p-p SCATTERING AT 90° AND HADRONIC STRUCTURE OF THE PROTON*

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We explain p-p differential scattering cross section at 90° and laboratory momentum 5-13.4 GeV/c in terms of two optical potentials and distorted-wave Born approximation of the corresponding amplitudes. The radial dependence of these potentials indicates two hadronic density distributions of the proton of root-mean-square radii 0.40 and 0.26 F.

Recently a sharp break in the p-p elastic differential scattering cross section was experimentally observed, and it was conjectured that this break might indicate the existence of two different inner regions of the proton.[1] Two theoretical explanations of the break have been proposed, both on the basis of singularities in the complex angular momentum plane.[2,3] In this note we propose a different explanation, which, in fact, supports the original conjecture regarding the structure of the proton.[1] The present explanation is based on an earlier work,[4] where it was pointed out from optical model considerations, that the nucleon appears to consist of a number of nucleon-matter (or hadronic) density distributions of increasing mean-square radii and that the inner distributions, which are associated with heavier quanta, dominate the large-momentum-transfer scattering.

We use the relativisitc eikonal description developed in I and assume that in the energy region of our interest, three different interactions are the most important. These interactions are represented by complex energy-dependent potentials. Of the three potentials, one is responsible for diffraction scattering. We do not explore the nature or form of this potential but parametrize its contribution in a standard way. The other two we take to be of the form

$$V_i(s,r)=g_i(s)\exp[-\mu_i(r^2+\beta_i^{\,2})^{1/2}]/(r^2+\beta_i^{\,2})^{1/2},$$
$$(i=1,2).$$

The radial dependence of this potential can be understood[4] as due to a hadronic density distribution of the proton which interacts with a similar distribution through a Yukawa potential $\exp(-\mu_i r)/r$ and whose finite size is determined by the parameter β_i.

The scattering amplitude for all energies and physical angles is represented by

$$f(s,\Delta)=ik\int_0^\infty bdb\, J_0(b\Delta)[1-e^{2i\delta(s,b)}], \tag{1}$$

where k is the c.m. momentum, $\Delta=2k\sin\frac{1}{2}\theta=(-t)^{1/2}$ is the momentum transfer, and s is the square of c.m. energy. The phase-shift function $\delta(s,b)$ is related to the "optical potential" $V(s,r)$ by

$$\delta(s,b)=-\frac{1}{2k}\int_b^\infty \frac{V(s,r)rdr}{(r^2-b^2)^{1/2}}. \tag{2}$$

If $\delta_0(s,b)$ represents the contribution of the diffraction potential to the phase-shift function, then we can write

$$\delta(s,b)=\delta_0(s,b)+\delta_1(s,b)+\delta_2(s,b), \tag{3}$$

where

$$\delta_i(s,b)=-\frac{1}{2k}\int_0^\infty V_i[s,(b^2+z^2)^{1/2}]dz$$
$$=-\frac{g_i(s)}{2k}K_0[\mu_i(b^2+\beta_i^{\,2})^{1/2}], \tag{4}$$

$(i=1,2)$. One expects[5] the diffraction potential to have an energy dependence such that $\delta_0(s,b)$ is independent of s for large values of s. Further, if $\mu_i\beta_i$ is much large than unity, then because of the sharp fall of the modified Bessel function $K_0(z)$, both $\delta_1(s,b)$ and $\delta_2(s,b)$ are going to be small quantities. Treating $\delta_1(s,b)$ and $\delta_2(s,b)$ as small, we obtain from (1), (3), and (4)

$$f(s,\Delta)=ik\int_0^\infty bdb\,J_0(b\Delta)[1-e^{2i\delta_0(b)}]-g_1(s)\int_0^\infty bdb\,J_0(b\Delta)e^{2i\delta_0(b)}K_0[\mu_1(b^2+\beta_1^{\,2})^{\frac{1}{2}}]$$
$$-g_2(s)\int_0^\infty bdb\,J_0(b\Delta)e^{2i\delta_0(b)}K_0[\mu_2(b^2+\beta_2^{\,2})^{\frac{1}{2}}]. \tag{5}$$

The three terms on the right-hand side of Eq. (5) have simple physical interpretations. The first term corresponds to the diffraction amplitude, which gives the exponential elastic diffraction peak for small momentum transfer. The second and the third terms correspond to distorted-wave Born approximation (DWBA) of the amplitudes due to potentials $V_1(s,r)$ and $V_2(s,r)$, respectively.[6-8] The factor $\exp[2i\delta_0(b)]$ occurring in the latter terms represents the absorptive correction coming from diffraction scattering.

