[1]A. Citron et al., Phys. Rev. 144, 1101 (1966); S. W. Kormanyos et al., Phys. Rev. Letters 26, 709 (1966).

[2]S. Chu and C. Tan, University of California Radiation Laboratory Report No. UCRL-17511, 1967 (unpublished).

[3]S. Mandelstam, University of California Radiation Report No. UCRL-17250, 1966 (unpublished).

[4]C. E. Jones and V. L. Teplitz, Phys. Rev. (to be published).

[5]S. Mandelstam, Nuovo Cimento 30, 1148 (1963).

[6]N. N. Khuri, "On the Possibility of an Infinite Sequence of Regge Recurrences" (to be published).

[7]In Ref. 6, it was assumed that $\gamma(s)=\beta(s)(s-4)^{-\alpha(s)}$ was bounded by $f(s)$. We are indebted to Professor Khuri for pointing out to us that his argument may be equally well carried out by assuming that $\beta(s)$ is bounded by $f(s)$. The additional convergence which appears to be present in the latter case because of the absence of the factor $(s-4)^{\alpha}$ is negated by using the fixed-$z$ limit for $P_\alpha(z)$ as we have done in Eq. (8), rather than the fixed-$t$ limit of Ref. 6.

[8]Hypotheses (iv) and (iii) appear superficially not to be independent. If, for example, $\beta(s)>f(s)$, unless there are special cancellations, $a(l,s)$ will also exceed $f(s)$, in which case normal partial-wave dispersion relations become invalid. More precise statements are difficult to make, and we will treat (iii) and (iv) as independent below. We are grateful to Professor Khuri for discussions on this point.

[9]S. Mandelstam, Ann. Phys. (N.Y.) 19, 254 (1962).

[10]See E. C. Titchmarsh, Theory of Functions (Oxford University Press, New York, 1939), Sec. 5.81.

[11]R. P. Boas, Jr., Entire Functions (Academic Press, Inc., New York, 1954).

[12]Ref. 4, and also, S. Mandelstam and L. Wang, Phys. Rev. (to be published).

[13]A. M. Jaffe, Phys. Rev. Letters 17, 661 (1966).

[14]J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, Phys. Rev. Letters 18, 32 (1967); and V. Singh, ibid. 18, 36 (1967).

---

# FORWARD PEAKS FROM THE EXCHANGE OF ODD-PARITY REGGE TRAJECTORIES*

R. F. Sawyer
University of California, Santa Barbara, California

(Received 5 June 1967)

It is shown that a parity-doubled (at $t=0$), $I=1$, odd-signature trajectory is favored by a simple dynamical mechanism. A set of experiments is described whereby one could settle decisively the question of which (if any) of the possible O(4) conspiracies at $t=0$ is realized.

(1) There has been considerable interest recently in the possibility that the exchange of a pion Regge trajectory could give rise to a forward peak in reactions such as $n+p\to p+n$, $\gamma+N\to\pi+N$.[1-4] Such a contribution of an odd-parity trajectory to forward processes must necessarily involve conspiracy between trajectories degenerate at $t=0$, of the kind predicted by O(4) symmetry.[5-8] If one of the leading conspirators is to be the pion trajectory, the conspiracy must be of class III, in the terminology of Ref. 8. This is the parity-doubled class, in which the pion $[G=P=(-1)^{J+1}]$ conspires with a trajectory for which $G=-P=(-1)^{J+1}$ (at the same leading point, $J=\alpha$).

However there is a grave theoretical difficulty with this idea, as can be seen from the simplest O(4) consideration. The two angular momenta $(j_1,j_2)$ which characterize class III differ by unity. ($j_1+j_2=p$, the index in which the amplitude is to be analytically continued; $|j_1-j_2|=M$, which is to remain fixed.[7,8]) For $M=1$, the minimum physical angular momentum in a representation is unity. Thus, if the pion trajectory were bound with $J=0$ at zero mass and were in class III, we would necessarily conclude that the pion was unobservable. Of course, away from zero energy the $T$ matrix will no longer be diagonal in the O(4) labels, but the pion mass is so small that the class-III trajectory should be nonsense choosing at $J=0$.

