ON THE DECAYS $\Sigma^{\pm} \rightarrow \Lambda^0 + e^{\pm} + \nu$†

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We have scanned 550 000 pictures of stopping K^- mesons in hydrogen for the decays $\Sigma^{\pm} \rightarrow \Lambda + e^{\pm} + \nu$, and find $\Sigma^- \rightarrow \Lambda + e^- + \nu$/all Σ^- decays to be $(0.64 \pm 0.12) \times 10^{-4}$ and $\Sigma^+ \rightarrow \Lambda + e^+\nu$/all Σ^+ decays to be $(0.20 \pm 0.08) \times 10^{-4}$. The rate for $\Sigma^- \rightarrow \Lambda + e^- + \nu$ yields a value of $D = 0.76 \pm 0.07$, and $\alpha = D/(D+F) = 0.64 \pm 0.06$. The observed electron-neutrino angular correlation together with the lambda polarization determines the magnitude of the ratio of vector to axial-vector coupling for this decay to be 0.31 ± 0.30, consistent with the hypothesis of conserved vector current, with the absence of second-class currents, and with time-reversal invariance.

We report here the results of a study of the decay modes $\Sigma^{\pm} \rightarrow \Lambda + e^{\pm} + \nu$.[1] The sigmas were produced via the reaction $K^- + p \rightarrow \Sigma^{\pm} + \pi^{\mp}$ in the 30-inch Brookhaven National Laboratory hydrogen bubble chamber, which was exposed to a stopping separated K^- beam at the Brookhaven alternating-gradient synchrotron. A total of 550 000 pictures were taken, and all have been scanned for any topology which might appear to be a charged Σ decaying into an electron and a Λ. It was not required that the Σ be produced by a K^- which had come to rest before interacting.

All such selected events were measured and put through the spatial reconstruction and kinematic fitting program TVGP-SQUAW and PACKAG. It was found that 51 Σ^- and 12 Σ^+ events fit the seven-constraint hypothesis

$$\begin{array}{l} K^- + p \rightarrow \Sigma^{\pm} + \pi^{\mp} \\ \qquad\quad \llcorner\!\rightarrow \Lambda^0 + e^{\pm} + \nu \\ \qquad\qquad\quad \llcorner\!\rightarrow \pi^- + p, \end{array}$$

with a χ^2 probability greater than 3%.

The largest sources of fake events which had been picked up in scanning were (a) those cases where a K^- interaction yields a Λ^0 and a Dalitz pair, with one of the electrons being fast and/or dipping so that it was mistakenly identified as a $\pi^{\mp}$ from the reaction $K^- + p \rightarrow \Sigma^{\pm} + \pi^{\mp}$ with the presumed Σ decaying immediately; (b) events of the type

$$\begin{array}{l} K^- + p \rightarrow \Sigma^+ + \pi^- \\ \qquad\quad \llcorner\!\rightarrow p + \pi^0 \\ \qquad\qquad\quad \llcorner\!\rightarrow \gamma + e^+ + e^-, \end{array}$$

