant in the calculation of the K_L^0 beam momentum became negative due to experimental errors were treated by setting this discriminant equal to 0. Figures 3(a) and 3(b) show the resultant charged-pion energy spectra divided by the prediction of the Monte Car-

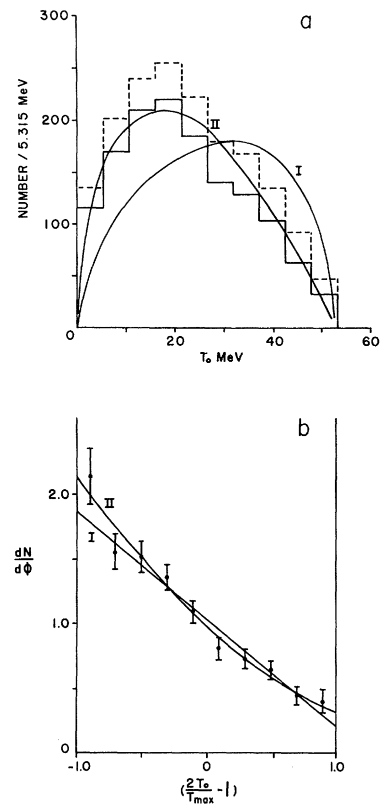

FIG. 2. (a) π^0 energy spectrum for events with $(p_0')^2 \geq -5000$ (MeV/c)^2. The broken histogram represents the data; the solid histogram, the data after subtraction of predicted leptonic background. The solid curve I represents phase space, and II, linear-spectrum approximation. (b) Number of events divided by phase space. The solid line I represents linear spectrum with $a_0 = -0.294$ and II, linear matrix element with $a_0 = -0.334$.

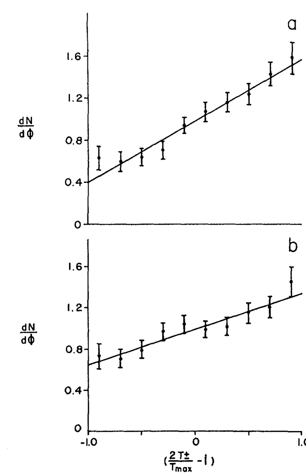

FIG. 3. (a) T_- spectrum divided by phase space. (b) T_+ spectrum divided by phase space. The straight lines are best-fit values giving $a_- = 0.21$ and $a_+ = 0.13$.

lo calculation for the uniform phase space. The expected kinematic reflection of the variation of the matrix element with T_0 is clearly visible; an independent fit of the form

$$\frac{dN}{d\varphi}=1+a_{\pm}\frac{2M_k}{M_\pi^2}T_{\max}\left(\frac{2T_\pm}{T_{\max}}-1\right)$$

to both π^+ and π^- spectra yields $a_+=0.13\pm0.02$, $a_-=0.21\pm0.02$, where the errors are purely statistical and do not take into account possible systematic effects from the kinematic ambiguity. In the absence of any CP-nonconserving effects we expect $a_+=a_-$; the statistical probability calculated from these errors that the two results are consistent is at the 1% level. We have combined the charged-pion data to give $a_{\rm ch}=0.168\pm0.011$; if there is no dependence of the matrix element on T_- or T_+, the the value[9] of $a_{\rm ch}$ deduced from a_0 is 0.154 ± 0.010, in good agreement with the observed value.

Previous measurements of the rate and spectra in $K_L^0\to\pi^++\pi^-+\pi^0$ have been summarized by Trilling.[7] The results in this paper replace our previous results quoted in his Ref. 32.[10] The branching ratio $(+-0)$/all charged $=0.161\pm0.005$ gives the rate $\Gamma_{+-0}=(2.50\pm0.12)\times10^6$ sec^{-1}, taking the K_L^0 mean life as $(5.15\pm0.14)\times10^{-8}$ sec,[1] and the branching (000)/all modes as 0.231 ± 0.02.[7] This ratio is in good agreement with the compilation of Ref. 7, excluding our previous figure, and with the more recent result of Hawkins,[11] 0.162 ± 0.015.

