Research.

[1]E. A. Guggenheim, J. Chem. Phys. 13, 253 (1945). More precisely, the left-hand side of Eq. (1) should be $(\rho_L-\rho_G)/2\rho_c$. The distinction is negligible for ^{3}He, for which the rectilinear diameter is constant. The form in the text simplifies data analysis, since values of ρ_L and ρ_G were not always measured at identical temperatures.

[2]For a comprehensive discussion, see M. E. Fisher, J. Math. Phys. 5, 944 (1964); L. P. Kadanoff et al., Rev. Mod. Phys. 39, 395 (1967).

[3]M. A. Weinberger and W. G. Schneider, Can. J. Chem. 30, 422 (1952).

[4]H. L. Lorentzen, Acta Chem. Scand. 7, 1335 (1953); in Statistical Mechanics of Equilibrium and Nonequilibrium, edited by M. Meixner (North-Holland Publishing Company, Amsterdam, 1965).

[5]M. H. Edwards and W. C. Woodbury, Phys. Rev. 129, 1911 (1963).

[6]R. H. Sherman, Phys. Rev. Letters 15, 141 (1965).

[7]H. M. Roder, D. E. Diller, L. A. Weber, and R. D. Goodwin, Cryogen. 3, 16 (1963).

[8]C. E. Chase and R. C. Williamson, in Critical Phenomena, edited by M. S. Green and J. V. Sengers (National Bureau of Standards, Washington, D. C., 1967), p. 197.

[9]R. H. Sherman and E. F. Hammel, Phys. Rev. Letters 15, 9 (1965).

[10]M. E. Fisher, Phys. Rev. Letters 16, 11 (1966).

[11]P. R. Roach and D. H. Douglass, Jr., Phys. Rev. Letters 17, 1083 (1966).

[12]C. E. Chase, E. Maxwell, and W. E. Millett, Physica 27, 1129 (1961); C. E. Chase and G. O. Zimmerman, Phys. Rev. Letters 15, 483 (1965).

[13]E. C. Kerr, Phys. Rev. 96, 551 (1954).

[14]E. C. Kerr and R. H. Sherman, in Proceedings of the Tenth International Conference on Low Temperature Physics, Moscow, 31 August-6 September, 1966 (to be published). We are grateful to Dr. Sherman for informing us privately of this problem well in advance of the conference.

[15]Kindly loaned by Dr. R. C. Williamson, National Aeronautics and Space Administration Electronics Research Center, Cambridge, Massachusetts

[16]H. van Dijk, M. Durieux, J. R. Clement, and J. K. Logan, Physica 24, S129 (1954).

[17]R. H. Sherman, S. G. Sydoriak, and T. R. Roberts, J. Res. Natl. Bur. Std. (U.S.) 68A, 579 (1964).

[18]We are grateful to Dr. Roach for sending us a copy of his computer program.

[19]S. G. Sydoriak and R. H. Sherman, J. Res. Natl. Bur. Std. 68A, 547 (1964).

STIMULATED EMISSION FROM THE UPPER-HYBRID RESONANCE IN A MAGNETOPLASMA*

R. M. Hill, D. E. Kaplan, and S. K. Ichiki
Lockheed Palo Alto Research Laboratory, Palo Alto, California

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We have observed coherent emission at the upper-hybrid resonance frequency from a magnetoplasma following excitation of the resonance by a microwave pulse. These results constitute the first quantitative measurement of this resonance as a function of electron density.[1] For propagation in the extraordinary mode across the static magnetic field B_0, the dispersion relation for a cold plasma predicts a resonance which is shifted relative to the free-electron cyclotron frequency $\omega_c = eB/m$. At low electron density the expression for this resonance, known as the upper hybrid, is given approximately by

$$\omega_{uh} \cong \omega_c + \omega_p^2 \sin^2\theta/2\omega_c, \tag{1}$$

where ω_p, the plasma frequency, is found from $\omega_p = (n_e e^2/m\epsilon_0)^{1/2}$ and θ is the angle between $\vec{B}_0$ and the wave vector $\vec{k}$. The frequency shift is thus proportional to the electron density n_e. At resonance the index of refraction for the plasma becomes infinite, and both phase and group velocity vanish. This should result in a peak in the absorption of a microwave signal propagating at this frequency. At the same time, the disturbance produced in the plasma at this frequency is stored locally before being dissipated by radiation, collision, or other losses.

The observation of the absorption peak has proved difficult in practice. In laboratory plasmas the radiation has to pass through regions of low electron density near the plasma boundary before penetrating the interior. Study of the dispersion relation shows that under these circumstances there exists a layer between the boundary and the resonance region in which the wave will not propagate. The upper hybrid is thus said to be inaccessible.[2] If this layer of evanescent propagation is narrow, some energy will tunnel through to the resonance region and upper-hybrid resonance is observed.[3] However, the amount of this tunneling is usually quite small, and explains the lack of quantitative observation of the resonance.

