pure gold the activation energy observed in all three experiments approaches  $0.90 \pm 0.04$ eV. This we believe is the migration energy of the single lattice vacancy.

(5) Very little of the data obtained in the present experiments yield the 0.69-eV energy seen in gold by Ytterhus and Balluffi.<sup>1</sup> Additional experiments have been done which show that slowly quenched specimens of high purity slowly quencied specimens of light purity<br> $(\rho_{4.2}^{\circ}$  K = 0.15 × 10<sup>-9</sup>  $\Omega$  cm, size corrected give the 0.69-eV energy. The rate of quench was about  $7 \times 10^3$  °C/sec. The rate used to obtain the data given in the table was  $(7 \pm 2) \times 10^4$  °C/ sec. Apparently the 0.69 eV is associated with tightly bound divacancies. Vacancy-divacancy equilibrium is probably not attained during the annealing of the fast quenched specimens because the dislocations introduced by the fast quench provide too many sinks. The binding energy of the divacancy is probably the 0.4-eV value of Ytterhus and Balluffi rather than O.l eV as claimed by Kino and Koehler.<sup>2</sup>

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 $<sup>1</sup>J$ . A. Ytterhus and R. W. Balluffi, Phil. Mag. 11,</sup> 707 (1965).

 $2$ T. Kino and J. S. Koehler, Phys. Rev. 162, 632 (1967).

## NONLINEAR PRESSURE DEPENDENCE OF ELASTIC CONSTANTS AND FOURTH-ORDER ELASTIC CONSTANTS OF CESIUM HALIDES

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The pressure dependence of the elastic constants of CsCl, CsBr, and CsI was measured up to 10 kbar by an ultrasonic technique and found to be nonlinear. The second pressure derivatives of the elastic constants and three linear combinations of the fourth-order elastic constants were determined.

The complete set of the single-crystalline, third-order elastic (TOE) constants or some of their linear combinations have been measured up to the present for about ten or 30 materials, respectively. No data, however, are as yet available on fourth-order elastic (FOE) constants, although their contribution to anharmonic effects may be comparable with that arising from the TOE constants.

The elastic constants of CsCI, CsBr, and CsI were measured at 298'K as a function of hydrostatic pressure up to 10 kbar by using the ultrasonic-pulse superposition technique. This extends previous measurements' on these materials to a higher pressure range and increased accuracy. Four independent runs referring to different propagation and polarization directions were made to determine the three elastic constants as a function of pressure and to obtain one internal check which indicated good self-consistency of all measurements. Figure 1 shows typical results for the square of the reciprocal transit time versus pressure. The change of the path length contributes at most one-half of this curvature in the case of  $c_{12}$ , and much less for  $c_{11}$  and  $c_{44}$ . It was taken into account by Cook's method, '



FIG. 1. Square of reciprocal transit time of transverse waves (propagating in [110] and polarized in  $[11\overline{0}]$ ) versus pressure for CsCl, CsBr, and CsI. The solid lines represent a least-sqaures fit to a parabola.

using the isothermal values for the bulk modulus and its first pressure derivative calculated according to Overton.<sup>3</sup> The adiabatic elastic constants can be represented as a quadratic function of the pressure. The elastic constants and their first and second derivatives obtained from a least-squares fit to the experimental data are compiled in Table I. The error shown arises from the standard deviation of the least-squares fit and, predominantly, from the uncertainty in the pressure reading. which was taken from a Foxboro recorder in connection with a Manganin cell and estimated at 1.5%. The quantities  $C_{ijkl}$  and  $C_{ijkl}$ " are the isothermal pressure derivatives of the adiabatic elastic constants.

From them the mixed adiabatic-isothermal partial contractions  $\Gamma_{ijk}l^{(3)} = C_{ijklmm}$  and  $\Gamma_{ijkl}^{(4)} = C_{ijklmmpp}$  of the tensors of the TOE and FOE

 $\overline{S}$ 

Table I. Adiabatic elastic constants  $C_{ijkl}$ <sup>S</sup> (in 10<sup>11</sup> dyn cm<sup>-2</sup>), their first isothermal pressure derivatives  $C_{ijkl}$ ' (dimensionless), and their second isothermal pressure derivatives  $C_{ijkl}$ " (in 10<sup>-11</sup> dyn<sup>-1</sup> cm<sup>2</sup>) for three cesium halides.