(2) It may nevertheless be of great interest to look at the possibility that a class-III, $I=1$ conspiracy does exist, with intercept $\alpha(0)\approx 0$,[9] but giving rise to no $J=0$ particles. We would then speculate about possible $J=1$ or $J=2$ intersections, depending on the signature. Note that which of the possible meson pairs can couple to the trajectories depends on the signature. Remembering that the distinctive feature of class-III exchange in forward scattering is the fact that the $s^\alpha$ contribution is pure spin double flip,[10] we see that observing which (if either) of the forward reactions $\pi+N\to\rho+N$ or $\pi+N\to\omega+N$ shows large spin double flip would indicate the signature of the trajectory.[11]

(3) There is a dynamical consideration which encourages taking seriously the class III, $I=1$, odd-signature possibility. Consider $\pi\omega$ scattering: We can obtain either a $P=(-1)^J$ or a $P=(-1)^{J+1}$ type of trajectory. The latter may be class II with a $J^P=1^+$, $I^G=1^+$ meson such as the (tenative) $B$ as the leading particle. However a trajectory with $P=(-1)^J$ is necessarily in class III. If $\pi\omega$ is bound to form a $J^P=1^-$ meson on a trajectory, this trajectory is necessarily parity doubled at $t=0$.[12] The argument is elementary; there are just two ways of combining the $\omega$ particle's O(4) spin $(\frac{1}{2},\frac{1}{2})$ with the orbital angular momentum $(\frac{1}{2}p,\frac{1}{2}p)$. One leads to the $1^+$ trajectory $(p\pm\frac{1}{2},\ p\pm\frac{1}{2})$; the other of the mixed parity case $(p+\frac{1}{2},p-\frac{1}{2})\oplus(p-\frac{1}{2},p+\frac{1}{2})$.

There are any number of candidates for our $1^-$ particle among the experimental bumps; $B$ is a prime candidate for the $1^+$ companion.[13] Of course, the trajectories need not have the same slopes. They merely must intersect at $t=0$.

(4) In an earlier note[10] we speculated on the implications of a class-II odd-signature, or class-III even-signature conspiracy, for some of the same processes that we consider here. Now let us allow class II or III of either signature. The leading quantum numbers of each of the four possibilities are given for $I=1$ as[14]

$$\begin{aligned}
&\text{II even}\quad P=-G=(-1)^{J+1},\quad G(\text{meson})=(+);\\
&\text{II odd}\quad P=-G=(-1)^{J+1},\quad G(\text{meson})=(-);\\
&\text{III even}\quad P=+G=(-1)^{J+1},\quad P=-G=(-1)^{J},\\
&\qquad\qquad G(\text{meson})=(-);\\
&\text{III odd}\quad P=+G=(-1)^{J+1},\quad P=-G=(-1)^{J},\\
&\qquad\qquad G(\text{meson})=(+).
\end{aligned}\tag{1}$$

Recalling that the $s^\alpha$ term in class III is pure spin flip and in class II, pure nonflip, we summarize in Table I the possible reactions off nucleons in which the detection of the meson polarization, or the mere presence of the forward peak, can give information as to which conspiracies are realized. It should be noted that class I (even-parity, nonconspiring) trajectories do not contribute to any of the first five forward reactions of Table I.

One simplification in the possible analysis of data results from the fact that in checking the predictions of Table I, all considerations on the existence of the peak or on the spin change in the meson system are independent of inelasticity at the nucleon vertex (provided always that the momentum transfer remains less than a pion mass, more or less).

(5) A clue as to whether both leading members of a class-III conspiracy should be observed as neighboring resonances, that is, whether both trajectories rise at about the same rate, would come only from detailed consideration of a theoretical model. For example, the $\pi\omega$ model (with a Bethe-Salpeter equation and $\rho$ exchange as the force) could be pursued as the possible source of our "$B$" conspiracy.

---

*Work supported in part by the National Science Foundation.

[1]L. Van Hove, summary report in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (University of California Press, Berkeley, 1967).

[2]E. Leader, unpublished.

[3]L. Durand, III, Phys. Rev. Letters 18, 58 (1967).

[4]M. B. Halpern, to be published.

[5]D. V. Volkov and V. N. Gribov, Zh. Eksperim. i Teor. Fiz. 44, 1068 (1963) [translation: Soviet Phys.—JETP 17, 720 (1963)].

