RISING TRAJECTORIES, FALLING RESIDUES, AND EXPERIMENT

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We attempt here a limited experimental test of a recent speculation that the Regge residue function, $\beta(s)$, shows an exponential or faster decrease with $\alpha(s)$ as $s\to\infty$ to insure polynomial boundedness for the total amplitude.

Khuri[1] has shown that if the scattering amplitude $f(s,z)$ is bounded by a polynomial in s at fixed z for $s\to\infty$, then we must have the condition[2]

$$\lim_{\substack{s\to\infty\\ z\text{ fixed}}} |\beta(s)P_{\alpha(s)}(z)|/s^N=0. \tag{1}$$

for any Regge contribution in the direct channel. This may be satisfied with a "rising" trajectory

$$\alpha(s)\xrightarrow[s\to\infty]{}\infty$$

if the residue function $\beta(s)$ satisfies (in the spinless case)[3]

$$\beta(s)\underset{s\to\infty}{\leq} e^{-b\alpha(s)}. \tag{2}$$

A model implying a high-s behavior

$$\beta(s)\xrightarrow[s\to\infty]{}\text{const}[\alpha(s)/e]^{-\alpha(s)}$$

was suggested by Jones and Teplitz[4] on the basis of incorporating the Mandelstam symmetry[5] into the theory. The aim of this communication is to correlate an analog of the requirement (2) for the πN case with experiment by relating the appropriate β to the partial widths of the known $B=1$, $Y=1$ resonances and attempt to see any tendency for (2) to be fulfilled.

We first extend the work of Khuri[1] in a straightforward way to the πN case, and write a bound analogous to (2) on the residue functions defined by

$$\beta(\sqrt{s})=\lim_{J\to\alpha(\sqrt{s})+\frac{1}{2}}[J-\tfrac{1}{2}-\alpha(\sqrt{s})]f(J,\sqrt{s}), \tag{3}$$

with

$$\operatorname{Im}f(J,\sqrt{s})=k\,|f(J,\sqrt{s})|^2 \tag{4}$$

for $J=$ half integer, $(m+\mu)^2\leq s\leq(m+2\mu)^2$, and $4k^2=s-2(m^2+\mu^2)+(m^2-\mu^2)^2/s$.

Subscripts on β, α, and f supplying isospin, parity, and signature are implicit.

By explicating the contribution of a given Regge pole to the scattering amplitudes f_1, f_2,[6] we can arrive at the proper prelude to the requirement (2) on β:

$$|\beta(\sqrt{s})P_{\alpha(\sqrt{s})+1}{}'(z) \pm\beta(-\sqrt{s})P_{\alpha(-\sqrt{s})+1}{}'(z)| \underset{\substack{s\to\infty\\ z\text{ fixed}}}{<} s^N, \tag{5}$$

which yields the analog of (2) (assuming no cancellations),

$$\ln\beta(\sqrt{s}) \underset{\substack{s\to\infty\\ \alpha\to\infty}}{<} -\text{const}\,\alpha(\sqrt{s}). \tag{6}$$

From Eqs. (3) and (4) we may derive the following relations at a resonance of mass $\sqrt{s_R}$:

$$\beta(\sqrt{s_R})=\left[\frac{d\operatorname{Re}\alpha(s)}{ds}\right]_{s=s_R}\frac{x s_R^{1/2}\Gamma}{k(s_R)}, \tag{7}$$

where $\Gamma=$ total width of the resonance, and $x=\Gamma_{\text{elastic}}/\Gamma=$ elasticity of resonance.

Let us examine the logarithm of β as given by Eq. (7). The variation of $\ln(s_R^{1/2}/k_R)$ is small compared to that of $\ln x$ in the energy range considered, and of course goes to zero as $s\to\infty$. Experiments[7] do not seem to indicate large variation of Γ with the resonance, and hence even less of $\ln\Gamma$.

Finally, present indications are that $d\alpha/ds\simeq$ const,[8] at least in the energy range considered. Hence Eqs. (6) and (7) yield the approximate requirement

$$\ln x \underset{\alpha\to\infty}{\leq} -b\alpha(\sqrt{s}),\quad b>0. \tag{8}$$

We will test this condition for values of α in the higher resonance region.