The prediction of the $|\Delta I|=\frac{1}{2}$ rule is that the rate $K_L^0\to\pi^++\pi+\pi^0$ is equal to twice the rate $K^+\to\pi^++\pi^0+\pi^0$, taking into account the relative phase-space factors. Using the phase-space factors of Ref. 7, and the rate $\Gamma_{+00}=(1.35\pm0.05)\times10^6$ sec^{-1}, we obtain

$$\frac{\Gamma_{+-0}/\varphi_{+-0}}{2\Gamma_{+00}/\varphi_{+00}}=0.94\pm0.06,$$

in excellent agreement with the prediction. We may alternatively express this result in terms of the amplitude ratio $r=S_1(\Delta I=\frac{3}{2})/S_1(\Delta I=\frac{1}{2})$, using the relation[8]

$$\Gamma_{(+-0)}/\Gamma_{(+00)}=2.06\left(\frac{S_1(\Delta I=\frac{1}{2})+S_1(\Delta I=\frac{3}{2})}{S_1(\Delta I=\frac{1}{2})-\frac{1}{2}S_1(\Delta I=\frac{3}{2})}\right)^2,$$

where S_1 indicates the transition amplitude to a symmetric final state with $I=1$, and obtain $r=0.02\pm0.02$. Weyers, Foldy, and Speiser[12] have used a current$\times$current Hamiltonian to calculate the ratio of the amplitudes in $K^+(+00)$ and $K_L^0(+-0)$, and find $A_2/2A_+=0.9$, that is, an intensity ratio 0.81. This differs by slightly more than two standard deviations from the results of this experiment. The $|\Delta I|=\frac{1}{2}$ rule also predicts that the parameter a_0 characterizing the variation of the matrix element with T_0 (T_+) is the same in $K_L^0(-0)$ decay and $K^+(+00)$ decay. There have been two recent determinations of this parameter in K^+ decay.[13,14] The result of this experiment, that $a_0=-0.294\pm0.018$, is not inconsistent with the linear spectrum value $a_0=-0.24\pm0.02$ of Ref. 13, and is in good agreement with the linear spectrum value of Ref. 14, $a_0=-0.30\pm0.05$. The fit by a linear matrix element, yielding $a_0=-0.334\pm0.023$, is in good agreement with the result of Ref. 14, that $a_0=-0.30^{+0.035}_{-0.045}$. Several authors[15,16] have recently derived theoretical expressions for the parameter a_0 using a theory of partially conserved axial-vector current. Abarbanel[15] derives $a_0=3M_\pi^2/(M_K^2-M_\pi^2)=-0.26$, and Hara and Nambu[16] give $S=-4_Q/M_K$ or $a_0=-0.24$. The result of this experiment favors the former value but cannot exclude the latter.

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ON PARTIAL SUMMETRY*

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In a recent Letter Schwinger[1] obtained the relation

$$\left|\frac{G_A}{G_V}\right| = \frac{5}{(3\sqrt{2})} \tag{1}$$

for the weak $n \to p$ transition, based on the juxtaposition of chiral SU(3)⊗SU(3) and of SU(6) summetry considerations. We wish to point out that this is easily understandable without referring to the specialized "partial symmetry" arguments of Ref. 1.

It is well known that chiral SU(3)⊗SU(3) symmetry leads to the relation[2]

$$\frac{g_{\pi N} m_\rho}{g_{\rho N} m_N} = \sqrt{2}\left|\frac{G_A}{G_V}\right|. \tag{2}$$

$SU(6)_w$, on the other hand, yields[3]

$$\frac{g_{\pi N} m_\rho}{g_{\rho N} m_N} = \frac{5}{3}. \tag{3}$$

Equation (2) is obtained in the "soft-pion" limit. As such, it is definitely incompatible with exact SU(6) summetry which requires $m_\rho^2 = m_\pi^2$. However, Eq. (3) is obtained in SU(6) theory with mass splittings included[3] when $m_\pi^2 \ll m_\rho^2$ is, of course, possible. It is therefore meaningful to require the compatibility of Eqs. (2) and (3), which leads precisely to Eq. (1).[4]

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[4]This fact was known to many people, including the author (see, e.g., B. W. Lee, in Proceedings of the Thirteenth International Conference on High Energy Physics (University of California Press, Berkeley, California, 1967), p. 60. It is easy to see that all further tests of partial symmetry suggested in Ref. 1 are identical to those obtained from standard $SU(6)_W$ and universality considerations.