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BOOTSTRAPLIKE CONDITIONS FROM SUPERCONVERGENCE

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We study the generalized superconvergent sum rules for a number of processes of the kind $P+P\to P+V$, where P and V stand for pseudoscalar and vector mesons, respectively. We find simple algebraic equations for the parameters of the leading Regge trajectories which are exchanged. The results are in good agreement with experiments. The limit of SU(3) symmetry is also considered.

Several authors[1] have shown that superconvergent sum rules can be generalized to cases in which the Regge trajectories are not low enough, by use of analytic continuation. On the other hand in a recent paper[2] we have emphasized the possible importance of the Regge-tail contribution even in the case of superconvergence. As a whole, in both cases, the program can be thought of as relating high-energy parameters (Regge trajectories) with low-lying resonances by means of equations of the form

$$\int_0^{\nu_0} \mathrm{Im}A(\nu,t)d\nu \propto \frac{\beta(t)}{\alpha(t)+1}. \qquad (1)$$

This procedure has successfully been checked[3] in some detail for πN scattering in a range of the momentum transfer t. Meson systems, even if less accessible to experimental data, can still offer, with suitable assumptions, interesting tests of these ideas.

In Ref. 2 we stressed the possibility that the continuum could be important in meson systems and we attempted saturation of vector-meson (V), pseudoscalar-meson (P) scattering sum rules including the Regge-tail contribution.

Here we consider $P+P\to P+V$ scattering. In fact, only the vector and tensor mesons[4] dominate the low-energy region. These same states are known to control the high-energy behavior when their trajectories cross into the physical t region. Furthermore, crossing adds new relations in terms of the same parameters, by exchanging the role of resonances and trajectories. Equation (1) may then be regarded as a consistency condition for the parameters of the Regge trajectories and, in this sense, can be thought of as a bootstraplike equation, a point of view emphasized by other groups as well. A number of problems still remain to be solved in order to bring this approach to the level of a consistent dynamical theory.[5] These mainly concern unitarity, t dependence of the residue functions, the number of equations for higher moments to be considered. Some of the parameters appearing in our approach as constants will turn out to be smooth but perhaps complicated functions of t.

We first restrict ourselves to isospin symmetry and consider a number of testable sum rules. In particular, from $\pi+\pi\to\pi+\omega$ we get very nice bootstrap conditions for the ρ and from $\pi+\eta\to\pi+\rho$ we obtain a very good result for both the ρ and A_2 trajectories. We also look at possible SU(3)-symmetric solutions and we find an interesting and unique solution including a mass formula among T, V, and P mesons and exchange degeneracy of V and T trajectories.

We consider the family of processes

$$P^\alpha(p_1)+P^\beta(p_2)\to P^\gamma(p_3)+V^\delta(q),$$

where $\alpha, \beta, \gamma, \delta$ are SU(3) indices. The scattering amplitude is of the form (e_μ is the polarization vector of the V meson)

$$T^{\alpha\beta\gamma\delta}(\nu,t)=\epsilon_{\mu\nu\rho\sigma}e_{\mu}p_{1\nu}p_{2\rho}p_{3\sigma}A^{\alpha\beta\gamma\delta}(\nu,t),$$

$$\nu=-\tfrac{1}{4}(p_1+q)(p_2+p_3)=\tfrac{1}{4}(s-u);\quad t=-(p_1-q)^2. \tag{2}$$

Conservation laws and Bose statistics imply that the intermediate states must have $J^{PC}+1^{--},2^{++}$, $3^{--},\cdots$ or, in the Regge language, the states lie on the well-known vector and tensor trajectories. The nontrivial, lowest moment sum rules are

$$\int_0^{\nu_1}\nu\,\mathrm{Im}A^{\alpha\beta\gamma\delta}(\nu,t)d\nu=\frac{\nu_1^{\,2}\beta^V(t)}{\alpha_V(t)+1}d_{\alpha\delta\lambda}f_{\lambda\beta\gamma}\ \text{(even crossing)}, \tag{3}$$

$$\int_0^{\nu_2}\mathrm{Im}A^{\alpha\beta\gamma\delta}(\nu,t)d\nu=\frac{\nu_2\beta^T(t)}{\alpha_T(t)}f_{\lambda\alpha\delta}d_{\lambda\beta\gamma}\ \text{(odd crossing)}, \tag{4}$$