Table I. Contributions of order $s^\alpha$ from various $I=1$ conspiracy classes to various forward reactions off nucleon targets. By $\sigma$ we mean a $J^P=0^+$, $I^G=0^+$ particle. Class III is pure double spin flip; class II, pure spin nonflip. Even or odd refers to signature.

| | III (even) ($\pi$ conspiracy) | III (odd) ($B$ conspiracy) | II (even) | II (odd) ($A_1$ conspiracy) |
|---|---|---|---|---|
| $\pi+N\to\rho+N$ | yes | no | no | yes |
| $\pi+N\to\omega+N$ | no | yes | yes | no |
| $\gamma+N\to\pi+N$ | yes | yes | no | no |
| $\pi+N\to\sigma+N$ | no | no | no | yes |
| $\gamma+N\to A_1+N$ | yes | yes | yes | yes |
| $N+N\to N+N$ | yes | yes | yes | yes |

[6]G. Domokos and P. Suranyi, Nucl. Phys. 54, 529 (1964).

[7]M. Toller, University of Rome Nota Interna 76 and 84, 1965 (unpublished).

[8]D. Z. Freedman and J. M. Wang, Phys. Rev. (to be published).

[9]To give the energy dependence of the $n+p\to p+n$ forward peak.

[10]R. F. Sawyer, to be published.

[11]Spin nonflip contributions to forward $\pi+N\to\rho+N$ can come from the exchange of an $A_1$ conspiracy (class II, see Ref. 10). There is some evidence that in the reactions $\pi^-+p\to\rho^0+n$ and $\pi^-+N\to\rho^0+(N+n\pi)$ at lab momentum 18 BeV/$c$, spin-nonflip dominates for the smallest momentum transfers. I am grateful to Professor D. Caldwell for showing me an analysis of his data, a preliminary account of which appeared in Jones et al., Phys. Letters 21, 590 (1966).

[12]The combination $\pi\omega$ has of course been suggested many times as a prime ingredient of a dynamical $\rho$ meson. However the $1^-$ member of our conspiracy cannot be a $\rho$ (since it becomes uncoupled from any spin-zero plus spin-zero system at zero energy). It would no doubt mix with the $\rho$, except at zero energy.

[13]A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

[14]This notation for quantum numbers may be somewhat unconventional. The given dependence of $G$ on $J$ is for the nucleon-antinucleon system. In $\bar{N}+N\to\bar{N}+N$ there is no connection between charge conjugation and signature. (The $st$ spectral region remains the $st$ spectral region under the operation $C$.) But for the process $\bar{N}+N\to$ two different bosons, charge conjugation interchanges the $st$ and $su$ spectral regions in such a way that the intrinsic $G$ parity of the trajectory, for purposes of coupling to mesons, is a signature-dependent constant along the trajectory. The way to remember the results is, of course, to demand that our $\bar{N}N$ $G$ parity agree with $G$(meson) at the right-signature integers.

# SUM RULES FOR THE SPECTRAL FUNCTIONS OF SU(3) ⊗ SU(3)*

S. L. Glashow
Harvard University, Cambridge, Massachusetts

and

Howard J. Schnitzer
Brandeis University, Waltham, Massachusetts

and

Steven Weinberg†
University of California, Berkeley, California

(Received 15 May 1967)

The spectral-function sum rules are given for a general Lie algebra, and then applied to the vector and axial-vector mesons coupled to the currents of SU(3) ⊗ SU(3).

The spectral-function sum rules[1] for chiral SU(2) ⊗ SU(2) have been used to relate the $\rho$ and $A1$ masses, and to calculate the $\pi^+$-$\pi^0$ mass difference.[2] In this note we shall state the corresponding sum rules for a general Lie algebra, and will then show that the sum rules for the SU(3) ⊗ SU(3) spectral functions can be used to make sense out of the masses and couplings of the observed vector and axial-vector mesons.

We will first consider a general multiplet of currents which satisfy the commutation relations

$$[J_\alpha{}^0(\vec{x};t), J_\beta{}^\nu(\vec{y},t)] = -iC_{\alpha\beta\gamma}\delta^3(\vec{x}-\vec{y})J_\gamma{}^\nu(\vec{x},t)+S.T., \tag{1}$$

where $C_{\alpha\beta\gamma}$ is the structure constant of a simple Lie algebra,[3] and $S.T.$ denotes a Schwinger term.[4] The spectral-function sum rules are

$$\int\rho_{\alpha\beta}{}^{(1)}(\mu^2)\mu^{-2}d\mu^2+\int\rho_{\alpha\beta}{}^{(0)}(\mu^2)d\mu^2=S\delta_{\alpha\beta}, \tag{2}$$

$$\int\rho_{\alpha\beta}{}^{(1)}(\mu^2)d\mu^2=Z\delta_{\alpha\beta}, \tag{3}$$