BACKWARD πN CHARGE-EXCHANGE SCATTERING AND POLE EXTRAPOLATIONS OF BARYON REGGE-EXCHANGE AMPLITUDES*

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High-energy $\pi^{\pm}p$ backward elastic-scattering data are analyzed in terms of N_α and Δ_δ Regge exchanges. Using the signs of the Regge residues found from extrapolations to the particle poles (which agree with the signs obtained from interference with direct-channel resonances), we predict the $\pi^- p \to \pi^0 n$ cross section near 180°.

Backward πN scattering at high energy provides a particularly suitable area for study of the fascinating properties of baryon Regge poles, e.g., (i) behavior of Regge-exchange amplitudes at wrong-signature nonsense points[1]; (ii) connection between exchange amplitudes and particle spectra (e.g., MacDowell symmetric parity doublets of resonances[2]); (iii) shrinkage of $d\sigma/du$[3]; (iv) asymptotic relations of the type[4]

$$\sigma(\pi^- p \to \pi^- p)_{\text{Backward}} = 2\sigma(\bar{p}p \to \pi^+\pi^-)_{\text{Forward}}.$$

Thus far, qualitative experimental indications have supported the general features expected from Reggeized baryon exchanges. The rapid increase in measurements[3,5] in these difficult experiments now makes a more detailed treatment possible.

In this Letter we present the results of an analysis of all available high-energy data[3,5] ($P_{\text{lab}} > 4$ BeV/c) on backward πN scattering using a simple Regge model proposed some time ago.[6] In essence, the model utilizes the following approximations: (a) N_α and Δ_δ exchanges only, (b) linear trajectories of the form $\alpha(\sqrt{u}) = a + bu$ as suggested by the N^* resonance spectrum,[6] (c) residues of minimal form[7]:

$$\gamma(\sqrt{u}) = \beta[1 + \delta(\sqrt{u}^{\frac{1}{2}})](1/s_0)^{\alpha(\sqrt{u}) - \frac{1}{2}},$$

(d) amplitudes that vanish at wrong-signature nonsense points.[8]

This construction possesses sufficient flexibility for investigation of Regge properties such as those outlined above. A least-squares fit to the $\pi^{\pm}p$ data has been used to determine the parameters (a, b, β, δ, and s_0) for each trajectory. Direct-channel resonances with fixed parameters and smoothly truncated tails were also included in the fit in an effort to represent the transition region where resonance effects are still non-negligible (at least up to 6 BeV/c).

In Fig. 1 the resultant fits are compared with experimental data at two representative momenta, 5.9 and 9.9 BeV/c. The model accounts reasonably well for all the high-energy data. The marked dip in the $\pi^+ p$ cross section near $u \simeq -0.2$ (BeV/c)^2 is interpreted as the wrong-signature nonsense zero of the N_α Regge amplitude at $\alpha_N(\sqrt{u}) = \frac{1}{2}$.[9] [At this point the $\pi^+ p$ cross section comes entirely from the Δ_δ exchange contribution in our model. Hence, near this u value, the model roughly predicts

$$\frac{d\sigma}{du}(\pi^- p) \simeq \frac{d\sigma}{du}(\pi^+ p)].$$

The values of the parameters of the exchange amplitudes obtained from the simultaneous analysis of the $\pi^{\pm}p$ backward scattering data with $P_{\text{lab}} > 4$ BeV/c are given below. Also shown are the trajectories obtained from straight-line extrapolations through the known resonance spectrum[6] and the values of the Regge residues at the physical-particle poles, $N_\alpha(938, \frac{1}{2}^+)$ and $\Delta_\delta(1236, \frac{3}{2}^+)$.[10]