For the diffraction amplitude, we now use the parametrization that $\delta_0(b)$ is completely imaginary and

$$1-e^{2i\delta_0(b)}=(\sigma_T{}^d/\pi R^2)e^{-2b^2/R^2},$$

where $\sigma_T{}^d$ is the total cross section due to diffraction scattering and R is the optical model radius. Inserting this formula in Eq. (5), we have

$$f(s,\Delta)=\frac{ik\sigma_T{}^d}{4\pi}\exp(-\tfrac{1}{8}R^2\Delta^2)-\sum_{i=1,2}g_i(s)\{\beta_iK_1[\beta_i(\Delta^2+\mu_i{}^2)^{1/2}](\Delta^2+\mu_i{}^2)^{-1/2}+\frac{\sigma_T{}^d}{\pi R^2}$$
$$\times\int_0^\infty bdb\,J_0(b\Delta)\exp(-2b^2/R^2)K_0[\mu_i(b^2+\beta_i{}^2)^{1/2}]\}. \qquad (6)$$

The elastic differential cross section[9] is given by

$$\frac{d\sigma}{dt}=\frac{\pi}{k^2}\frac{d\sigma}{d\Omega}=\frac{\pi}{k^2}|f(s,\Delta)|^2.$$

There are ten parameters occurring in (6); namely, $\sigma_T{}^d$, R, μ_i, β_i, Reg_i, Img_i $(i=1,2)$. We have ignored the energy dependence of g_1 and g_2 and have considered R as approximately known. Using Eq. (6), we have made a fit to the 90° fixed-angle elastic differential cross-section data of Akerlof et al.[1] Our calculated cross section is shown by the solid curve in Fig. 1 together with experimental points. The elastic differential cross sections which we obtain separately from the $i=1$ term (dash curve) and $i=2$ term (dot-dash curve) in Eq. (6) are also shown. We find the diffraction contribution, i.e., the first term on the right-hand side in (6), completely negligible. The values of the parameters obtained by us are given in Table I. The calculated curve follows the experimental points throughout the whole energy region, and the quantitative agreement is quite reasonable. We also find that the two amplitudes $i=1$ and $i=2$ in (6) interfere strongly and destructively throughout the whole range of k^2. The inteference is strongest near the region of experimental break where the two amplitudes as well as their sum have nearly the same magnitudes. The possibility that such

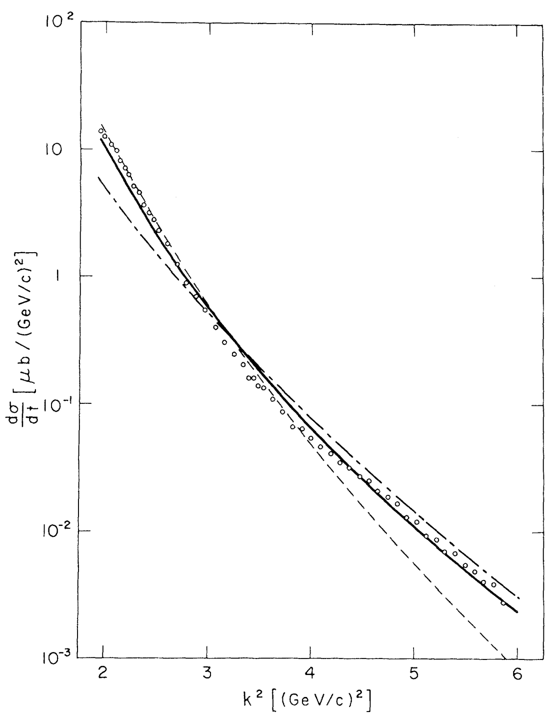

FIG. 1. Solid curve represents the calculated elastic differential cross sections. The circles are the experimental points of Akerlof et al. (Ref. 1). Dash line (dot-dash line) represents the elastic differential cross sections we obtained from the $i=1$ ($i=2$) term in Eq. (6).