As an alternative technique the plasma may be excited by an intense short pulse near resonance. A short time after the end of the pulse the disturbance in the plasma will have died down except in a very narrow frequency band about the resonance. In spite of the weak coupling between the interaction region and the outside, we have been able to observe this "ringing" as a sharp, coherent emission signal.

The experiment is conducted in an argon afterglow plasma contained in a cubic glass bottle, 4 cm on a side. The magnetic field variation $\Delta B/B_0$ over the plasma volume is less than 0.03 %. Electron densities in the range 10^9-$10^{11}/\mathrm{cm}^3$ are selected by varying the time in the afterglow at which the excitation pulse is initiated. A microwave interferometer operated far above ω_c is used to measure electron density. The resonance is excited with a 10-W microwave pulse, 10 nsec in duration, and the resultant emission signal with a power level of order 1 μW is presented both on an oscilloscope and spectrum analyzer. For excitation in the extraordinary mode ($\vec{E}\perp B_0$), the principal results are as follows:

(a) For propagation normal to $\vec{B}_0$ ($\vec{k}\perp\vec{B}_0$), the experimentally determined frequency of the upper-hybrid resonance closely follows Eq. (1). For the cyclotron frequency equal to 9.144 GHz, the frequency difference $(\omega_{\mathrm{uh}}-\omega_c)/2\pi$ is given by $\Delta\nu = 4.403\times 10^{-3} n_e$ Hz. The data are shown in Fig. 1 and are seen to agree with Eq. (1) within experimental error.

(b) As the angle of propagation θ is decreased from $\pi/2$, the deviation of the upper-hybrid resonance from ω_c is expected to follow the equation $\Delta\nu \cong (\omega_p^2/4\pi\omega_c)\sin^2\theta$. We find experimentally that the frequency deviation does decrease monotonically with decreasing θ but more rapidly than expected. The explanation of this behavior remains obscure, but it may be due to depolarization effects.

(c) When the exciting pulse is applied at ω_c, intense stimulated emission at ω_c is observed at all electron densities. This is compatible with strong coupling to the low density of electrons very near the glass wall. The amount of energy stored and reradiated at ω_{uh} is about 10^{-3} of that observed at ω_c, thus showing the inaccessibility condition clearly.

(d) Stimulated emission at ω_c has been observed when the exciting frequency is at ω_{uh}. This indicates some coupling between the plasma disturbance at these two frequencies. This result is obtained most clearly when the frequency difference $\Delta\nu$ is in excess of 100 MHz. There is then a finite frequency region between ω_c and ω_{uh} in which an exciting pulse results in no observable emitted frequencies. As the frequency of the microwave pulse approaches ω_{uh}, and emission becomes observable, emission at ω_c reappears. The amplitude of the signal at ω_c reappears. The amplitude of the signal at ω_c is now proportional to the amplitude of ω_{uh}. This unexpected result will be studied further.

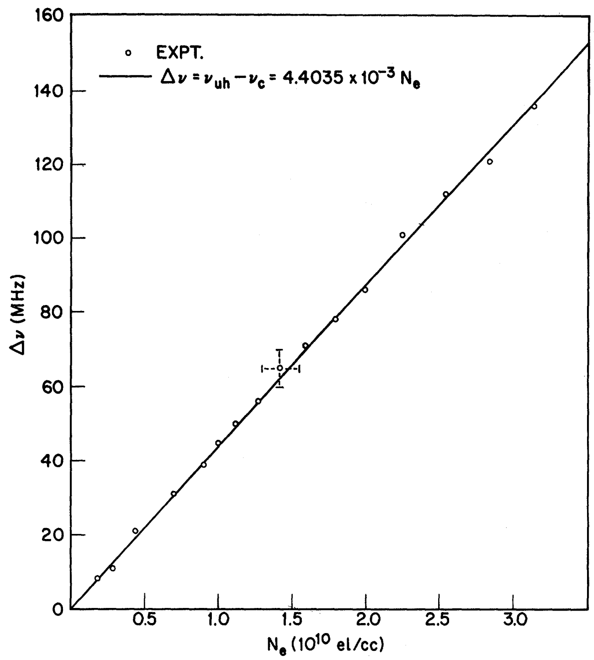

FIG. 1. The frequency deviation $\Delta\nu = \omega_{\mathrm{uh}}-\omega_c/2\pi$ as a function of electron density in units of $10^{10}/\mathrm{cm}^3$. The cyclotron frequency was 9.144 GHz. The error bars are typical for all data points.

The experiment illustrates a new and useful means for the study of collective phenomena in the neighborhood of the upper-hybrid resonance. The technique employed may also be readily adapted as a means for determination of electron-density conditions such that ω_{uh} is displaced from ω_c by at least one resonance linewidth.