In Fig. 1 are plotted the presently known or estimated values of $\ln x$ for the N_γ and Δ_δ trajectories against values of $\alpha=J-\frac{1}{2}$ assumed by Barger and Cline[8] in their Chew-Frautschi plots. The values of x are those given in Column 6 of Table I, Ref. 8. These have been derived from measurements on elastic cross sections using the spin assignments in Ref. 8 [since

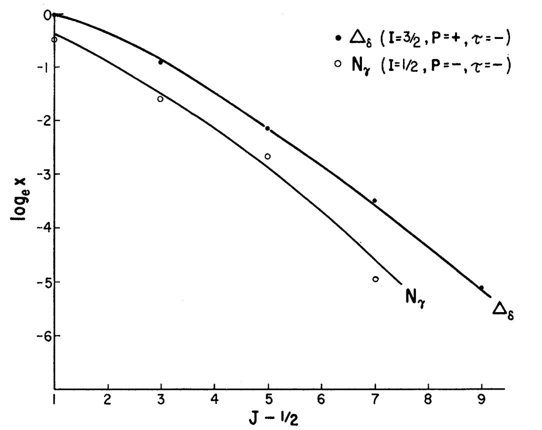

FIG. 1. Logarithm of the inelasticity x as a function of $J-\frac{1}{2}=\alpha$. Experimental points are obtained from Ref. 8.

cross-section measurements determine only $(J+\frac{1}{2})x$ for a resonance]. Where there are elasticities available as a result of phase-shift analyses, these have been averaged with the cross-section values. Note that the values of x used do not depend on the resonance–Regge-pole model used by Barger and Cline.

Since the experimental data available on the N_α resonances are too sparse, we have omitted them from consideration.

Results.–Although each datum point in the figure should be accompanied by large, unknown error bars, and s is hardly infinite, yet three interesting facts seem to be roughly indicated:

(1) The inelasticity x (and hence β; see previous paragraph) falls at least as fast as $e^{-b\alpha}$, $b>0$.

(2) There is some approximate kind of universality in the fall-off. If one insists on an average exponential fall at the highest J values, we might say that

$$\beta \approx \mathrm{const}\, e^{-0.8\alpha} \tag{9}$$

for the N_γ and Δ_δ trajectories. It is amusing to speculate whether the constancy of $d\ln\beta/d\alpha$ is correlated to the constancy of $d\alpha/ds$ for the same trajectories.

(3) The detailed behavior $\beta \sim 1/\Gamma(\alpha+\frac{3}{2})$ suggested in Ref. 4 seems to predict too rapid a drop of β, at least within the limitations of the present data. For instance, if one normalizes this form to give the observed inelasticity at $\alpha=7$ for the Δ_δ, the inelasticity is off from the measured value by a factor of 20 at $\alpha=9$. However, the data are still much too limited to make a definite statement.

To conclude: We have demonstrated that experimental elasticities make plausible a general conjecture of Jones and Teplitz[4] about the rapid fall-off of $\beta(s)$, enabling α to approach ∞ and remain consistent with certain boundedness assumptions on the total amplitude. It would be of interest to test these results on the N_α trajectory when there is sufficient experimental information.

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[1]N. N. Khuri, Phys. Rev. Letters 18, 1094 (1967).

[2]This bound may also be derived by demanding the existence of partial-wave dispersion relations with a finite number of subtractions. A treatment of this point will appear elsewhere.

[3]The essential point in the proof is that $|P_\alpha(z)| < e^{b\alpha}$, $\alpha \to \infty$ with $b>0$.

[4]C. E. Jones and V. L. Teplitz, Phys. Rev. Letters 19, 135 (1967).

[5]S. Mandelstam, Ann. Phys. (N.Y.) 19, 254 (1962).

[6]V. Singh, Phys. Rev. 129, 1889 (1963).

[7]See, for instance, A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, Matts Roos, Paul Soding, W. J. Willis, and C. G. Wohl, Rev. Mod. Phys. 39, 1 (1967).

[8]V. Barger and D. Cline, Phys. Rev. 155, 1792 (1967).