where the e^- looks like a π^- which, together with the proton, fakes a Λ decay; and (c) events where a Σ^- is produced and interacts with a proton, in order to give a Σ^0 which in turn decays into a Λ^0 and a gamma ray, the latter producing a Compton electron within a short distance of the interaction point. The kinematic fitting procedure removed most of these fake events. The remainder were removed by requiring that the $\Sigma^{\pm}$ and the Λ each have a projected length greater than 1 mm, and that the fitted Σ^- momentum be greater than 80 MeV/c at decay. The events used in determining rates for the processes $\Sigma^{\pm} \rightarrow \Lambda + e^{\pm} + \nu$ were also required to satisfy two additional criteria: The absolute value of the sine of the dip angle of all charged tracks must be less than 0.96, and the projected length of the proton from the Λ decay must be greater than 1 mm. These last two cuts were applied because of unknown detection efficiency for events with such configurations. Of the events which had fit kinematics, 35 Σ^- and six Σ^+ events remained after cuts. These numbers must be corrected according to the results of a Monte Carlo calculation which showed that 44% of all Σ^- decays and 55% of all Σ^+ decays would have survived these cuts. In addition, we must correct our observed number of events for a scanning efficiency of 0.80 ± 0.08 and for the probability of 0.664 ± 0.011 for the Λ to decay into $\pi^- p$.[2] The scanning efficiency was determined by rescanning one quarter of the film. Both real $\Sigma^{\pm} \rightarrow \Lambda + e^{\pm} + \nu$ events, of the similar topology, $K^- + p \rightarrow \Lambda +$ Dalitz pair, where one of the electrons is fast, were used in estimating the efficiency. In order to check that the scanning was unbiased with respect to electron laboratory momentum, we have plotted that variable in Fig. 1. Superimposed on this is the expected electron-momentum distribution for a Σ^- of momentum 150 MeV/c. No momentum-dependent bias is apparent.

The over-all correction factors, including all the above effects, are 4.28 for Σ^- and 3.42 for Σ^+.

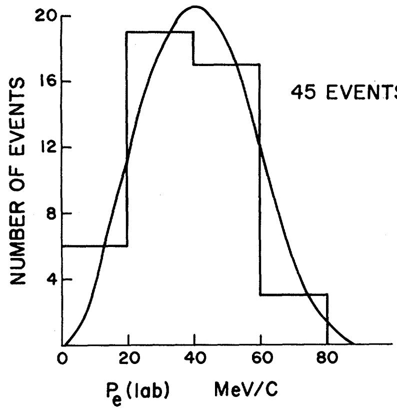

FIG. 1. Observed laboratory-momentum spectrum of the electron from the decay $\Sigma^- \to \Lambda + e^- + \nu$. The smooth curve is the expected spectrum for a Σ^- of momentum 150 MeV/c at decay.

The total number of Σ^+'s in our film was determined by counting the number of Σ^+'s decaying into protons on 5800 frames distributed throughout the run. In order to correct this number for zero-length Λ's from the reactions

$$K^- + p\begin{Bmatrix}\Lambda^0\\ \Sigma^0\end{Bmatrix} + \pi^0$$

which might fake a zero length $\Sigma^+ \to p + \pi^0$, all events with a Λ of projected length less than 2 mm were included in the nominal $\Sigma^+ \to p + \pi^0$ sample. Those Λ's with length 2 to 5 mm were also recorded, and from the number of such events, together with the expected distribution in Λ length as obtained from a Monte Carlo calculation, it was possible to predict the number of Λ's with length 0 to 2 mm which had been included in our sample of Σ^+ events. No distinction was made between Σ's produced by K's at rest or in flight. The number of Σ^-'s was determined by counting all other two-prong events and subtracting from this the expected number of $\Sigma^+ \to \pi^+ + n$ decays. This gives a denominator of $(2.35 \pm 0.12) \times 10^6$ Σ^-'s and $(1.04 \pm 0.07) \times 10^6$ Σ^+'s. We find the branching ratio $\Sigma^- \to \Lambda + e^- + \nu$/all Σ^- decays to be $(0.64 \pm 0.12) \times 10^{-4}$ and $\Sigma^+ \to \Lambda + e^+ + \nu$/all Σ^+ decays to be $(0.20 \pm 0.08) \times 10^{-4}$. These values are in agreement with the measurements of $(0.75 \pm 0.28) \times 10^{-4}$ and $(0.66 \pm 0.35) \times 10^{-4}$ for the Σ^- and Σ^+ branching ratios reported by Courant et al.[3]

It has been predicted that in the absence of second-class currents, the ratio of the rates $\Gamma(\Sigma^+ \to \Lambda + e^+ + \nu)/\Gamma(\Sigma^- \to \Lambda + e^- + \nu)$ should be just the phase-space ratio for the two decays, or $1/1.63 = 0.61$.[4] We observe this ratio to be 0.64 ± 0.28, consistent with the absence of second-class currents.

