SIMPLE PROPERTY OF ELECTRON-ELECTRON COLLISIONS IN TRANSITION METALS

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The electrical and thermal resistivities observed by White and Tainsh[1] on very pure nickel provide a clear confirmation of theoretical estimates[2,3] that electron-magnon,[4,5] as well as electron-phonon, scattering should be much less important than electron-electron scattering at very low temperatures. The latter mechanism is known to yield an electrical resistivity[6-8] ρ proportional to T^2, and a thermal resistivity[8-10] W proportional (as for the case[11,12] of He^3) to T; these behaviors are observed[13] for nickel below 20°K.

The purpose of this Letter is to point out that these latest data on the electrical and thermal resistivities of nickel provide a test, though of course only a single one, for a simple and plausible hypothesis concerning the nature of electron-electron scattering in nickel and similar metals, a hypothesis which if correct can greatly facilitate the decipherment of a wide variety of electronic transport phenomena. Let $\vec{k}, s$ represent reduced wave vector and spin, respectively, for an electronic quasiparticle. A collision is described by $(\vec{k}, s; \vec{k}', s) \rightarrow (\vec{k}+\vec{q}, s; \vec{k}'-\vec{q}, s')$. The significant collisions are those for which all four wave vectors lie in a thermal neighborhood of the Fermi surface. For given $\vec{k}, \vec{k}+\vec{q}$ on the Fermi surface, the locus of possible $\vec{k}', s$ giving a $\vec{k}'-\vec{q}$ on the Fermi surface is some curve or curves on the Fermi surface. If we were to limit ourselves to non-Umklapp collisions on a spherical Fermi surface, there would be just one such curve. But for the many-sheeted Fermi surface of ferromagnetic nickel,[14] with Umklapp allowed, actual constructions for typical values of $\vec{k}$ and $\vec{q}$ have shown that there are usually 10 to 15 such curves, or even more. (Umklapp collisions should usually have sizable matrix elements because of the considerable hybridization[15] of the wave functions.) This suggests that the collision operator of the linearized quasiparticle transport equation[11,12] will pretty well wipe out any preference for a particular direction of velocity that may be possessed by the distribution function on which it acts.

The property just described leads to a very convenient mathematical formulation. In an obvious generalization of the familiar Fermi-liquid transport theory,[11,12] let the perturbed occupation number of the quasiparticle state $\vec{k}$ (including band and spin indices) be written as

$$n_k = f(\epsilon_k, T) + (\partial f/\partial \epsilon_k)\psi_k, \tag{1}$$

where f is the Fermi function, ϵ_k in the first term is the quasiparticle energy in the perturbed distribution, and ψ_k is infinitesimal. Then, to the first order in the perturbing fields it is easily shown that

$$\left(\frac{\partial n_k}{\partial t}\right)_{\text{collisions}} = \sum_{k'} (L_{kk'}\psi_{k'} - L_{k'k}\psi_k), \tag{2}$$

where $L_{kk'}$ describes the increment in the rate of arrival or removal at $\vec{k}$ due to the increments in the $n_{k'}$ for $\vec{k}' \neq \vec{k}$. The mathematical statement of our assumption is that for the ψ_k's that occur in electrical and thermal conduction,

$$\sum_{k'} L_{kk'}\psi_{k'} \approx 0. \tag{3}$$

In such case the electrical and thermal conduction are described by a $\vec{k}$-dependent relaxation time τ_k, where

$$\frac{1}{\tau_k} = \left(\frac{\partial f}{\partial \epsilon_k}\right)^{-1} \sum_{k'} L_{k'k}. \tag{4}$$

The condition $kT \ll$ bandwidths and Fermi energies allows one to separate τ_k into energy- and direction-dependent factors, the former being of the form familiar in Fermi-liquid theory,[11,12]

$$1/\tau_k = [(\pi kT)^2 + (\epsilon_k - \mu)^2]\Lambda_k, \tag{5}$$

where Λ_k is a function of direction, band, and spin, defined over the Fermi surface, and μ is the Fermi level.

The driving term $(\partial n_k/\partial t)_{\text{fields}}$, contains $(\partial f/\partial \epsilon_k)v_k$ dotted into $\vec{E}$ or $(\epsilon_k - \mu)\nabla T$, for the electrical or thermal case, respectively; $\vec{v}_k$ is the velocity of state $\vec{k}$. The coefficient of the perturbation of n_k in the expression for the electrical or thermal flux is proportional to $\vec{v}_k$ or $(\epsilon_k - \mu)\vec{v}_k$, respectively. Therefore, the

electrical and thermal conductivities, if evaluated to the leading order at low temperatures, involve the function Λ_k in the same manner, and the Lorenz ratio is given by a ratio of two energy integrals as

$$\frac{\rho}{WT} = \left(\frac{k}{e}\right)^2 \frac{\int_{-\infty}^{\infty} x^2(\pi^2+x^2)^{-1}(\partial f/\partial x)dx}{\int_{-\infty}^{\infty}(\pi^2+x^2)^{-1}(\partial f/\partial x)dx} = 1.45(k/e)^2 = 1.08\times 10^{-8}\ \mathrm{W}\ \Omega/\mathrm{deg}^2, \quad (6)$$

where $x = (\epsilon-\mu)/kT$. Note that no assumption other than (3) has been needed: The relative participation of different bands, etc., may be anything.

When the assumption (3) is not made, the Lorenz number can be quite dependent on the angular distribution of the scattering. The prediction (6) accords nicely with the observed value[1] 1.0×10^{-8} W Ω/deg^2, and this may be taken as significant, though of course not conclusive, evidence for the validity of (3). It would therefore be appropriate to attempt to correlate other transport data, for transition metals in the range where $\rho \propto T^2$, using a relaxation time of the form (5).

In the data of White and Tainsh[1] it is noteworthy that although $\rho \propto T^2$ only below 20°K, the Lorenz ratio is constant up to about 50°K. This would be understandable if the most important of the scattering mechanisms supplementing electron-electron scattering at the higher temperatures are also describable by a relaxation time with the energy dependence (5). This is, in fact, the case for any phonon- or magnon-scattering processes that have the property of of effectively destroying the initial velocity, either by Umklapp or by transitions to a high-mass band.[16,17]

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