α_V and α_T being the trajectory functions and the β functions being defined through the equations

$$A^{\text{even}}(\nu,t)\underset{\nu\to\infty}{\longrightarrow}\frac{\beta^V(t)}{\sin\pi\alpha_V(t)}[1-e^{-\pi\alpha_V(t)}](\nu/\nu_1)^{\alpha_V(t)-1}d_{\alpha\delta\lambda}f_{\lambda\beta\gamma}, \tag{5}$$

$$A^{\text{odd}}(v,t)\underset{\nu\to\infty}{\longrightarrow}-\frac{\beta^T(t)}{\sin\pi\alpha_T(t)}[e^{-i\pi\alpha_T(t)}+1](\nu/\nu_2)^{\alpha_T(t)-1}f_{\lambda\alpha\delta}d_{\lambda\beta\gamma}. \tag{6}$$

The couplings of the resonances are defined as

$$\mathcal{L}(V^\lambda P_1^{\ \alpha}P_2^{\ \beta})=if_{\alpha\beta\lambda}g_{PPV}(p_2-p_1)_\mu e_\mu, \tag{7a}$$

$$\mathcal{L}(V_1^{\ \delta}V_2^{\ \lambda}P^\gamma)=id_{\lambda\gamma\delta}g_{VVP}\epsilon_{\mu\nu\rho\sigma}e_\mu^{\ 1}e_\nu^{\ 2}q_\rho p_{3\sigma}, \tag{7b}$$

$$\mathcal{L}(T^\lambda P_1^{\ \alpha}P_2^{\ \beta})=d_{\alpha\beta\lambda}g_{PPT}e_{\mu\nu}^{\ \ T}(p_1-p_2)_\mu(p_1-p_2)_\nu, \tag{7c}$$

$$\mathcal{L}(T^\lambda P^\gamma V^\delta)=f_{\lambda\gamma\delta}g_{TPV}\epsilon_{\lambda\mu\nu\rho}q_\lambda e_\mu p_{3\nu}(p_3-q)_\sigma e_{\rho\sigma}^{\ \ T}. \tag{7d}$$

With the available formalism we start by considering only SU(2) symmetry. We start with the reaction $\pi+\pi\to\pi+\omega$. There is only one isospin amplitude, and use of Eq. (3) leads to

$$-g_{\rho\pi\pi}g_{\rho\pi\omega}(2m_\rho^{\ 2}-m_\omega^{\ 2}-3m_\pi^{\ 2}+t)=\frac{4}{\pi}\frac{\beta^\rho\nu_1^{\ 2}}{(\alpha_\rho+1)}. \tag{8}$$

Since the left-hand side of (8) vanishes at $t=-2m_\rho^2+m_\omega^2+3m_\pi^2\simeq-0.53\ \mathrm{BeV}^2$, we predict that the other side vanishes at this point as well. The most natural interpretation is to say that β contains a factor $\alpha_\rho(t)$ because of the helicity flip and our prediction is then $\alpha_\rho(-0.53)=0$. This corresponds to the dip point known to be present in πN charge-exchange scattering at about $t=-0.59\ \mathrm{BeV}^2$ as a consequence of the exchange of the same trajectory. We can also use Eq. (8) at $t=m_\rho^2$. By means of Eqs. (5) and (7) we have

$$\beta^\rho(m_\rho^{\ 2})=-\pi\alpha_\rho'(m_\rho^{\ 2})g_{\rho\pi\pi}g_{\rho\pi\omega}, \tag{9}$$

and Eq. (8) gives

$$\nu_1^{\ 2}=\tfrac{1}{2}(3m_\rho^{\ 2}-m_\omega^{\ 2}-3m_\pi^{\ 2})/\alpha_\rho'(m_\rho^{\ 2}). \tag{10}$$

