PHOTODISINTEGRATION OF THREE-PARTICLE NUCLEI*

I. M. Barbour†
Department of Natural Philosophy, The University of Glasgow, Glasgow, Scotland

and

A. C. Phillips†
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 17 October 1967)

Most attempts[1] to obtain a theory that can accommodate both the two-body [$He^3(\gamma,p)d$] and three-body [$He^3(\gamma,n)2p$] photodisintegrations of He^3 have been unsuccessful[1]; the predicted cross sections for the three-body disintegration have been a factor of 2 or 3 too large. These reactions are known to be dominated by electric dipole transitions, and the main defects of the above calculations must be inadequate representations of the initial bound state and final scattering states of the three-nucleon system. In particular, the electric dipole operator has the effect of emphasizing the role of the asymptotic structure of the three-nucleon bound-state wave function.[2-4] As a consequence of this, it has been shown[3,4] that the use of a wave function with the correct asymptotic behavior reduces the ratio of the three- to the two-body photodisintegration cross sections and leads to a closer agreement with experiment. But it is impossible to assess the implications of these results without a reliable theory of the final-state wave functions.

Hitherto the three-particle aspects of the nucleon-deuteron and three-nucleon final states have been neglected. For the nucleon-deuteron state the distortion of the deuteron is usually neglected, and for the three-nucleon state only final-state interactions between nucleon pairs in S states have been taken into account in first rescattering approximations.[3,4] Such techniques violate the three-particle unitarity relation which connects the two-body and three-body states of the three-nucleon system. This is not the case if one uses the Faddeev equations[5] with a separable approximation for the two-nucleon off-the-energy-shell scattering amplitudes.[6] The Fadeev approach treats the three-particle aspects of the problem exactly, and has been applied with considerable success to the neutron-deuteron scattering problem.[7,8]

The chief advantage of this method is that the two- and three-body photodisintegration channels are dynamically related. In this connection Barton[9] has shown that the particular two-nucleon interaction which is responsible for the deuteron, and hence allows the existence of the two-body photodisintegration channel, must have a large effect via the final-state interactions on the three-body channel. For example, if the two-nucleon interaction is weakened so that the deuteron is no longer bound, then the disappearance of the two-body photodisintegration channel must be accompanied by a large increase in the three-body cross section. This result, which is based on a new decomposition of the standard bremsstrahlung-weighted electric dipole sum rule, rests only upon the assumptions of a totally symmetric S state for the three-nucleon bound state and electric dipole absorption dominance.

In this Letter we introduce a new three-nucleon bound-state wave function, and for the continuum states we adopt the Faddeev equations and a low-energy separable approximation.[6,8] Only electric dipole contributions to the photodisintegration cross sections are included. We use the usual dipole operator, "z." Hence, in the region of validity of Siegert's theorem (i.e, photon energies less than 50 MeV) we need not include the modifications of the electromagnetic operator that arise because of the existance of nonlocal and exchange nuclear forces. Further details and results will be published elsewhere.[10]

The final states are determined by the numerical solution of the Faddeev equations.[11] The separable approximation to the $^1S_0$ and $^3S_1$ two-nucleon scattering amplitudes corresponds to an effective-range parametrization of the on-shell amplitudes.[12] The $^1S_0$ state is specified by the scattering length $a_s = -20.34$ F and effective range $r_s = 2.7$ F, and the $^3S_1$ state by the scattering length $a_d = 5.397$ F and the deuteron binding energy $\epsilon_d = 2.225$ MeV. The net result is the inclusion of only those virtual three-particle intermediate states which have the configuration of a nucleon and a $^1S_0$ two-nucleon subsystem or a nucleon and a $^3S_1$ two-nucleon subsystem. Thus, in this approximation, physically interesting information may be obtained from the amplitudes for the production of these configurations and the functions that represent the propagation and decay of the virtual $^3S_1$ and $^1S_0$ states.

For the initial state we adopt a wave function that emphasizes the asymptotic behavior in configuration space. Defining p_α to be the relative momentum of the nucleons β and γ, and q_α to be the momentum of nucleon α with respect to the β-γ subsystem, we have

$$\Psi(p_\alpha, q_\alpha) = (p_\alpha{}^2 + \tfrac{3}{4}q_\alpha{}^2 + E_B)^{-1} \sum_{n=d,s} \sum_{\beta=1}^{3} \left\{ \frac{h_n}{(q_\beta{}^2 + \frac{4}{3}E_n)(q_\beta{}^2 + \nu_n{}^2)^2 (p_\beta{}^2 + \mu_n{}^2)} \times |I_{\gamma\alpha} = I_n, I_\beta = \tfrac{1}{2}, I = \tfrac{1}{2}, I_z\rangle |S_{\gamma\alpha} = S_n, S_\beta = \tfrac{1}{2}, S = \tfrac{1}{2}, S_z\rangle \right\}, \quad (1)$$