If one assumes that the decay $\Sigma^- \to \Lambda + e^- + \nu$ is governed only by the large axial-vector term in the interaction,[5] then in the framework of the Cabibbo theory for hyperon decay,[6] the branching ratio for this process may be written[7]

$$\frac{\Sigma^- \to \Lambda + e^- + \nu}{\text{all } \Sigma \text{ decays}} = 0.60 \times 10^{-4} \cos^2\theta (\tfrac{2}{3}) 3D^2.$$

Taking the Cabibbo angle θ to be 0.268,[8] our rate yields a value of $D = 0.76 \pm 0.07$. This, combined with the sum of $D + F = 1.18$ as obtained from neutron β decay,[9] gives

$$\alpha = D/(D+F) = 0.64 \pm 0.06.$$

This value for α is in agreement with several previous determinations from hyperon decay.[3,8,10,11] What is somewhat different, however, is that our determination of this parameter essentially depends only on the magnitudes of the $\Delta S = 0$ currents, since large percentage changes in the $\Delta S = 1$ currents will not affect α for small θ. Most theoretical predictions for α, ranging from SU(6) to approximate bootstrap calculations, are in agreement with the value measured here. More precise data on $\Delta S = 1$ hyperon leptonic decays are required to test the goodness of SU(3) invariance for the axial-vector currents, i.e., to compare $F_A(\Delta S = 1)$ and $D_A(\Delta S = 1)$ with the values of $F_A(\Delta S = 0)$ and $D_A(\Delta S = 0)$ given in this paper. In this connection, the recent report of Kim[12] that the KYN strong interactions satisfy SU(3) invariance with $\alpha = 0.61 \pm 0.02$ is of great interest.

The matrix element for the process $\Sigma \to \Lambda + e + \nu$ may be written, if we assume no second-class currents and neglect weak magnetism and induced pseudoscalar terms, as

$$\bar{u}_e \gamma_\mu (1 + \gamma_5) u_\nu \bar{u}_\Lambda \gamma_\mu (f + g\gamma_5) u_\Sigma.$$

According to the theory of conserved vector current, the form factor f should vanish at zero momentum transfer. If we square the matrix element and sum over all variables except the electron-neutrino opening angle and Λ spin assuming constant form factors f and g, we find that in the limit of nonrelativistic lambdas (p_Λ/m_Λ is at most 0.07), we may write the transition rate[13]

$$R \propto |f|^2+3|g|^2+(|f|^2-|g|^2)\cos\theta_{e\nu} + 2\hat{S}_\Lambda\cdot\{(\mathrm{Re}fg^*+|g|^2)\hat{k}_e+(\mathrm{Re}fg^*-|g|^2)\hat{k}_\nu + \mathrm{Im}fg^*(\hat{k}_e\times\hat{k}_\nu)\}.$$

Here, $\hat{k}_e$, $\hat{k}_\nu$, and $\hat{S}_\Lambda$ are unit vectors in the direction of electron momentum, neutrino momentum, and lambda spin in the Σ center of mass, respectively.

If we ignore information on the spin of the lambda, we may get one estimate of the parameter $x \equiv f/g$ by examining the distribution in $\cos\theta_{e\nu}$, which is given by

$$F(\cos\theta_{e\nu}) = N_{\text{events}}\frac{[x^2+3+(x^2-1)\cos\theta_{e\nu}]}{2(x^2+3)}.$$

We have plotted our experimental distribution in $\cos\theta_{e\nu}$ for 45 events in Fig. 2(a). For the purpose of examining the structure of the matrix element of the decay, an additional ten events have been used here which had been cut in determining the rate. They are good $\Sigma^- \rightarrow \Lambda+e^-+\nu$ events, and were cut previously only because of unknown detection efficiency for their peculiar topologies. A Monte Carlo calculation has shown that the dependence of detection efficiency on configuration in the laboratory has no noticeable effect on the center-of-mass distributions, and so these events have been included in this portion of the experiment. By doing a linear fit to our data, we find the best value of $|x| = 0.82 \pm 0.53$. Because F is a function of x^2 only, it cannot be used to determine the sign of x.