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[1]A density-dependent absorption resonance was ob-

served qualitatively by G. Bekefi, J. D. Coccoli, E. B. Hooper, and S. J. Buchsbaum [Phys. Rev. Letters 9, 6 (1962)]. A. F. Kuckes and A. Y. Wong [Phys. Fluids 8, 1161 (1965)] predicted that the density-dependent peak in emission and absorption would occur at the cut-off frequency for which the dielectric constant = 0, as observed by S. J. Tetenbaum, to be published. The upper-hybrid resonance has been identified in solid-state plasmas at fixed density for frequencies in the far infrared, and $\omega_p > \omega_c$. S. Isawa, Y. Sawada, E. Burstein, and E. D. Palik [J. Phys. Soc. Japan Suppl. 201, 1742 (1966)], J. Phys. Soc. Japan Suppl. 21, 7 (1966).

[2]T. H. Stix, The Theory of Plasma Waves (McGraw Hill Book Company, Inc., New York, 1962), Vol. 6, p. 60-66. In problem 9 on p. 66, Stix points out that an increasing B in the direction of the observer can remove this inaccessibility. A. Y. Wong and A. F. Kuckes [Phys. Rev. Letters 13, 306 (1964)] showed this in a qualitative way.

[3]K. G. Budden, Radio Waves in the Ionosphere (University Press, Cambridge, 1961).

EFFECTIVE COORDINATE-SPACE POTENTIAL BETWEEN He^3 ATOMS IN SUPERFLUID He^4 †

L. J. Campbell
Department of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois
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Starting from the helium interatomic potential for isolated atoms and the difference between the volume per atom of He^3 and He^4 in solution, an effective He^3-He^3 interaction is derived and used to calculate the solution properties (specific heat, magnetic susceptibility, He^3 "sound velocity," spin diffusion, thermal conductivity, viscosity, and maximum solubility) for small concentrations of He^3.

A phenomenological momentum-space potential V_k was introduced recently by Bardeen, Baym, and Pines[1] (BBP) to account for newly measured properties[2] of dilute solutions of He^3 in superfluid He^4. A more microscopic calculation of the He^3-He^4 solution properties is outlined here. The calculation is phenomenological in that (1) the experimental values of the volume per atom in solution and the average nearest-neighbor distance are used, (2) an unknown He^4 effective mass was treated as a phenomenological parameter, and (3) following BBP, a self-consistent He^3 effective mass was chosen at zero concentration. The results, in general, support the potential V_k of BBP.

At very low temperatures, where almost all the He^4 is superfluid, the He^3 atoms are regarded as a gas in a neutral, nonviscous medium of constant density. Because of their lower mass and hence larger zero-point excursions, the He^3 atoms carve out of the liquid a larger volume per atom $\omega_3 = (1+\alpha)\omega_4$ than that of the He^4 atoms (ω_4); from Kerr's data[3] $\alpha \cong 0.28$. Both the effective mass of a single He^3 atom m_{03}^* and the effective interatomic potential $v_{\rm eff}$ between two He^3 atoms are influenced by the He^4 background.

For two He^3 atoms at $\vec{r}$ and $\vec{r}'$, $v_{\rm eff}(\vec{r}-\vec{r}')$ is a sum of three terms: (1) the induced potential arising from each He^3 atom interacting with the He^4 background through the bare He^3-He^4 potential v_{34}, (2) the induced potential stemming from the He^4 background interacting with itself through the bare He^4-He^4 potential v_{44}, and (3) the bare He^3-He^3 potential v_{33}. These three terms are individually large but mutually cancel to order α^2 for $|\vec{r}-\vec{r}'|$ larger than a couple of interparticle spacings. For small $|\vec{r}-\vec{r}'|$, the repulsive core of the bare He^3-He^3 potential dominates. Of course, since the above bare potentials are assumed to be those between isolated He atoms, they are all equal; so the subscripts will be dropped later.

The contribution of v_{34} to $v_{\rm eff}$ is found by integrating over the background the potential density $v_{34}(\vec{r}-\vec{s})d^3s/\omega_4$ of a He^3 atom at $\vec{r}$ (or $\vec{r}'$) interacting with a small volume d^3s of the background at $\vec{s}$. That is, the integral in $\vec{s}$ is over the volume of the system excluding the volumes, of size ω_3 each, occupied by the two He^3 atoms at $\vec{r}$ and $\vec{r}'$. The result is a constant plus a function of $\vec{r}-\vec{r}'$ which is the contribution of v_{34} to $v_{\rm eff}(\vec{r}-\vec{r}')$. This function is

$$-2(1+\alpha)v_{34}([\vec{r}-\vec{r}']_{34})g(\vec{r}-\vec{r}'), \quad (1)$$

where $[\vec{r}-\vec{r}]_{34}$ = maximum of $|\vec{r}-\vec{r}'|$ and the average nearest-neighbor distance between He^3