For the (N_α, N_β) trajectory, the fitted values are

$$\text{Re}\,\alpha(\sqrt{u}) = -0.34 + 0.74u,$$

$$|\gamma(\sqrt{u})| = 20\left(1 + \frac{1.1}{M_N}\sqrt{u}\right) \text{ (BeV)}^{-1},$$

$$s_0 = 0.51 \text{ (BeV)}^2;$$

the Chew-Frautschi plot and Born pole give

$$\text{Re}\,\alpha(\sqrt{u}) = -0.39 + 1.01u,$$

$$\gamma(M_N) = -34 \text{ (BeV)}^{-1}.$$

For the $(\Delta_\delta, \Delta_\gamma)$ trajectory, the fitted values are

$$\text{Re}\,\alpha(\sqrt{u}) = 0.24 + 0.56u,$$

$$|\gamma(\sqrt{u})| = 0.13[1 + (1.0/M_{33})\sqrt{u}] \text{ (BeV)}^{-1},$$

$$s_0 = 0.93 \text{ (BeV)}^2;$$

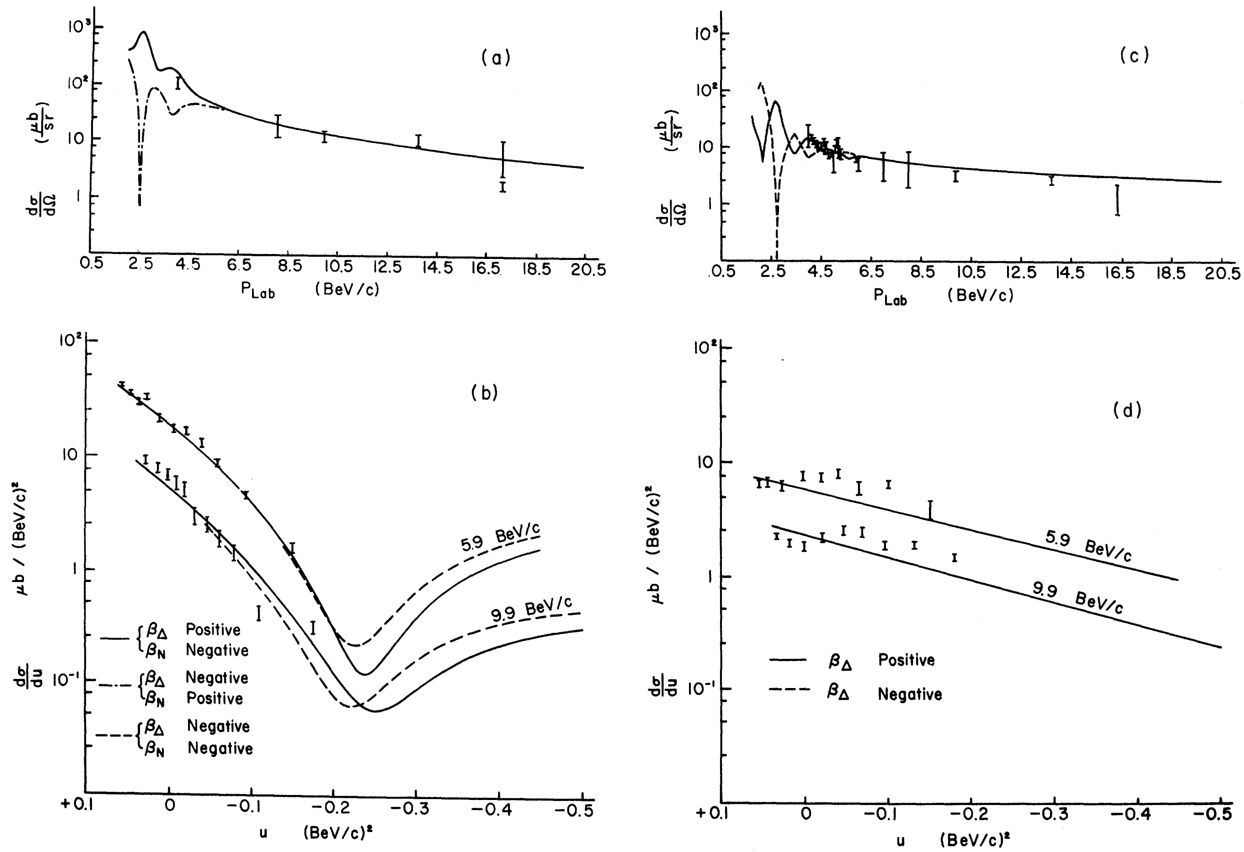

FIG. 1. N_α, Δ_δ Regge-exchange fits to $\pi^\pm p$ backward-scattering data. (a) $(d\sigma/d\Omega)_{\pi^+p}$ energy dependence at $\cos\theta = -1$ (data for $\cos\theta < -0.997$). (b) $(d\sigma/du)_{\pi^+p}$ vs u at $P_{\rm lab} = 5.9$ BeV/c and $P_{\rm lab} = 9.9$ BeV/c. The solid curves refer to the preferred solution (β_Δ positive and β_N negative—see text). Some alternative sign choices for β_Δ, β_N are also shown for comparison (cf., Ref. 12). Note that the π^+p solutions at high energy are insensitive to the sign of β_Δ/β_N. (c) $(d\sigma/d\Omega)_{\pi^-p}$ energy dependence at $\cos\theta = -1$ (data for $\cos\theta < -0.997$). The solid and dashed curves represent the two sign choices for β_Δ. Above 6 BeV/c, these curves coincide. (d) $(d\sigma/du)_{\pi^-p}$ vs u at $P_{\rm lab} = 5.9$ BeV/c and $P_{\rm lab} = 9.9$ BeV/c. Data on π^+p energy dependence are taken from Refs. 3 and 5; plotted data on angular distributions are taken from Ref. 3.

the Chew-Frautschi plot and Born pole give

$$\mathrm{Re}\,\alpha(\sqrt{u}) = 0.15 + 0.90u,$$

$$\gamma(M_{33}) = 2.6\ (\mathrm{BeV})^{-1}.$$

The following features are apparent:

(i) The trajectory intercepts at $u = 0$ are approximately the same as the extrapolations on Chew-Frautschi plots to $u = 0$.[6]

(ii) Both $\pi^\pm p$ show considerable shrinkage,[3] but not as much as expected from the slopes of Chew-Frautschi plots. Hence the trajectories have some curvature for $u < 0$ (analogous to the curvature of the ρ-meson trajectory for $t < 0$).