In Fig. 2(b) we have plotted the distribution in $T_\Lambda/T_{\Lambda\mathrm{max}}$, where T_Λ is the lambda kinetic energy in the sigma center of mass, a variable which is strongly correlated with $\cos\theta_{e\nu}$. The curved lines are the predicted spectra, assuming pure vector ($x=\infty$), pure axial vector ($x=0$), and equal amounts of V and A ($x=\pm1$). As in the case of the fit using $\cos\theta_{e\nu}$ alone, we find that although $x=\infty$ is ruled out, it is difficult to distinguish between $|x|=0$ and $|x|=1$. This is more clearly done by using information on the polarization of the lambda. We have constructed the likelihood function for the lambda decay distribution, given our distribution in $\cos\theta_{e\nu}$,

$$\mathcal{L}_x(\hat{q}_\pi|\cos\theta_{e\nu}) = \prod_{i=1}^{45}\tfrac{1}{2}\left\{1+\frac{2\alpha\hat{q}_{\pi_i}\cdot[(x+1)\hat{k}_{e_i}+(x-1)\hat{k}_{\nu_i}]}{x^2+3+(x^2-1)\cos\theta_{e\nu_i}}\right\},$$

where $q_{\pi_i} = \pi^-$ momentum in the Λ center of mass and $\alpha = \Lambda$ asymmetry parameter $= -0.66$.[2] It has been assumed in writing this expression that x is real, as required by time-reversal invariance. The natural logarithm of $\mathcal{L}_x(q_\pi|\cos\theta_{e\nu})$ is plotted in Fig. 3, yielding for f/g the result $x = -0.07 \pm 0.37$. The two independent determi-

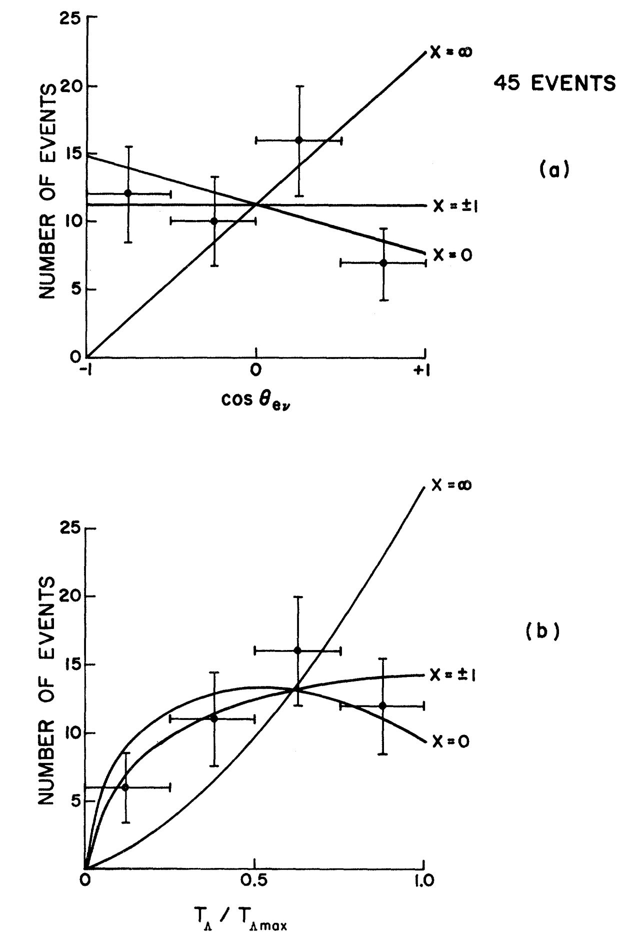

FIG. 2. (a) Distribution in electron-neutrino opening angle in the Σ^- center of mass for the decay $\Sigma^- \rightarrow \Lambda^+ + e^- + \nu$. (b) Distribution in lambda kinetic energy in the Σ^- center of mass divided by its maximum allowed value. The solid lines are the expected distributions for pure vector ($x=\infty$), pure axial vector ($x=0$), and equal amounts of V and A ($x=\pm1$).

