$$|\eta_{00}| = (4.17 \pm 0.30) \times 10^{-3}, \qquad (19)$$

and we then obtain

$$\text{from } K_L^0 \to \pi^{\pm} + e^{\mp} + \nu\text{: } \mathrm{Re}x = 0.14 \pm 0.06, \qquad (20)$$

$$\text{from } K_L^0 \to \pi^{\pm} + \mu^{\mp} + \nu\text{: } \mathrm{Re}x = -0.16 \pm 0.24. \qquad (21)$$

We have the following comments on results (20) and (21):

(a) A change in θ_{00} less than or equal to 5° implies a change in Rex less than or equal to 0.01. Hence Rex is rather insensitive to a change in θ_{00}.

(b) In the phenomenological analysis in Ref. 6 the sign of the pion-pion phase shift $\delta_2-\delta_0$ is opposite to the sign for this phase shift obtained from the analysis of pion production in pion-nucleon scattering. In Ref. 6 it was therefore proposed to take another solution to the data with $m_L - m_S < 0$. The phases θ_{+-} and θ_{00} then change their sign but the absolute values are not changed. Since Rex only depends on the cosine of the phases this sign ambiguity has no influence on our results.

(c) The two values (20) and (21) for Rex are in agreement within the error limits. The reason why Rex obtained from $K_L^0 \to \pi^{\pm} + \mu^{\mp} + \nu$ has a large error is that the experimental error in δ quoted in Ref. 3 is much larger than the experimental error quoted in Ref. 4. The weighted average of (20) and (21) is

$$\mathrm{Re}x = 0.12 \pm 0.06, \qquad (22)$$

suggesting that the selection rule $\Delta S = \Delta Q$ is violated by ~10%. This means that the weak hadronic current should contain a term with $\Delta Y/\Delta Q = -1$.

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SUM RULES FOR MULTIPION PRODUCTION PROCESSES*

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An additive quark model (together with an isospin distribution of charge configurations) is used to establish sum rules connecting pion-nucleon with nucleon-nucleon induced multipion production cross sections. The resulting relations are shown to be well satisfied by present experimental data.

Multipion production experiments have up to now shown that the distribution of an s-particle production reaction over the various charge configurations is essentially determined by isospin coefficients.[1] Given the cross section $\sigma(\pi^-p \to p2\pi^-\pi^+\pi^0)$, we write

$$\sigma(\pi^-p \to p2\pi^-\pi^+\pi^0) = \sigma(\pi N \to N4\pi)[\tfrac{2}{3}C(\tfrac{1}{2}, -\tfrac{1}{2}\,|p2\pi^-\pi^+\pi^0) + \tfrac{1}{3}C(\tfrac{3}{2}, -\tfrac{1}{2}\,|p2\pi^-\pi^+\pi^0)] \qquad (1)$$

to define $\sigma(\pi N \to N4\pi)$, the cross section for particles without charge and isospin. The coefficients[2]

$C(I, I_3 | Q_i^S)$ determine the projection of a particular s-particle charge configuration Q_i^S onto total isospin (I, I_3), with the sum overall (physically) different charge configurations normalized to unity

$$\sum_{\{i\}} C(I, I_3 | Q_i^S) = 1. \tag{2}$$

Expression (1) assumes no interference, the isospin independence of cross sections $(\sigma_{1/2} = \sigma_{3/2})$, and that the effects of resonances can at least for total cross sections be neglected.[1]

The cross sections for different charge configurations then are simply related by the isospin coefficients $C(I, I_3 | Q_i^S)$; we have, e.g.,

$$\frac{\sigma(\pi^- p \to p 2\pi^-\pi^+\pi^0)}{\sigma(\pi^- p \to n 2\pi^+ 2\pi^-)} = \frac{2C(\frac{1}{2}, -\frac{1}{2} | p 2\pi^-\pi^+\pi^0) + C(\frac{3}{2}, -\frac{1}{2} | p 2\pi^-\pi^+\pi^0)}{2C(\frac{1}{2}, -\frac{1}{2} | n 2\pi^+ 2\pi^-) + C(\frac{3}{2}, -\frac{1}{2} | n 2\pi^+ 2\pi^-)} = 1.69, \tag{3}$$

and similarly for other charge configurations with both π^+ and π^- as incident particles. These relations are generally well satisfied by experimental data[1]; we shall later on illustrate this for the particular case (3).

