We can compare the results with the single-particle estimate of Moszkowski[4]:

$$\frac{B_I(L\to 0)}{e^2}=\frac{1}{4\pi}\left(\frac{3}{3+L}\right)^2 R^{2L}.$$

The value $G=B_{\exp}/B_I$ gives a measure of the collective character of the excitation. To take into account the deformation of the wave function in the vicinity of the nucleus, we must introduce in formulas (1) and (2) the exact value of R given by the phase-shift analysis of the elastic data. Our results are given in Table I and compared with those given by Coulomb excitation[5] and by nuclear resonance scattering of γ rays.[6,7]

Our study emphasizes the electromagnetic properties in spherical nuclei with an incomplete neutron shell. These properties are proportional to the components of the proton wave functions in the closed shells and vary appreciably from one isotope to another. From this point of view, although electromagnetic and nuclear interactions are different, it is interesting to compare collective multipole excitations given by inelastic scattering of electrons and α particles.[8]

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QUANTITATIVE STUDIES OF NUCLEAR STRUCTURE THROUGH ISOBARIC ANALOG RESONANCES*

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Since the discovery of isobaric analog resonances in heavy nuclei,[1] it has been realized that important nuclear-structure information could be derived from a study of the various decay widths of these resonances.[2] In particular, the elastic proton width is expected to be simply related to the spectroscopic factor of the corresponding parent analog state. In this Letter we present a method suitable for the quantitative extraction of spectroscopic factors and apply it to study the low-lying states in ^{209}Pb. The analysis is done in two steps: First, a single-particle potential is found that gives an adequate description of the low-lying states in ^{209}Pb. This potential, together with a charge-exchange term of known strength, completely determines the potentials occurring in the Lane equations,

$$\{K+U_0+\Delta_C+\tfrac{1}{2}(T_0-1)V_1-E_p\}\varphi_{nA}=-(\tfrac{1}{2}T_0)^{1/2}V_1\varphi_{pC}, \tag{1a}$$

$$[K+U_0+V_C-\tfrac{1}{2}T_0V_1-E_p]\varphi_{pC}=-(\tfrac{1}{2}T_0)^{1/2}V_1\varphi_{nA}. \tag{1b}$$

Here U_0 stands for a Saxon-Woods potential with a spin-orbit term. The charge-exchange interaction is taken to be $(\vec{t}\cdot\vec{T}_0)V_1(r)$ with $\vec{t}$ and $\vec{T}_0$ denoting the isospin operators of the extra nucleon and core nucleus, respectively. Finally, the Coulomb displacement energy is denoted by Δ_C and the Coulomb potential by $V_C(r)$. These equations have been solved numerically by a number of authors.[3]

Instead of resorting directly to numerical methods, we first replace the coupled equations by the

corresponding integral equation. This is easily done with Feshbach's projection-operator technique[4]:

$$\varphi_{pC}^{(+)} = \chi_{pC}^{(+)} + G_{pC}^{(+)} U_1 G_{nA} U_1 \varphi_{pC}^{(+)}, \quad (2)$$

with $U_1 = (\vec{t}\cdot\vec{T})V_1$. In obtaining this expression, due account has been taken of the fact that at proton energies of interest the (nA) channel is at negative energies; furthermore, we are considering a particular partial wave. $\chi_{pC}^{(+)}$ is a solution of the homogenous part of (1b) and $G_{pC}^{(+)}$ is the Green's function of the same equation. The Green's function G_{nA} for the homogeneous part of (1a) can be expanded in terms of the bound and continuum states of the corresponding Hamiltonian. We retain only one of the terms due to bound states and delete the contributions from the (nA) continuum entirely:

$$G_{nA}^{(+)} \cong \varphi_{nA}^{(m)} \rangle \langle \varphi_{nA}^{(m)} / (E_p - E_m).$$

The justification for this approximation is that states of the same spin and parity are generally well separated, and the contributions from the (nA) continuum are expected to be small in general. This expression for $G_{nA}^{(+)}$ and the solution of the homogeneous part of (1b) suffice to calculate $\varphi_{pC}^{(+)}$ explicitly. Finally, the scattering function for the proton channel takes the form

$$S_p = e^{2i\eta}\left[1 - \frac{i\Gamma^{(A)}}{E - E_0 + \frac{1}{2}i\Gamma^{(A)}}\right].$$

η is the phase shift occuring in the asymptotic form of $\chi_{pC}^{(+)}$ and E_0 is the resonance energy, including the level shift. The quantity of primary interest is the elastic proton width of the isobaric analog resonance. In the present framework, it is also the total width and can readily be calculated[4]:

$$\Gamma^{(A)} = (k_p/E_p) T_0 \langle \varphi_{nA} | V_1 | \chi_{pC}^{(+)} \rangle^2. \quad (3)$$