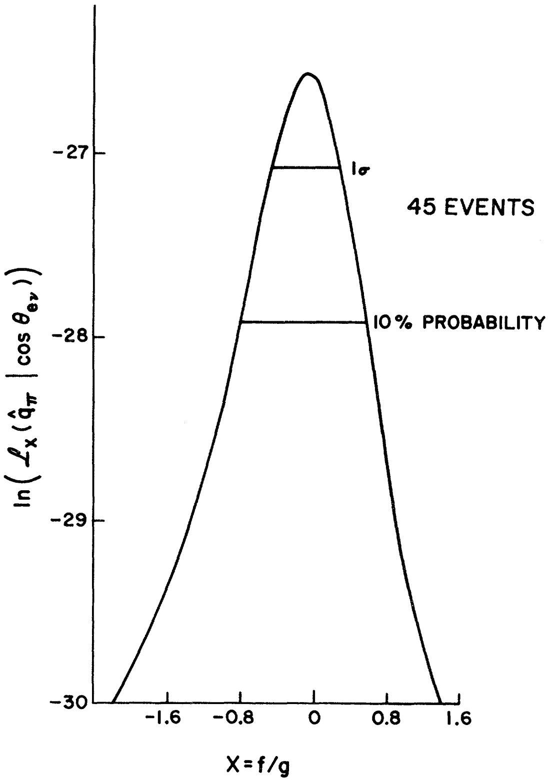

FIG. 3. Natural logarithm of the likelihood function for the lambda decay distribution versus the ratio X of vector to axial-vector currents. The result is $x = -0.07 \pm 0.37$.

nations of x, one using $\cos\theta_{e\nu}$ alone, and the other using Λ polarization, are consistent within their errors, and if we combine them to arrive at a best value for $|x|$, we find

$$|x| = |f/g| = 0.31 \pm 0.30.$$

In writing the likelihood function, we have used the total polarization vector of the Λ for each event. If we average over $\cos\theta_{e\nu}$ and measure the average component of Λ polarization along each of the three orthogonal directions defined by

$$\hat{\alpha} = \frac{\hat{k}_e + \hat{k}_\nu}{|\hat{k}_e + \hat{k}_\nu|}, \quad \hat{\beta} = \frac{\hat{k}_e \;\; \hat{k}_\nu}{|\hat{k}_e - \hat{k}_\nu|}, \quad \hat{\gamma} = \frac{\hat{k}_e \times \hat{k}_\nu}{|\hat{k}_e \times \hat{k}_\nu|},$$

we now compare our experimental data with the expected values

$$\langle \hat{\alpha} \cdot \hat{S}_\Lambda \rangle = \frac{8}{3} \frac{\mathrm{Re}(x)}{|x|^2 + 3},$$

$$\langle \hat{\beta} \cdot \hat{S}_\Lambda \rangle = \frac{8}{3} \frac{1}{|x|^2 + 3},$$

$$\langle \hat{\gamma} \cdot \hat{S}_\Lambda \rangle = \frac{\pi}{2} \frac{\mathrm{Im}(x)}{|x|^2 + 3}.$$

Experimentally, we find 0.10 ± 0.39, 1.25 ± 0.38, and 0.07 ± 0.39 for these three orthogonal components of polarization, this being in agreement with both the real and imaginary parts of x being zero. Our data are thus in agreement with the predictions of time-reversal invariance and the hypothesis of conserved vector current theory for this decay.