Whereas the isospin distributions of charge configurations gives, with $\sigma(\pi N \to N s\pi)$, connections between all s-particle production processes induced by πN interaction, the quark concept[3-6] allows an extension of such relations to couple reactions initiated by members of different SU(3) multiplets, e.g., by πN, ρN, or NN interactions.

It has been shown[7] that one can in this way predict multipion photoproduction cross sections from $\pi^\pm p$ data, making use of the vector-meson-dominance model to couple γ with ρ, ω, and then appealing to a quark picture to relate ρ, ω, and π. The predictions thus obtained also agree very well with experiment.[7]

In the present note we want to apply the additive quark picture for production processes[6] to connect $\sigma(\pi N \to N s\pi)$ with $\sigma(NN \to NN(s-1)\pi)$. Taking such processes (as in Ref. 6) to be given as (random phase) sum of two-quark interactions leading to production

$$qq' \to \tilde{q}\tilde{q}' + (s-1)\pi, \tag{4}$$

we immediately obtain the ratio 3:2 for NN to πN induced reactions: With nucleons as (qqq) systems and pions as $(q\bar{q})$, there are nine terms for NN and six for πN interactions; we assume the high-energy equality of all nonstrange qq and $q\bar{q}$ interactions. The laboratory energies at which to compare cross sections we choose in the ratio[4] $p_{\text{lab}}^N / p_{\text{lab}}^\pi = \frac{3}{2}$; for an equidistribution of energy among the three (two) quarks of the incident nucleon (pion), we compare in

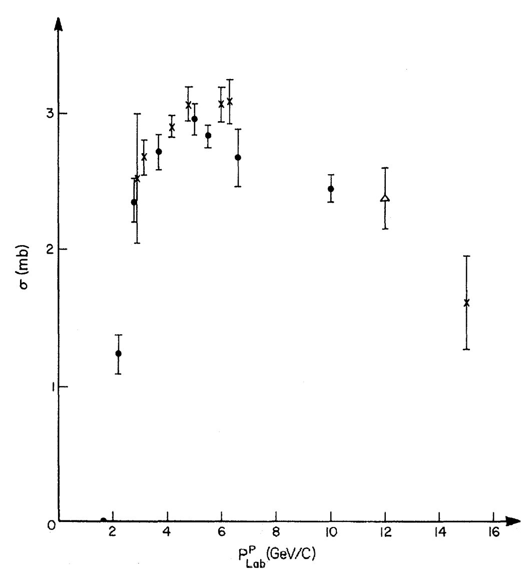

FIG. 1. Measured values of $\sigma(pp \to pp\pi^+\pi^-)$ (closed circles) compared with predictions calculated (crosses) from $\sigma(\pi^- p \to p\pi^+ 2\pi^-)$, (triangles) from $\sigma(\pi^+ p \to p 2\pi^+\pi^-)$ data.

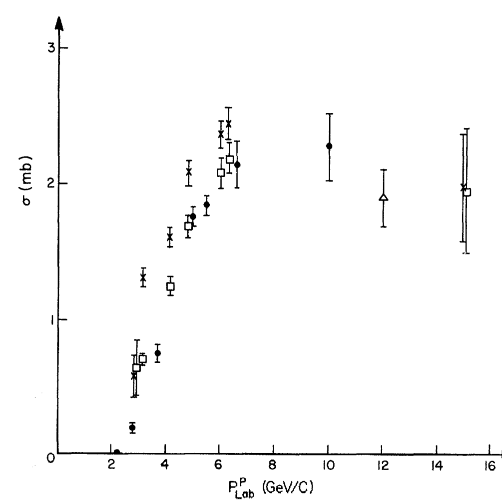

FIG. 2. Measured values of $\sigma(pp \to pp\pi^+\pi^-\pi^0)$ (closed circles) compared with predictions calculated (crosses) from $\sigma(\pi^- p \to p\pi^+ 2\pi^-\pi^0)$, (squares) from $\sigma(\pi^- p \to n 2\pi^+ 2\pi^-)$, (triangles) from $\sigma(\pi^+ p \to n 3\pi^+\pi^-)$ data.