The second step in our analysis consists in obtaining from the experimental data the elastic proton widths of isobaric analog states in ^{209}Bi. A theory of isobaric analog resonances that includes the effects arising from the $T_<$ states was first given by Robson[2] using the R-matrix theory. In a series of papers, Weidenmüller and Maxauh[5] have recently developed a theory of isobaric analog resonances starting from a shell-model type of description of nuclear reactions. In addition to the isospin-forbidden neutron decays, these theories take into account the isospin-allowed inelastic proton decay of the isobaric analog resonances. We use the expressions for the energy-averaged scattering matrix obtained from Ref. 5. For elastic scattering from spin-zero targets, we can write

$$\langle S_\lambda \rangle = \exp(2i\delta_\lambda) - i\exp(2i\alpha_\lambda)\frac{\Gamma_\lambda}{E - E_R + \frac{1}{2}i\Gamma}, \quad (4)$$

where

$$\Gamma_\lambda = s\Gamma_\lambda^{(A)}\left[\frac{1 + 2iY_\lambda(\tilde{\Delta}_\lambda/\tilde{\Gamma}_\lambda)}{1 + Y_\lambda}\right]^2$$

and

$$\exp(2i\delta_\lambda) = \frac{1 - Y_\lambda}{1 + Y_\lambda}\exp(2i\alpha_\lambda).$$

Here δ_λ is the optical-model phase shift and α_λ is its real part. s is the spectroscopic factor of the parent analog state. $\tilde{\Gamma}_\lambda$ is the width of the single-particle resonance displayed by solutions of the equation

$$[K + U_0 + V_C + \tfrac{1}{2}T_0 V_1 - E_p]\psi_p = 0.$$

Finally $\tilde{\Delta}_\lambda$ is the amount by which this single-particle resonance is shifted from the energy commonly known in the literature as $E_>$.[2] It is the eigenvalue of the homogeneous part of (1a).

We found an optical potential U_0 that reproduced the binding energies of the $g_{9/2}$, $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$ states in ^{209}Pb. The observed and calculated binding energies of these states are compared in Fig. 1. The parameters characterizing this potential are $V_0 = 50.9$ MeV, $r_0 = 1.19$ F, $a = 0.75$ F, and $V_{S.O.} = 5.8$ MeV. The form factor for the charge-exchange potential V_1 was chosen to be of volume type with a strength of 0.5 MeV. This corresponds to a symmetry term $26[(N-Z)/A]$ MeV in the proton potential and is in agreement with the values obtained from the analysis of charge-exchange (p,n) reactions.[6] The functions φ_{nA} and $\chi_{pC}^{(+)}$ were obtained by the numerical integration of the homogeneous parts of Eqs. (1a) and (1b). These functions were corrected for the nonlocality of the optical potential.[7] The nonlocality length used was 0.85 F. Finally, the the-

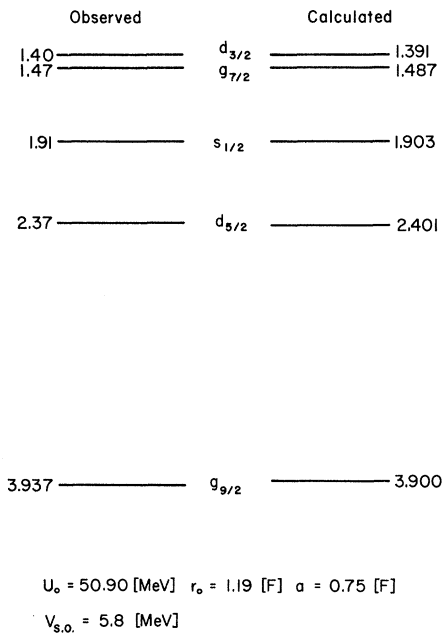

FIG. 1. Comparison of the observed and calculated binding energies of the $g_{9/2}$, $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$ states in ^{209}Pb.

oretical partical widths $\Gamma^{(A)}$ were calculated using expression (3). The theoretical excitation functions were generated using expression (4) for the scattering matrix. The depth and geometry of the real part of the proton potential were fixed in accordance with the neutron potential and symmetry term. A surface type of imaginary potential was used for the protons, and its diffuseness and strength changed until the observed background was well reproduced at all four scattering angles of 90°, 125°, 150°, and 170°. Total widths and resonances energies were treated as parameters and their values adjusted to obtain agreement between calculated excitation functions and experimental data.[8] The calculated excitation functions are compared with the elastic scattering data in Fig. 2. Values of the spectroscopic factors necessary to obtain a fit to the data are given in the second row of Table I.