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DECAY RATE AND SPECTRA IN $K_L^0 \to \pi^+ + \pi^- + \pi^0$ †

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In a study of 9629 decays of K_L^0, the branching ratio $[K_L^0 \to \pi^+ + \pi^- + \pi^0 / K_L^0 \to$ all charged modes$]$ has been measured as 0.161 ± 0.005. These decays have been selected from the data with a background from the K_L^0 leptonic modes of approximately 17%, and the energy spectrum of the π^0 meson fitted to a decay matrix element of the form $M = 1 - (a_0/M_\pi^2)(S_3 - S_0)$. The best fit value of the parameter a_0 was -0.249 ± 0.018.

The Brookhaven National Laboratory 14-in. liquid-hydrogen bubble chamber was exposed to a cleared neutral beam taken at 21° from the Brookhaven alternating-gradient synchrotron. Approximately 150 000 good quality photographs were obtained. From the number of K_L^0 decays observed in the fiducial volume and the known K_L^0 lifetime of 5.15×10^{-8} sec,[1] it was determined that approximately 18 long-lived neutral K mesons passed through the fiducial volume on each pulse, together with about six neutrons per K_L^0. Neutron interactions in hydrogen do not produce a background, and are not considered further.

All neutral decays (V's) observed in the chamber were measured on digitized microscopes and projectors, and reconstructed using the geometrical reconstruction program TRED.[2] All calculations were based on the measured variables, in that the beam momentum and the direction and momentum of the neutral decay product were not known, so that there were insufficient constraints to allow a kinematic fit, other than the zero-constraint variety. Approximately 18 000 such events were measured, of which 10 985 were inside a fiducial region chosen to reduce the background from such processes as $K_L^0 + p \to K_S^0 + p$ and $K_L^0 + p \to \Lambda^0 + \pi^+$. The final fiducial volume was 15.0 cm long, 4.5 cm wide, and 5.25 cm deep.

Three cuts were made on the data to purify the sample further. (1) The effective mass of each V interpreted as $\pi^- p$ was calculated, and those with 1108.86 MeV $< M_{\text{eff}} \leq$ 1121.86 MeV, consistent with the mass of the Λ^0, removed. (2) The effective mass of each V interpreted as $\pi^+\pi^-$ was calculated, and those with 469.2 MeV $< M_{\text{eff}} \leq$ 529.2 MeV, consistent with K_S^0 decay to two pions, removed. (3) All V's with opening angle less than 2° or greater than 165° were removed, to eliminate electron pairs and decay of charged background particles in flight.

For each event remaining after cuts (1) and (3), three kinematic variables were calculated: (1) the total visible momentum in the decay $\vec{p}_{\text{vis}}$ [Fig. 1(b)]; (2) the Q value interpreting each charged track as a pion [Fig. 1(c)]; and (3) the variable

$$(p_0')^2 = \frac{(M_K^2 - M_0^2 - M_{12}^2)^2 - 4(M_0^2 M_{12}^2 + P_T^2 M_K^2)}{4(P_T^2 + M_{12}^2)},$$

where $M_K = K_L^0$ mass, $M_0 = \pi^0$ mass, M_{12} equals effective mass of the V interpreted as $\pi^+\pi^-$, and P_T equals transverse momentum of $\pi^+\pi^-$ [Fig. 1(d)].[3] Figure 1(c) clearly shows the peak of decays $K_S^0 \to \pi^+ + \pi^-$ with Q value 218.8 MeV; the shaded region has been removed and is not included in Figs. 1(b) or 1(d).

In order to understand the effects of the experimental resolution on the data, and to determine the beam momentum spectrum, a Monte Carlo calculation simulating the experiment was carried out. In this calculation the incoming K_2^0 beam momentum spectrum was taken as a variable to be determined by comparison with the data, as were the relative rates $K_{e3}:K_{\mu3}:K_{+-0}$. The K_{e3} and $K_{\mu3}$ decay interactions were assumed to be pure vector with no form factor variation,[4] and the ratio of the form factors[5] ξ was taken to be 0.9. A linear variation with T_0 in the squared matrix element in the K_{+-0} decay was assumed, with parameter $a_0 = -0.24$. The Monte Carlo program generated decays in the bubble chamber, from