	CsCl	CsBr	CsI
$c_{\rm 111}^{\phantom{00}S}$ $C_{1122}S$ $C_{1212}$ $c_{\scriptscriptstyle 1111}'$ $C_{\bf 1122}'$ $C_{1212}$ $C_{1111}''$	$3.670 \pm 0.017$ $0.889 \pm 0.013$ $0.808 \pm 0.009$ $7.01 \pm 0.23$ $5.14 \pm 0.22$ $3.69 \pm 0.15$ $-6.8 \pm 1.2$ $-1.8 \pm 1.1$	$3.063 \pm 0.017$ $0.809 \pm 0.018$ $0.753 \pm 0.010$ $6.71 \pm 0.26$ $5.21 \pm 0.31$ $3.81 \pm 0.19$ $-7.5 \pm 1.5$ $-2.4 \pm 1.7$	$2.452 \pm 0.015$ $0.667 \pm 0.016$ $0.628 \pm 0.010$ $6.72 \pm 0.25$ $5.12 \pm 0.30$ $3.84 \pm 0.18$ $-9.2 \pm 1.6$
$C^{}_{1122}$ " $C_{\,1212}$ ″	$-2.5 \pm 0.7$	$-2.2 \pm 1.0$	$-3.0 \pm 1.7$ $-2.5 \pm 1.0$

constants can be determined by means of Birch's formula for the zero-pressure value of the first pressure derivative.<sup>4,5</sup>

$$
C_{ijkl}' = \left(\frac{\partial C_{ijkl}}{\partial p}\right)_T = -\frac{1}{3B^T} (C_{ijkl}^S + \Gamma_{ijkl}^{(3)}) + (\delta_{ij}\delta_{kl} - \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}),\tag{1}
$$

and its extension to the zero-pressure value of the second pressure derivative.<sup>6</sup>

$$
C_{ijkl}^{\prime\prime} = \left(\frac{\partial^2 C_{ijkl}}{\partial \rho^2}\right)_{T} = \frac{1}{9(B^T)^2} [(1+3B')C_{ijkl}^{\prime S} + (4+3B')\Gamma_{ijkl}^{(3)} + \Gamma_{ijkl}^{(4)}].
$$
 (2)

Here  $B^T = \frac{1}{9}C_{ijij}T$  is the isothermal bulk mod-<br>ulus,  $B' = \frac{1}{9}C_{ijij}$  its isothermal pressure derivative, both at zero pressure, and δ the Kronecker symbol. The numerical data are listed in Table II. It is apparent that the partial contractions of the TOE constants are about 10 to 25 times, and the partial contractions of the FOE constants about 200 to 500 times larger than

Table II. Partial contractions  $\Gamma_{ijkl}^{(3)} = C_{ijklmm}$  and  $\Gamma$  ijkl  $^{(4)}$  = Cijklmmpp of intermediate adiabatic-isother-<br>mal TOE and FOE elastic constants (in 10<sup>11</sup> dyn cm<sup>-2</sup>).

	CsCl	CsBr	$_{\rm CsI}$
$\Gamma_{1111}^{(3)}$	$-43.9 \pm 2.9$	$-36.2 \pm 2.7$	$-30.0 \pm 1.9$
$\Gamma_{1122}^{(3)}$	$-21.7 \pm 2.0$	$-18.9 \pm 2.2$	$-15.4 \pm 1.6$
$\Gamma_{1212}^{(3)}$	$-24.4 \pm 1.8$	$-21.4 \pm 1.8$	$-17.9 \pm 1.3$
$\Gamma_{1111}^{(4)}$	$730 \pm 130$	$600 \pm 130$	$490 \pm 100$
$\Gamma_{1122}^{(4)}$	$410 \pm 90$	$360 \pm 100$	$290 \pm 70$
$\Gamma_{1212}^{(4)}$	$460 \pm 80$	$420 \pm 80$	$350 \pm 60$

the corresponding second-order elastic constants. This illustrates the slow convergence of the Taylor expansion of the internal energy with respect to the Lagrangian strains. All partial contractions are negative for the TOE constants. but positive for the FOE constants. Validity of the Cauchy relations for the TOE and FOE constants would require that  $\Gamma_{1122}{}^{(3)}\text{ = }\Gamma_{1212}{}^{(3)}$ and  $\Gamma_{1122}^{(4)} = \Gamma_{1212}^{(4)}$ . Obviously these relations are approximately fulfilled both for the TOE and for the FOE constants with the deviation increasing from about  $10\%$  for CsCl to about 20% for CsI.

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<sup>&</sup>lt;sup>3</sup>W. C. Overton, Jr., J. Chem. Phys. 37, 117 (1962).

<sup>&</sup>lt;sup>4</sup>F. Birch, Phys. Rev.  $71$ , 809 (1949).

 ${}^5$ G. R. Barsch, Phys. Status Solidi 19, 129 (1967).

<sup>&</sup>lt;sup>6</sup>G. R. Barsch and Z. P. Chang, to be published.