Table I. Calculated values of the parameters.

i	μ_i (GeV)	β_i (F)	Reg_i (GeV/c)	Img_i (GeV/c)	$\sigma_T{}^d$ (mb)	R (F)
1	0.712	0.485	15.01	−2.90	36.2	1.1
2	1.559	0.400	−9.80	−11.78		

a strong destructive interference occurs in this region was noticed by Akerlof et al.[1] The need for destructive interference indicates that models where the amplitude is completely imaginary[10] will not produce a sharp break, even if the impact parameter amplitude $[1-e^{2i\delta(s,b)}]$ is taken as a sum of Gaussian functions in b^2.

When Δ becomes very large, because of the oscillations of the Bessel function, the main contribution to the integrals in Eq. (6) will come from small values of b, and so, as a good approximation, we can put the Guassian factors inside the integrals equal to unity. In that case the scattering amplitudes due to potentials $V_1(s,r)$ and $V_2(s,r)$ become

$$-g_i(s)(1+\sigma_T{}^d/\pi R^2)\beta_i K_1 \times[\beta_i(\Delta^2+\mu_i{}^2)]^{\frac{1}{2}}/(\Delta^2+\mu_i{}^2)^{\frac{1}{2}},\quad i=1,2,$$

i.e., exactly the same as the Born amplitudes except for a multiplicative factor. We now note that for Δ sufficiently large, the amplitude will behave as[11] $\sim\Delta^{-3/2}\exp(-\beta_2\Delta)$ and, therefore, will obey the Cerulus-Martin bound.[12] On the other hand, as noted in Ref. 1, Gaussian source functions will mean $d\sigma/dt\sim e^{-cs}$ and thus violation of this analyticity bound.

Figure 1 indicates that for small k^2 the amplitude due to potential $V_1(s,r)$ dominates, while for large k^2 that due to $V_2(s,r)$ dominates.[13] From our values of μ_i's and β_i's we find that the rms radii of the hadronic density distributions corresponding to the two potentials are $(\langle r^2\rangle)^{1/2}=0.40$ and 0.26 F, respectively. Following I, this will imply that for small k^2 the elastic scattering is dominated by a hadronic density distribution of the proton of rms radius 0.40 F, which interacts by exchanging a meson of mass ~700 MeV; for large k^2 the scattering is dominated by a second hadronic density distribution[14] of the proton of rms radius 0.26 F, which interacts by exchanging a much heavier meson of mass ~1500 MeV.

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[3] I. A. Sakmar and J. H. Wojtaszek, to be published.

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[5] This also underlies the earlier work of Serber on p-p scattering using an optical potential; see R. Serber, Phys. Rev. Letters 10, 357 (1963); Rev. Mod. Phys. 36, 649 (1964).

[6] K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964).

[7] R. C. Arnold, Phys. Rev. 153, 1523 (1967).

[8] The distorted-wave asymmetric Born approximation amplitude obtained here differs from the DWBA amplitude obtained from a single-particle-exchange potential in three important respects. In the present case, the coupling constant is complex and energy dependent, and the Born amplitude has form factors. In the other case, the coupling constant is real and energy independent, and the Born amplitude has no form factor. We feel that some of the difficulties of the single-particle DWBA amplitude at high energies can be removed if these modifications are done.

[9] We have disregarded spin complications. Also worth noting is that no symmetrization is needed for 90° scattering. [See, for example, D. Avison, Phys. Rev. 154, 1570 (1967)].

[10] For example, A. D. Krisch, Phys. Rev. 135, B1456 (1964); Lectures in Theoretical Physics (University of Colorado Press, Boulder, Colorado, 1966), Vol. IX.

[11] This follows from the asymptotic behavior of the modified Bessel function; namely, $K_1(z)\approx(\pi/2z)^{1/2}\times\exp(-z)$ when $z\to\infty$.

[12] F. Cerulus and A. Martin, Phys. Letters 8, 80 (1964).

[13] The repulsive real part of the potential $V_1(s,r)$ together with the fact that $\mathrm{Re}g_1$ is large and $\mu_1\sim700$ MeV suggests that it may be connected with the ω-meson exchange potential.

[14] It is interesting to note that Schopper, by assuming that the electromagnetic structure and the hadronic structure of the proton may be similar, has examined the magnetic form-factor data and finds indications of different inner regions of the proton (H. Schopper, to be published).