where $I_{\gamma\alpha}$, I_β, and I ($S_{\gamma\alpha}$, S_β, and S) are the isospins (spins) of the $\gamma\alpha$ subsystem, of the particle β, and of all three nucleons, respectively. The parameter E_B is the three-nucleon binding energy. The asymptotic behavior when one of the nucleons is distant from the other two is determined by $E_d (= E_B - \epsilon_d)$ or $E_s (= E_B)$. Other parameters, which were determined by a rough fit to the wave function obtained by applying the separable approximation and the Faddeev equations to the bound-state problem, are $\nu_s = 7.0\ \mathrm{F}^{-1}$, $\mu_s = 1.173\ \mathrm{F}^{-1}$, $\nu_d = 4.0\ \mathrm{F}^{-1}$, and $\mu_d = 1.427\ \mathrm{F}^{-1}$. For He^3, $h_s = -1.830$ and $h_d = +1.375$, and for H^3, $h_s = -1.948$ and $h_d = 1.517$. This wave function has an S' probability density $\simeq 1.6\%$, and, if proton and neutron mean-squared radii of $0.64\ \mathrm{F}^2$ and $-0.12\ \mathrm{F}^2$ are assumed, the charge radii are $R(\mathrm{He}^3) = 1.85$ F and $R(\mathrm{H}^3) = 1.62$ F. In our calculation of the photodisintegration cross sections only the completely spatially symmetric part of (1) is retained. In addition the parameters corresponding to the He^3 nuclei are used.

Theoretical results for the total cross section for the three-body channel are shown in Fig. 1. Curves a, c, and e represent the cross section corresponding to the complete solution, the first-rescattering approximation, and zero-rescattering approximation, respectively. It is evident that the summation of the multiple-rescattering series is necessary. This is also the case for the calculation of the two-body cross section. [See Fig. 2(a).]

The influence of the deuteron bound state on the three-body channel may be deduced from Fig. 1. The curves b and d correspond, respectively, to the complete and first-rescattering solutions when the $^3S_1$, two-nucleon interaction has been weakened, so that it is identical to the $^1S_0$ interaction and hence can no longer bind the deuteron. Inspection of Fig. 1 shows that the disappearance of the deuteron and the two-body photodisintegration channel is accompanied by a large increase in the three-body photodisintegration cross section. This effect occurs both in the complete solution (curves a and b) and in the first rescattering approximation (curves c and d).

The comparison with the experimental data[1,13] is given in Figs. 2(a) and 2(b). The bremsstrahlung-weighted cross section can also be found:

$$\int_{5.49}^{40} \sigma_2 dE_\gamma / E_\gamma = 1.13\ \mathrm{mb}$$

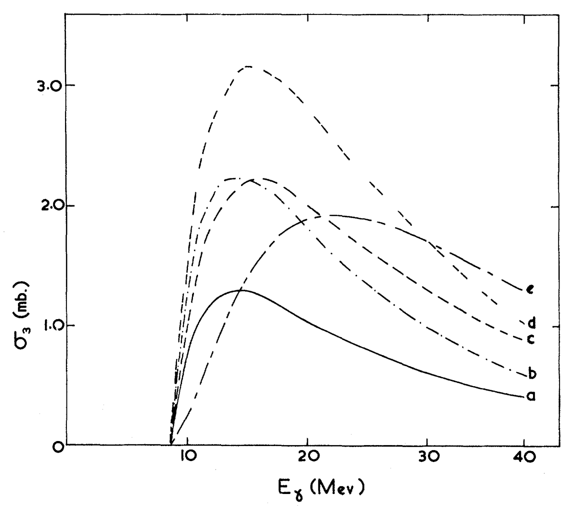

FIG. 1. The dependence of the total cross section for $\mathrm{He}^3(\gamma, n)2p$ on the structure of the final scattering state. Curves a, c, and e correspond, respectively, to the complete, the first-rescattering, and the zero-rescattering solutions, with inclusion of the $^1S_0$ antibound state and the $^3S_1$ deuteron bound state. Curves b and d correspond, respectively, to the complete and first-rescattering solutions when the $^3S_1$ interaction is put equal to the $^1S_0$ interaction.