We can go one step further assuming linearity for the trajectory function, $\alpha_\rho(t)=at+b$, and the t dependence of β as[6] $\alpha_\rho(t)[\alpha_\rho(t)+1]\bar{\beta}$, where $\bar{\beta}$ is practically constant in the interval under consideration. Expressing the mass through $m_\rho^2=(1-b)/a$, we get from Eq. (8) two algebraic equations in terms of the three parameters a, b, and ν_1:

$$\nu_1^2=1/2a^2, \tag{11a}$$

$$(m_\omega^2+3m_\pi^2)a+3b=2. \tag{11b}$$

If we insert $m_\rho^2=0.6$ BeV^2, we obtain $a=0.89$ BeV^{-2} and $b=0.47$ which are very good, and $\nu_1=0.8$ BeV^2, which is also quite reasonable.

The analog of Eq. (8) for φ scattering still has the ρ trajectory contributing to the right-hand side. Since the mass of the φ is much different from that of the ω, consistency with the previous results has the only solution $g_{\rho\pi\varphi}=0$, in agreement with experiment.

We analyze now the reaction $\pi+\eta\to\pi+\rho$ in the various channels having either ρ- or A_2-trajectory exchange. This yields three sum rules that read

$$-g_{\rho\pi A_2}g_{\eta\pi A_2}(t+2m_{A_2}^2-m_\rho^2-m_\eta^2-2m_\pi^2)(M_+^2+2t)=4\beta^\rho\nu_1^2/\pi(\alpha_\rho+1), \tag{12a}$$

$$g_{\rho\pi\pi}g_{\eta\rho\rho}+g_{\rho\pi A_2}g_{\eta\pi A_2}(M_-^2+2t)=2\beta_{A_2}\nu_2/\pi\alpha_{A_2}, \tag{12b}$$

$$(t+2m_{A_2}^2-m_\rho^2-m_\eta^2-2m_\pi^2)(M_-^2+2t)g_{\rho\pi A_2}g_{\eta\pi A_2}=(t+m_\rho^2-m_\eta^2-2m_\pi^2)g_{\rho\pi\pi}g_{\rho\rho\eta}, \tag{12c}$$

where

$$M_\pm^2=[m_{A_2}^2-(m_\rho^2+m_\eta^2-2m_\pi^2)\pm(m_\rho^2-m_\pi^2)(m_\eta^2-m_\pi^2)/m_{A_2}^2].$$

Linearizing as before, the resulting system is essentially satisfied by the following solution[7]:

$$\alpha_\rho(t)=0.42+0.96t;\quad \nu_1=(\sqrt{2}\,\alpha_\rho')^{-1}\cong 0.75\text{ BeV}^2;$$

$$\alpha_{A_2}(t)=0.35+0.95t;\quad \nu_2=\sqrt{3}\,(\sqrt{2}\,\alpha_{A_2}')^{-1}\cong 1.25\text{ BeV}^2;\quad g_{\rho\pi\pi}g_{\eta\rho\rho}/g_{\rho\pi A_2}g_{\eta\pi A_2}\cong 5\text{ BeV}^2. \tag{13}$$

We consider now reactions involving strange particles. In general, they lead to relations between coupling constants; however, in a few cases all the resonance contributions vanish at some value of t, predicting a sense-nonsense zero in the residue function. In particular, the sum rules corresponding to $\pi+K\to\omega+K$, $\pi+K\to\rho+K$, and $\pi+K\to\pi+K^*$ are, respectively (in units BeV^2),

$$-g_{K^*K\pi}g_{K^*K\omega}\tfrac{1}{4}(t+0.47)+g_{K_2^*K\omega}g_{K_2^*K\pi}\tfrac{1}{2}(t+2.86)(t+0.40)=\beta^\rho\nu_1^2/\pi(\alpha_\rho+1), \tag{14a}$$