Table I. Spectroscopic factors of the $g_{9/2}$, $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$ states in ^{209}Pb for three different values of the radius parameter r_0. The diffuseness and spin-orbit strength were held fixed at the values in Fig. 1. The values in the second row correspond to a geometry that reproduces the observed binding energy of these states very well.

r_0	$g_{9/2}$	$d_{5/2}$	$s_{1/2}$	$g_{7/2}$	$d_{3/2}$
1.17	1.07	0.88	0.93	0.89	0.88
1.19	0.97	0.85	0.90	0.84	0.86
1.23	0.79	0.78	0.84	0.73	0.80

The calculated partial width $\Gamma_\lambda^{(A)}$ depends to some extent on the geometry of the single-particle potential chosen to describe the parent analog state. Consequently, the extracted spectroscopic factors also depend upon it. The first and third rows in Table I show spectroscopic factors obtained for the radius parameter $r_0 = 1.17$ and $r_0 = 1.23$. Values obtained for the $g_{9/2}$ state depend rather strongly on the choice of the radius parameter r_0. Penetration fac-

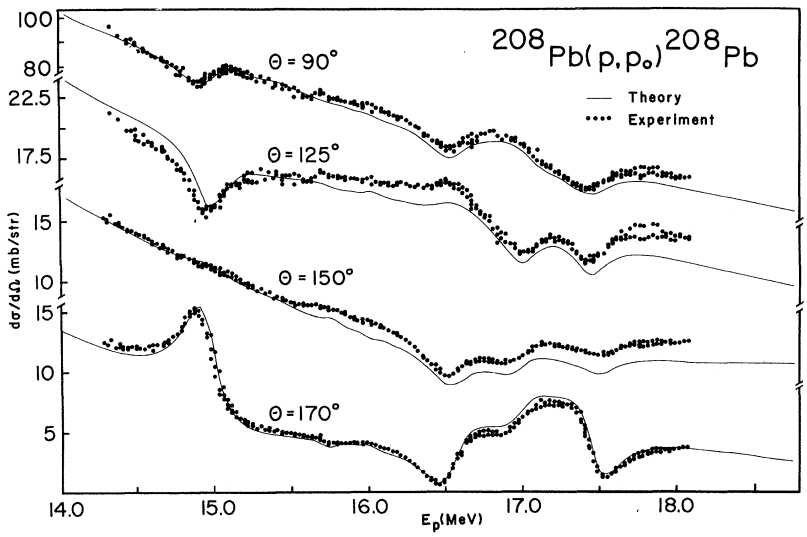

FIG. 2. Comparison of the calculated excitation functions with the experimental data of Ref. 8.

tors for large values of angular momentum and energies close to the Coulomb barrier are strongly dependent upon the radius chosen for the nuclear surface. This dependence is reflected in our analysis and is quite pronounced for the $g_{9/2}$ state, although we do not factorize the partial width into a reduced width and a penetration factor. Similarly, the spectroscopic factors also depend weakly upon the strength of V_1. For the five states in ^{209}Pb that we studied, this dependence is well described by

$$\frac{(\Delta S)/S}{(\Delta V_1)/V_1} \cong -0.7.$$

Uncertainties in the extracted spectroscopic factors arising from the above-mentioned dependences are much larger than those due to other optical model parameters.

In conclusion, we note that the above method for extracting spectroscopic factors of the parent analog states appears very promising since the results do not depend strongly on the choice of optical-model parameters. This method can easily be modified to analyze the inelastic proton scattering through the isobaric analog resonances. It is simple to use and one retains the advantages offered by the Breit-Wigner single level formula. A method similar in spirit to the one presented has recently been reported.[9] Similar calculations are in progress for a number of other nuclei.

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SEARCH FOR EXCITED STATES IN ^{3}He†

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A search for ^{3}He excited states in the reaction ^{3}He (p,p') ^{3}He* with proton bombarding energy of 30.9 MeV has been made. No inelastic peaks were observed.

Since the discovery of ^{4}He excited states, the $A=3$ nuclear system remains the only isobaric sequence in which no excited states are known to exist. In principle, knowledge of excited states in the mass-three system could directly enrich our understanding of nuclear forces. For this reason, a report by Kim et al.[1] of the observation of ^{3}He excited states prompted us and several other groups to re-examine the ^{3}He nucleus.

Kim et al.[1] observed peaks in the spectrum of protons inelastically scattered from ^{3}He at 30.2-MeV bombarding energy. The peaks, which were observed at two laboratory angles, were interpreted as indications of ^{3}He states ($\Gamma \simeq 0.9$ MeV) at approximately 10.2- and 12.6-MeV excitation. They reported laboratory cross sections of 2.0 and ~0.6 mb/sr, respectively,