REGGE-POLE MODELS FOR PION EXCHANGE REACTIONS*

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The problem of obtaining a satisfactory Regge-pole model for the pion-exchange reactions $np \to pn$, $\pi N \to \rho N$, $\pi N \to \rho N^*$, $\gamma p \to \pi^+ n$, and $\bar{p}p \to \bar{N}^* N^*$, is discussed with particular emphasis on the dynamics as seen in the s channel. It is pointed out that (i) satisfactory models can be obtained using a modified conspiracy model; (ii) pion exchange should provide the dominant contribution to the forward cross sections for incident momenta up to 30-50 GeV/c; and (iii) the energy dependence of the differential cross sections should change from that characteristic of pion exchange for $t \sim 0$, to that characteristic of the exchange of vector or tensor mesons at large momentum transfers $t \lesssim -0.5$ (GeV/c)^2.

It has been evident for some time that the simpler versions of the Regge-pole exchange model do not provide an adequate description of those high-energy reactions which are expected to proceed primarily through the exchange of a single pion. Examples are provided by forward np charge exchange ($np \to pn$), ρ-meson production in πN collisions ($\pi N \to \rho N, \pi N \to \rho N^*$), photoproduction of charged pions at forward angles ($\gamma p \to \pi^+ n$), and isobar production in nucleon-antinucleon collisions ($\bar{p}p \to \bar{N}^* N^*$). It is notable that such details of these processes as the absolute magnitude, energy dependence, and shape of the differential reaction cross sections at small momentum transfers, the decay angular distributions of the unstable particles produced, and the joint decay correlations in the case of double resonance production, are described quite well for present energies by the modified single-pion-exchange model (absorptive model).[1] In contrast, the simple Regge-pole model for single pion exchange fails in each case to account for one or more features of the experimental data. Models which involve conspiracies[2-4] between the pion or the A_1 meson and other Regge trajectories have been somewhat more successful, but have generally been regarded as artificial and overly complicated.

In the present paper, we wish to indicate how a satisfactory and relatively simple description of the foregoing reactions can be obtained with the framework of Regge-type theories. It will be advantageous first to discuss the problem as seen in the s channel, and in particular, the reasons for the success of the modified single-pion-exchange model. The difficulties encountered with the usual Regge-pole models are then easily understood, as is their resolution. For simplicity, we will restrict our detailed comments to the np charge-exchange reaction, and only indicate briefly the relevance of our results to the rest of the reactions noted above.

The differential np charge-exchange cross section is characterized for momenta of 2-8 GeV/c by an extremely sharp forward peak with a width $\lesssim m_\pi^2$ in terms of the momentum-transfer variable t, and an energy dependence close to that expected for a pion-exchange reaction.[5] These features of the cross section suggest that the long-range part of the interaction, hence, the small-angle cross section, is associated with the exchange of a single pion. The s-channel helicity amplitudes $M(\lambda_p', \lambda_n'; \lambda_n, \lambda_p)$ for this process are given in Born approximation for single pion exchange by

$$M(-\tfrac{1}{2},-\tfrac{1}{2};\ \tfrac{1}{2},\tfrac{1}{2}) = M(-\tfrac{1}{2},\tfrac{1}{2};\ \tfrac{1}{2},-\tfrac{1}{2}) = M(\tfrac{1}{2},-\tfrac{1}{2};\ -\tfrac{1}{2},\tfrac{1}{2}) = M(\tfrac{1}{2},\tfrac{1}{2};\ -\tfrac{1}{2},-\tfrac{1}{2}) = -(2p/\sqrt{s})(g_{\pi NN}{}^2/4\pi)t/(m_\pi{}^2-t), \tag{1a}$$

$$M(\lambda_p', \lambda_n'; \lambda_n, \lambda_p) = 0, \text{ all other helicities.} \tag{1b}$$

The normalization is such that the partial-wave transition amplitudes are bounded by unity, $|M_j| \leqslant 1$ (unitarity). The differential scattering cross section is given by

$$d\sigma/d\Omega = (16p^2)^{-1} \textstyle\sum_{[\lambda]} |M_{[\lambda]}|^2.$$

Because of the factor of t which appears in each of the transition amplitudes in Eq. (1a), the Born approximation for the charge-exchange cross section vanishes in the forward direction ($t=0$), and the forward peak observed in current experiments is not reproduced. The same problem is present in the Regge-pole model: If the only exchange is that of a Reggeized pion, the predicted charge-exchange cross section vanishes at $t=0$.