(iii) The N_α residue prefers a zero[11] at $\sqrt{u} \approx -M_N$ (presumably corresponding to the absence of a $\frac{1}{2}^-$ stable particle[2,9]) and the magnitude extrapolates approximately to the pole at $\sqrt{u} = M_N$, $\alpha = \frac{1}{2}$ [41 (BeV)$^{-1}$ compared with 34 (BeV)$^{-1}$].

(iv) The extrapolated magnitude of the Δ_δ residue at the pole ($\sqrt{u} = 1236$, $\alpha = \frac{3}{2}$) is much too small. Although this is a long extrapolation in the J plane, this result nevertheless suggests the desirability of an improved parametrization of the residue or trajectory function in this case. (The π^-p data at high energies show a hint of a turnover near 180° which is also not accounted for in the present parametrization.)

The high-energy $\pi^\pm p$ data are insensitive to the signs of the residues β_N and β_Δ of the N_α and Δ_δ amplitudes [see (c) above]. Three independent methods can be used in fixing the signs of these residues: (A) extrapolation of the N_α amplitude to the $(938, \frac{1}{2}^+)$ pole and the

Δ_δ amplitude to the $(1236, \frac{3}{2}^+)$ pole, assuming that the residues have no zeros in the $\sqrt{u}$ region of extrapolation; (B) interference of the exchange amplitudes with direct-channel resonances at intermediate energies[6]; (C) measurement of the backward $\pi^- p \to n\pi^0$ cross section (to determine the relative sign of the N_α and Δ_δ residues). The signs for β_N and β_Δ obtained from method A are reported above. In method B the nature of the dip-bump structure in the interference region (2-5 BeV/c) gives a qualitative determination of the residue signs, independent of quantitative details. The residue signs obtained from structure in backward $\pi^\pm p$ elastic scattering in this manner agree with the results of method A.[12] A very important consistency check on these signs can be obtained from the measurement of high-energy backward charge exchange (method C).