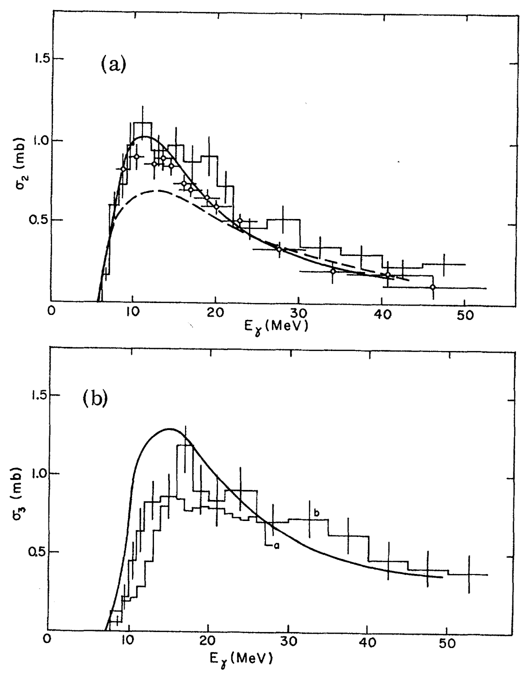

FIG. 2. (a) The total cross section for $He^3(\gamma,d)p$. The histogram is from Ref. (1) and the points from Ref. 13. The solid curve is the complete solution and the broken curve corresponds to a plane-wave approximation for the nucleon-deuteron state. (b) The total cross section for $He^3(\gamma,n)2p$. Histogram (a) is from Ref. 13 and histogram (b) from Ref. (1). The curve corresponds to the complete solution with distinct $^3S_1$ and $^1S_0$ two-nucleon interactions (i.e., curve a of Fig. 1).

and

$$\int_{7.72}^{40} \sigma_3 dE_\gamma/E_\gamma = 1.37 \text{ mb}.$$

The experimental results[1] are

$$\int_{5.49}^{170} \sigma_2 dE_\gamma/E_\gamma = 1.34 \pm 0.05 \text{ mb}$$

and

$$\int_{7.72}^{170} \sigma_3 dE_\gamma/E_\gamma = 1.47 \pm 0.07 \text{ mb}.$$

We have assumed electric dipole dominance, a spatially symmetric three-nucleon bound state given by Eq. (1), and a separable two-nucleon interaction in the final state. In view of these assumptions and also the neglect of Coulomb forces, short-range correlations, and noncentral forces, the agreement with experiment is quite satisfactory. In addition the neutron spectra (not shown here) in $He^3(\gamma,n)2p$ are also in agreement, both in shape and magnitude, with experiment.

In conclusion we emphasize that adequate treatment of the multiple rescattering in the three-nucleon system is essential if any meaningful comparison with experiment is to be made.

We would like to thank Dr. G. Barton and Dr. L. M. Delves for useful discussions. We would also like to thank Professor R. J. Blin-Stoyle for providing the opportunity to work at the University of Sussex.

---

*This work is supported in part through funds provided by the Atomic Energy Commission under Contract No. At(30-1) 2098.

†The main part of the work was done when the authors were at the School of Mathematical and Physical Sciences, Univ. of Sussex, Sussex, England.

[1]For a review of the nuclear photoeffect on three-particle nuclei, see V. N. Fetisov, A. N. Gorbunov, and A. T. Varfolomeev, Nucl. Phys. 71, 305 (1965).

[2]L. M. Delves, Phys. Rev. 118, 1318 (1960).

[3]V. N. Fetisov, Phys. Letters 21, 52 (1966); Nucl. Phys. 3, 322 (1967).

[4]J. M. Knight, J. S. O'Connell, and F. Prats, Phys. Letters 3, 322 (1966).

[5]L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. 39, 1459 (1960) [translation: Soviet Phys.—JETP 12, 1014 (1961)]; Izv. Akad. Nauk. SSSR 138, 565 (1961); 145, 301 (1962) [translation: Soviet Phys.—Dokl. 6, 384 (1961); 7, 600 (1963)].

[6]C. Lovelace, Phys. Rev. 135, B1225 (1964); in Strong Interactions and High-Energy Physics, edited by R. G. Moorhouse (Oliver and Boyd Publishers, London, 1964).

[7]R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. 140, B1291 (1965); R. Aaron and R. D. Amado, ibid. 150, 857 (1966).

[8]A. C. Phillips, Phys. Rev. 142, 984 (1966).

[9]G. Barton, to be published.

[10]The radiative neutron-deuteron problem has recently been considered using a similar formalism: A. C. Phillips, to be published.

[11]J. H. Hetherington and L. M. Schick, Phys. Rev. 156, 1647 (1967); I. M. Barbour and R. L. Schult, Phys. Rev. 155, 1712 (1967).

[12]Y. Yamaguchi, Phys. Rev. 95, 1628, 1635 (1954).

[13]H. M. Gerstenberg and J. S. O'Connell, Phys. Rev. 144, 834 (1966).