The foregoing difficulties are directly connected with the assumption that the t-channel exchanges are confined to the singlet spin channel of the $\bar{N}N$ system.[2,3] For such exchanges, the s-channel transition amplitude has the structure indicated in Eq. (1), with all except the four helicity amplitudes in Eq. (1a) zero, and those four equal. Since the amplitudes $M(-\frac{1}{2}, \frac{1}{2}; \frac{1}{2}, -\frac{1}{2})$ and $M(\frac{1}{2}, -\frac{1}{2}; -\frac{1}{2}, \frac{1}{2})$ for a total helicity change $|\lambda_n-\lambda_p-\lambda_p{}'-\lambda_n{}'|=2$ necessarily vanish in the forward direction ($x=1$) as $1-x=-t/2p^2$,[6] corresponding zeros appear in the amplitudes $M(-\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{1}{2})$ and $M(\frac{1}{2}, \frac{1}{2}; -\frac{1}{2}, -\frac{1}{2})$. The zeros in the latter are specific to the models considered, and are neither required by general principles,[6] nor expected to present in a complete dynamical theory. In fact, if the function $t/(m_\pi^2-t)$ in Eq. (1) is rewritten as

$$t/(m_\pi^2-t)=m_\pi^2/(m_\pi^2-t)-1, \tag{3}$$

it is evident that the forward zeros in the Born approximation result from a cancellation between an anomalous $j=0$ term $(2p/\sqrt{s})(g_{\pi NN}^2/4\pi)$ and the remainder of the amplitude. For large s, the anomalous term is energy independent, and many times larger than the absolute unitarity limit $g_{\pi NN}^2/4\pi \sim 14 \gg 1$. If this term is reduced to an acceptable magnitude, as by the absorptive corrections in the modified single-pion-exchange model,[1] the cancellation noted above is eliminated, and the amplitudes $M(-\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{1}{2})$ and $M(\frac{1}{2}, \frac{1}{2}; -\frac{1}{2}, -\frac{1}{2})$ peak strongly for $|t| \lesssim m_\pi^2$. The cross section predicted by this model agrees quite well with the experimental results at small momentum transfers.[1] The success of the modified single-pion-exchange model in the description of the reactions $\pi N \to \rho N$, $\pi N \to \rho N^*$, $\gamma p \to \pi^+ n$, and $\bar{p}p \to N^*N^*$, including such details as the resonance decay correlations, can also be traced to the elimination of unphysical contributions in the low partial waves.

It should be emphasized, first, that the modified pion-exchange amplitudes for these reactions do not describe pure 0^- exchange in the t channel, and second, that the precise manner in which the low partial-wave amplitudes are modified is relatively unimportant for $t \sim 0$, provided that unitarity is enforced. In this case, the fact that the pion-exchange amplitude contains significant contributions from very high partial waves,[7] $j \sim p/m_\pi$, while the contributions from such short-range processes as vector-meson exchange are confined to low partial waves, $j \lesssim p/m_V$, is crucial. If the exchanges are of comparable strength in the low partial waves, the pion-exchange contribution to the forward-scattering amplitude will be enhanced relative to that of the massive exchange by a geometrical factor $\sim m_V^2/m_\pi^2$. It is readily shown that pion exchange is strong in this sense for the reactions $np \to pn$, $\pi N \to \rho N$, $\pi p \to \rho N^*$, $\gamma p \to \pi^+ n$, and $\bar{p}p \to \bar{N}^*N^*$ at momenta of a few GeV/c. As a consequence, pion exchange is the dominant process in the dynamical description of these reactions at small momentum transfers. Moreover, this dominance should persist to quite high energies, despite the rapid decrease of the pion-exchange amplitudes with increasing energy (as s^{-1} relative to elementary vector exchange, or as $\sim s^{-1/2}$ relative to Reggeized vector or tensor exchange). For example, in the reactions $\pi^\pm p \to \rho^\pm p$, the ω- and π-exchange contributions to the low partial-wave amplitudes are of comparable magnitude for momenta of ~2 GeV/c. Even with the conservative assumptions of an s^{-1} energy dependence of the π-exchange amplitudes relative to those for ω exchange, and a much reduced enhancement factor $m_\omega^2/6m_\pi^2 \sim 5$, the π- and ω-exchange contributions to the forward ρ-production cross section would first become equal to an incident momentum ~12 GeV/c. A more reasonable estimate suggests that pion exchange will remain the dominant process for $t \sim 0$ for momenta less than 30-50 GeV/c.[8]

