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EVIDENCE FOR A PEELING MODEL OF VORTEX RING FORMATION BY IONS IN LIQUID HELIUM*

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In this paper an experiment will be described which indicates that the ion–vortex-ring transition is smoothly connected with no discontinuity in the drift velocity of the ion complex when the critical velocity for vortex-ring formation is reached.

Earlier work[1] has shown charged vortex rings may be produced in liquid helium by fast-moving ions, either positive or negative. These vortex rings are singly charged and have quantized circulation equal to $\oint v_s dl = h/m$, where m is the mass of a helium atom and h is Planck's constant. The nature of vortex-ring formation by ions has been the subject of some speculation.[1-4] To the best of the author's knowledge, all of the models which have been proposed assume that a complete ring is formed and, subsequently, the ion is trapped or hops onto the ring. If the steady-state drift velocity of the ion complex is measured as a function of applied field, a sudden reduction in velocity should occur when the ion reaches the critical velocity for vortex-ring formation.[2,3] Quite simply, this occurs because the cross section for quasiparticle scattering is increased if a vortex ring comparable in size with the ion is formed. The actual numerical change in viscous force on the ion complex is difficult to estimate when a small vortex ring is added to the bare ion because of the perturbation to the velocity field of the vortex ring by the ion. The experimental data presented in this paper indicate, in the author's opinion, that the vortex line associated with the formation of the vortex ring is slowly peeled away from the ion in the form of a steadily growing loop as the electric field is increased. Thus, no sudden change in drift velocity is observed nor is the ion required to hop onto the ring since it is always trapped on the core (or, conversely, one may say the vortex line formed is always stuck to the ion).

The experimental method used to determine the drift velocity as a function of electric field has been described elsewhere and is essentially a two-gate velocity spectrometer.[5,6] Po^{210} is used for producing the ions, and a drift space is provided before the velocity-measuring region so that the velocity of the ion complex can reach a steady-state value determined by the electric field through the apparatus.

If the quasiparticles responsible for the viscous force on the positive ion are thermal excitations (rotons), the velocity of the positive-ion complex as a function of applied electric field is shown in Fig. 1. Data pertaining to curves of this type have been presented earlier.[5] In a curve like this, one cannot be sure whether the transition is continuous or discontinuous because, when the field is large enough to push an ion to the critical velocity, it is also large enough to support a rather large ring, and any small rings which are formed immediately grow into a large ring. In order to trace out the vortex-ring spectrum shown in this figure, the vortex rings were produced in the source region and brought to equilibrium at a lower value of field, and their velocity was measured. For more details see Ref. 5. If the transition is continuous, the vortex-ring curve should extrapolate along a curve like the dotted one in Fig. 1. Unfortunately, experimentally tracing out a curve like this is troublesom since it is double-valued and the smaller ion complexes always grow into large rings. If the viscous force on the ion could be kept constant while increasing the viscous force on the vortex ring, one has hope that the complete curve could be traced out. The effect would be to shift the vortex-ring spectrum to the right and produce a curve that is single-valued. This can be accomplished by replacing the thermally excited quasiparticles with He^3 impurities. The He^3 atoms have a larger vortex-line–scattering cross section[1] but a smaller ion-scattering cross section[7] than thermally excited quasiparticles.

Figure 2 shows the results obtained for positive ions when 1 part He^3 is added to 5.35×10^3 parts He^4 and the temperature is reduced to 0.30°K which effectively eliminates roton scattering. The vortex-ring curve is now shifted relative to the ion curve, as expected. The shift is sufficient to make the curve single-valued rather than double-valued as in Fig. 1. Figure 2 indicates that the ion–vortex-ring transition is continuous, favoring the model in which vortex line is peeled away from the ion.

Based on other data,[5,6] the peeling out of vortex line begins when the velocity field of the superfluid in the neighborhood of the ion satisfies $\oint v\cdot dl = h/m$. Nucleation is probably not due to roton formation since other experiments