Figure 2 shows the cross-section predictions for πN charge exchange based on the Regge parameters given above. The results are surprisingly sensitive to the relative sign between the N_α and Δ_δ residues. The preferred signs from methods A and B above yield a charge-exchange cross section that is smaller than the $\pi^- p$ elastic cross section. In contrast, a positive sign for β_N/β_Δ leads to a $\pi^- p \to n\pi^0$ cross section as large as the $\pi^+ p$ elastic cross section.

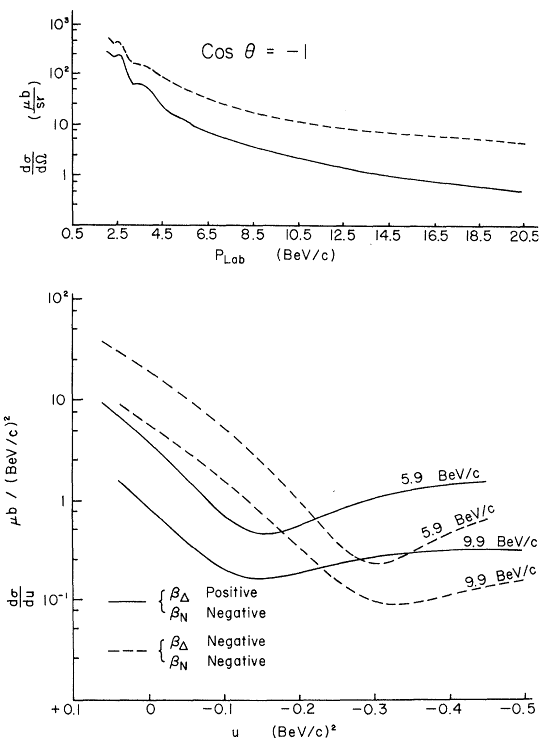

FIG. 2. Representative predictions for the $\pi^- p \to n\pi^0$ differential cross section near 180° as a function of momentum and angle. The preferred solution is denoted by the solid curves.

Perhaps the easiest experiment which could check the above prediction (and thus break the sign ambiguity) is

$$\pi^+ d \to p\pi^0[p]_S \text{ or } p\pi^+[n]_S,$$

where only the fast proton is observed. Here, $[N]_S$ denotes the spectator nucleon. The preferred solution gives a cross-section relation of

$$d\sigma(\pi^+ d)/d\Omega \simeq d\sigma(\pi^+ p)/d\Omega \text{ for } \cos\theta \simeq -1,$$

whereas the positive relative sign gives

$$d\sigma(\pi^+ d)/d\Omega \simeq 2 d\sigma(\pi^+ p)/d\Omega \text{ for } \cos\theta = -1.$$

The $\pi^+ d$ experiment does not suffer from normalization problems and may therefore be the best way to resolve the ambiguity.

The model presented in this Letter is undoubtedly somewhat oversimplified. However, we have explored other variations of the model (such as including N_γ exchange) and found nearly the same results near the backward direction where data are presently available; the predictions for backward charge-exchange scattering remained roughly the same near 180° as those given in Fig. 2.

As might be expected, the position and depth of the minima in $\pi^+ p$ are very sensitive to the precise details of the model. Thus, data through this u region for both $\pi^+ p \to p\pi^+$ and $\pi^- p \to n\pi^0$ should be extremely useful in studying the properties of the N_γ trajectory or additional $\sqrt{u}$ dependences of the N_α and Δ_δ amplitudes.