The situation is quite different at large momentum transfers ($t \lesssim -m_V^2$). Since the behavior of the scattering amplitude in this region is determined primarily by low partial waves, the geometrical enhancement factor in favor of pion exchange is not operative, and the rapid decrease of the pion-exchange amplitude with increasing energy can lead even at moderate energies to the dominance of massive exchanges.

It is clear that the difficulties which have been encountered with simple Regge-pole models for pion-exchange reactions result from the assumption that pion exchange may be treated independently of whatever other exchanges are present. This assumption conflicts with the conclusions of the arguments above, that extra exchanges must be present, and that these must be correlated with the basic pion exchange, if the dynamical effects of pion exchange are to be properly described. An alternative to the simple Regge-pole model is provided by the conspiracy model proposed by Volkov and Gribov.[2] Those authors observed that the kinematic constraints on the s-channel helicity amplitudes which lead, for example, to the unwanted zeros in the pion-exchange amplitude in np scattering, could be satisfied by a cancellation among the contributions of several Regge poles, each of which by itself escapes the kinematic restrictions. The two types of conspiracy which are possible in NN scattering[4] have both been considered in connection with the np charge-exchange reaction. The first, of Type II in the nomenclature of Ref. 4, involves the trajectory associated with the A_1 meson, and a daughter trajectory which, like the pion, appears in the singlet spin channel in $\bar{N}N$ scattering. Since the residue of the daughter pole need not vanish at $t=0$, it was suggested in Ref. 3 that the forward peak in the np charge-exchange reaction could be explained by destructive interference between the amplitudes associated with the daughter trajectory and the pion.[9] The model has not proved satisfactory.[10] A second model, suggested by the work of Freedman and Wang,[4] is based on the assumption that the pion trajectory is involved in a Type-III conspiracy. In this case, the pion residue function can remain finite at $t=0$. The remaining members of the conspiracy are trajectories which would be associated with 0^+ and 1^+ particles were they to reach physical values of the angular momentum. Although this model can be made to work, the results which have been obtained[11] have seemed somewhat suspect, as it is not possible to obtain the correct magnitude for the forward charge-exchange cross section without the introduction of what appears to be an unphysically rapid variation of the Regge-residue functions between $t=m_\pi^2$ and $t=0$.

We wish to point out two possible resolutions of the foregoing problems, both suggested by dynamical considerations. It should be noted, first, that conspiracies of both types arise naturally in perturbation calculations, and are probably a necessary ingredient of unitary theories. For example, in the case of the charge-exchange reaction, the terms in the scattering amplitude associated with the box diagram in which pion exchange is preceded or followed by the exchange of a vector meson (ρ, ω, or Pomeranchuk exchange) satisfy the Type-III conspiracy conditions.[12] [This diagram is associated, not surprisingly,[1] with a rescattering correction to the pion-exchange term.] It is of course expected that the replacement of the box by a sum of ladder diagrams will result in the usual fashion in the Reggeization of these amplitudes, hence, generate conspiring Regge poles. Because of the short range of the exchanges involved, it is clear that the resulting amplitudes will vary slowly near $t=0$. Two possibilities exist. First, the 0^+ object in this conspiracy may have nothing to do with the pion. In this case, we obtain a simple and flexible model in which destructive interference between the amplitude for the exchange of this 0^- object and the Reggeized pion-exchange amplitude can explain the forward peak observed in the charge-exchange cross section.[9] [A similar model which involves conspiracy among Regge cuts has been suggested by Huang and Muzinich.[13]] Alternatively, if the 0^- object is in fact the pion, the slow variation of the extra terms in the complete amplitude near $t=0$ implies that the pion residue function must vary rapidly in this region, in accord with the empirical results of Ref. 11. Similar results can be obtained for the reactions $\pi N\rightarrow\rho N$, $\pi N\rightarrow\rho N^*$, $\gamma p\rightarrow\pi^+n$, and $\bar{p}p\rightarrow\bar{N}^*N^*$. Since, in each case, the addition of a Type-III conspiracy to the normal pion-exchange amplitude reproduces the basic features of the modified single pion-exchange model, it is clear that the satisfactory models can be constructed for these reactions.