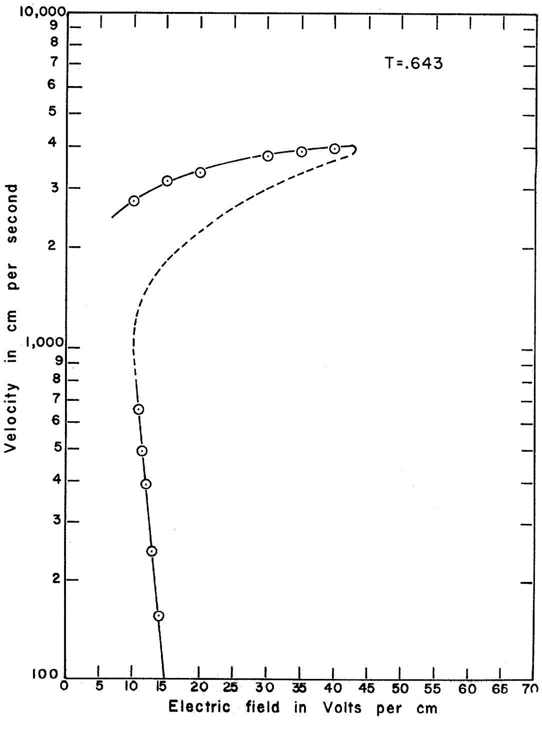

FIG. 1. Velocity versus electric field for positive-ion complexes with roton scattering.

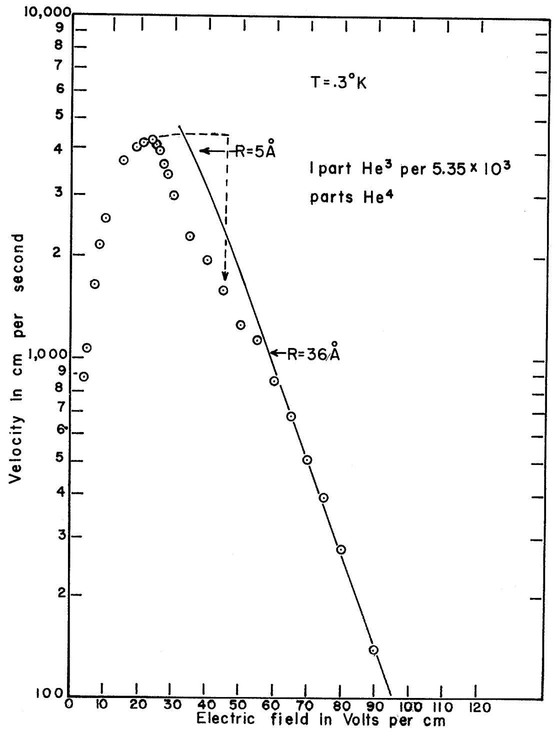

FIG. 2. Velocity versus electric field for positive-ion complexes with He^3 scattering. The theoretical vortex-ring radius at each of two different velocities is also indicated.

indicate the critical velocity for vortex-ring formation is not associated with roton creation.[5,6]

In the case of the negative ion, the viscous force on the ion is larger and the curve of velocity versus electric field looks similar to that for positive ions when the quasiparticles responsible for the viscous force are rotons.

If a complete vortex is formed when the ion reaches the critical velocity, a discontinuous curve like the dotted one in Fig. 2 should result.

The shape of the velocity curve in Fig. 2 is also interesting. The velocity of a vortex ring as a function of electric field is easily determined. Assume the usual expression for vortex-ring velocity, $v=(\kappa/4\pi r)[\ln(8r/a)-\frac{1}{2}]$, and set the frictional force on the vortex ring $\mathfrak{F}=\text{const}\times v\times 2\pi r=q\mathcal{E}$,[1,5,8] where r is the vortex-ring radius and $\mathcal{E}$ is the applied electric field. On substitution for v, the expression may be written $\mathcal{E}=\alpha[\ln(8r/a)-\frac{1}{2}]$, where α is a constant which has previously been determined.[1] Plotting $\ln v$ vs $\mathcal{E}$ results in the almost straight solid line shown in Fig. 2. For ion complexes having a velocity greater than 1000 cm/sec, corresponding to a vortex-ring radius of about 36 Å, significant deviations from the hydrodynamic equations are found.[8] Moreover, the deviation is to the left rather than the right, indicating that the viscous force on the complex is reduced by the presence of the ion. The presence of the ion on the vortex line should not greatly change the velocity of a ring of radius R, but the removal of a length of vortex line equal to the diameter of the ion from a small vortex ring could appreciably affect the quasiparticle scattering and reduce the frictional force on the vortex rings.

Determining the dispersion relation for the ion–vortex-ring complex near the critical velocity is rather difficult since, at best, the velocity field is a superposition of the flow fields for the ion and vortex ring.

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