$$-g_{K^*K\pi}g_{K^*K\rho}\tfrac{1}{4}(t+0.49)-g_{K_2^*K\pi}g_{K_2^*K\omega}\tfrac{1}{2}(t+2.88)(t+0.42)=\beta^\omega\nu_1^2/\pi(\alpha_\omega+1), \tag{14b}$$

$$-g_{K^*K\pi}g_{K^*K^*\pi}\tfrac{1}{4}(t+0.56)+g_{K_2^*K\pi}g_{K_2^*K^*\pi}\tfrac{1}{2}(t+2.95)(t+0.50)=\beta^\rho\nu_1^2/\pi(\alpha_\rho+1). \tag{14c}$$

We now turn to consider the interesting limit of SU(3) symmetry. We find that three independent sum rules exist (one for V exchange and two for T exchange) if the external multiplets are supposed to be unitary octets. After linearization the β's can be eliminated and we get the system

$$t+m_V^2-3m_P^2=2\nu_1^2\alpha_V(t)\alpha_V', \tag{15a}$$

$$g_{PPV}g_{VVP}+(2t+m_T^2-m_V^2-3m_P^2)g_{PPT}g_{PVT}=\tfrac{4}{3}\nu_2^2(\alpha_T+1)\alpha_T'g_{PPT}g_{PVT}, \tag{15b}$$

$$(t+m_V^2-3m_P^2)g_{VVP}g_{PPV}=+(t+2m_T^2-m_V^2-3m_P^2)(2t+m_T^2-m_V^2-3m_P^2)g_{PPT}g_{PVT}. \tag{15c}$$

The solution of Eqs. (15) contains as a consequence exchange degeneracy and, more precisely, all the parameters are expressed in terms of m_V and m_P in the following way:

$$\alpha_V(t) = \alpha_T(t) = \frac{1-3x}{2-3x} + \frac{t}{2m_V^2}(1-\tfrac{3}{2}x)^{-1}, \quad x = (m_P/m_V)^2; \tag{16a}$$

$$\nu_1 = (\sqrt{2}\,\alpha_V')^{-1}; \quad \nu_2 = \frac{\sqrt{3}}{\sqrt{2}}\alpha_T'^{-1}; \quad \frac{g_{PPV}g_{VVP}}{g_{PPT}g_{PVT}} = -2(t+2m_T^2-m_V^2-3m_P^2). \tag{16b}$$

The present work, as well as the results of Ref. 3, confirm this point: The interference model would not satisfy our equations and would yield double-counting effects.

Finally, we want to comment on the validity of Schwarz sum rules[8] deduced under the hypothesis of the absence of fixed poles at the wrong-signature integers. In cases we can check their validity, with the present scheme they are not satisfied. A similar result was found in Ref. 3.

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LOW-ENERGY K^--α ELASTIC SCATTERING*

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The differential cross section for K^--α elastic scattering for K^- laboratory momenta in the interval 110-160 MeV/c has been measured in a helium bubble chamber. The data, together with measurements of the total absorption cross section in the same region, have been fitted with a partial-wave analysis involving s- and p-wave scattering with complex phase shifts. These parameters are used for predicting the expected results of a K-mesonic x-ray experiment, viz., that >98% of the K^- capture is from the $2P$ state.

We report here a first measurement of low-energy K^--α elastic scattering. We analyzed film from the Argonne National Laboratory–Carnegie 25-cm helium bubble chamber,[1] which was placed in a separated, stopping K^- beam[2] at the zero-gradient synchrotron (ZGS). The magnetic field in the chamber was produced by a superconducting magnet with a central field of ~41 kG.[3]

The film was scanned at least twice in two of three views by searching for the scatter of any beam track >10° in either view. Each scattered beam track was then followed and required to decay or interact inside the chamber and have an additional track or tracks present at the second vertex.[4] This requirement yields a sample of K^--α scatter candidates with the large background of K^- decays and high-momentum scatters removed. This is a consequence of the fact that a K^--decay secondary