this way quark-quark interactions at the same energy in both cases. Thus we have

$$\sigma_{NN\to NN(s-1)\pi}(P_{\rm lab}{}^{N}) = \tfrac{3}{2}\sigma_{\pi N\to Ns\pi}(P_{\rm lab}{}^{\pi} = \tfrac{2}{3}P_{\rm lab}{}^{N}). \qquad (5)$$

By taking into account the appropriate isospin coefficients, we obtain relations between specific charge configurations; e.g.,

$$\frac{\sigma(pp\to pp\pi^+\pi^-\pi^0)}{\sigma(\pi^-p\to p2\pi^-\pi^+\pi^0)} = \frac{3}{2}\,\frac{C(1,1\,|pp\pi^+\pi^-\pi^0)}{\frac{2}{3}C(\frac{1}{2},-\frac{1}{2}\,|p2\pi^-\pi^+\pi^0)+\frac{1}{3}C(\frac{3}{2},-\frac{1}{2}\,|p\pi^-\pi^+\pi^0)} = 1.12 \qquad (6)$$

at energies as given in (5). Possible deviations at low energies and/or low particle numbers can occur because of significant resonance contributions or because of (nonadditive) baryon exchange contributions[5] in the πN system.

Multipion proton-proton data are presently available for the reactions[8]

$$pp\to pp\pi^+\pi^-$$
$$\to pp\pi^+\pi^-\pi^0$$
$$\to pn2\pi^+\pi^-$$

up to 10 GeV/c $P_{\rm lab}{}^{p}$. Figures 1-3 show these data together with our predictions from πN data.[9] In all channels the agreement is found to be very good. The approximate equality of the predictions from different πN charge configurations incidentally shows how well the isospin distribution assumption (3) is satisfied.

In conclusion: The additive quark model for production processes together with the isospin distribution of charge configurations give one-to-one connections between multipion final states from NN, πN, and (with vector-meson dominance) γN connections which in all cases are remarkably well fulfilled by experimental data, and which can hardly be explained without a quark picture. We find this to be yet another argument calling for a better understanding of the quark concept.

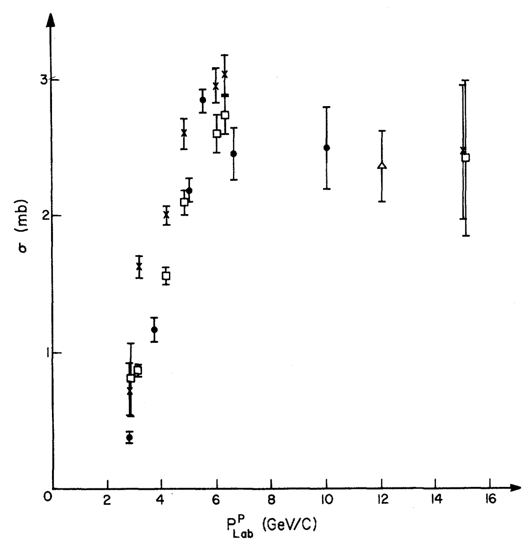

FIG. 3. Measured values of $\sigma(pp\to pn2\pi^+\pi^-)$ (closed circles) compared with predictions calculated (crosses) from $\sigma(\pi^-p\to p\pi^+2\pi^-\pi^0)$, (squares) from $\sigma(\pi^-p\to n2\pi^+2\pi^-)$, (triangles) from $\sigma(\pi^+p\to n3\pi^+\pi^-)$ data.

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