Present evidence strongly favors a negative relative sign of the N_α and Δ_δ residues. If, instead, charge-exchange measurements indicate that this relative sign is positive, then our present concepts regarding these Regge amplitudes would have to be significantly modified. A positive relative sign would also bring into question the validity of the interference model.

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[1]In the absence of fixed poles, the Regge amplitude vanishes at wrong-signature nonsense values of J [cf. S. Mandelstam and L. L. Wang, Phys. Rev. 160, 1490 (1967)].

[2]For discussion and references, see V. Barger and D. Cline, to be published.

[3]A. Ashmore, C. J. S. Damerell, W. R. Frisken, R. Rubinstein, J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan, and D. H. White, Phys. Rev. Letters 19, 460 (1967).

[4]A. Białas and O. Czyzewski, Phys. Letters 13, 337 (1964); V. Barger and D. Cline, Phys. Letters 25B, 415 (1967).

[5]W. R. Frisken et al., Phys. Rev. Letters 15, 313 (1965), and Phys. Rev. 152, 1162 (1966); T. Dobrowolski et al., Phys. Letters 24B, 203 (1967); C. T. Coffin et al., to be published; S. W. Kormanyos, Phys. Rev. Letters 16, 709 (1966); H. Brody et al., Phys. Rev. Letters 16, 828 (1966); A. S. Vovenko et al., to be published.

[6]V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966), and Phys. Rev. 155, 1792 (1967).

[7]Assuming single Regge-pole dominance for nonasymptotic values of u, unitarity in the u channel requires a change in sign of the residue between $\sqrt{u} = M+\mu$ and $\sqrt{u} = -(M+\mu)$ [cf. B. R. Desai, Phys. Rev. Letters 17, 498 (1966)]. The residue form $(1+\delta\sqrt{u})$ can accommodate such a sign change provided that $|\delta| > (M+\mu)^{-1}$.

[8]The explicit forms of the u-channel amplitudes are given in Eq. (15) of Ref. 6. All amplitudes contain the factors

$$(\alpha+\tfrac{1}{2})(\alpha+\tfrac{3}{2})\gamma\frac{1+i\tau e^{-i\pi\alpha}}{\cos\pi\alpha}(s)^{\alpha-\frac{1}{2}},$$

where τ is the signature of the trajectory. Note that for $\tau=+1$, the amplitude vanishes at $\alpha=-\frac{1}{2}$. This is a wrong-signature nonsense zero of the N_α amplitude.

[9]C. B. Chiu and J. D. Stack, Phys. Rev. Letters 19, 460 (1967).

[10]The residues at the poles are related to the πN coupling constant $(g^2/4\pi)=14.4$ by

$$\beta_N(1+\delta_N M_N) = -(3\pi/4)M_N\epsilon_N(g^2/4\pi) \quad \text{for } N_\alpha,$$

$$\beta_\Delta(1+\delta_\Delta M_\Delta) \simeq (3\pi/64)\epsilon_\Delta(s_0/M_\Delta)(g^2/4\pi) \quad \text{for } \Delta_\delta,$$

where $\epsilon_R = d\alpha/du$.

[11]Although the solution $\delta_N \approx 1\ \text{BeV}^{-1}$ was obtained at χ^2 minimum, acceptable fits were also found by fixing $\delta=0$. However, for $\delta=0$ the extrapolation to the nucleon pole is off by a factor of 2 and the condition in Ref. 7 is not satisfied.

[12]V. Barger and D. Cline, unpublished. For illustration of the application of method B, note the variation in shape of the solid and dashed curves in the intermediate momentum range (2-5 BeV/c) in Fig. 1. For this momentum range, the four sign choices $\beta_\Delta=\pm$, $\beta_N=\pm$ all give different solutions for π^+p because of resonance interference. At high energy ($P_{\text{lab}} > 6$ BeV/c), the fits are sensitive only to the sign of β_Δ/β_N and therefore only two different solutions are obtained. The present data on π^+p angular distributions at high energy do not distinguish the sign of β_Δ/β_N.