The main implications of the foregoing discussion can be summarized as follows:

(i) Models based on the existence of a Type-III conspiracy can provide reasonable parametrizations for pion-exchange reactions. It is not necessary that the pion trajectory participate in the conspiracy. However, detailed quantitative descriptions of these reactions may be rather intricate, and involve Type-II conspiracies, and conspiring cuts,[13] as well as the Type-III conspiracy suggested above.

(ii) In those reactions in which pion exchange is strong (in the dynamical sense), the main contributions to the forward-scattering amplitude should be determined up to quite high incident momenta (25-50 GeV/c, depending on the reaction) by the conspiracy described above. The contributions from vector or tensor exchanges should be prominent at intermediate energies only in the scattering at large momentum transfers, $t \lesssim -0.5$ (GeV/c)^2.[8]

(iii) The effective energy dependence of the differential cross sections for the foregoing reactions at intermediate energies should change from the $\sim s^{-1}$ behavior characteristic of pion exchange near $t \sim 0$, to that typical of the exchange of vector or tensor mesons at large momentum transfers.

(iv) We wish to emphasize, finally, that dynamical information, whether obtained from consideration of the reaction in question in the s channel, or from simple models, can be of great value in the formulation of Regge-pole models.

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[1] The theoretical background of the model is discussed by L. Durand, III, and Y. T. Chiu, Phys. Rev. 139, B646 (1965). Applications to the reactions above are discussed in the following papers, among others: $np \rightarrow pn$, E. M. Henley and I. J. Muzinich, Phys. Rev. 136, B1783 (1964); L. Durand, III, and Y. T. Chiu, Phys. Rev. 137, B1530 (1964); G. A. Ringland and R. J. N. Phillips, Phys. Letters 12, 62 (1964). $\pi N \rightarrow \rho N$, $\pi N \rightarrow \rho N^*$, J. D. Jackson et al., Phys. Rev. 139, B428 (1965). $\gamma p \rightarrow \pi^+ n$, L. Durand, III, unpublished; B. Richter, report to the International Symposium on Electron and Photon Interactions at High Energies, 1967 (to be published). $\bar{p}p \rightarrow \bar{N}^* N^*$, L. Durand, III, and Y. T. Chiu, unpublished; B. E. Y. Svensson, Nuovo Cimento 37, 714 (1965).

[2] D. V. Volkov and V. N. Gribov, Zh. Eksperim. i Teor. Fiz. 44, 1068 (1961) [translation: Soviet Phys. -JETP 17, 720 (1963)].

[3] L. Durand, III, Phys. Rev. Letters 18, 58 (1967).

[4] D. Z. Freedman and J.-M. Wang, Phys. Rev. Letters 18, 863 (1967); Phys. Rev. 160, 1560 (1967).

[5] H. Palevsky et al., Phys. Rev. Letters 9, 509 (1962); G. Manning et al., Nuovo Cimento 41A, 167 (1966).

[6] The rotation coefficients $d_{i,-i}{}^{j}(x)$ which appear in the partial-wave expansion of the double-helicity-flip amplitudes $M(-\frac{1}{2},\frac{1}{2};\frac{1}{2},-\frac{1}{2})$ and $M(\frac{1}{2},-\frac{1}{2};\frac{1}{2},-\frac{1}{2})$ contain $1-x$ as a common factor. In contrast, the partial-wave expansion of the amplitudes $M(-\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2})$ and $M(\frac{1}{2},\frac{1}{2};-\frac{1}{2},-\frac{1}{2})$ involves the ordinary Legendre functions. [Cf. M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959).] Any zeros in the latter are consequently of dynamical rather than kinematical origin.

[7] The high partial-wave amplitudes should be described properly by the Born approximation for single-pion exchange up to quite high energies: These amplitudes can be modified only by exchanges with ranges $\sim m_\pi^{-1}$.

[8] The situation is rather different for such SU(3) analogs of the reactions above as $\gamma N \rightarrow K\Lambda$, $\bar{p}p \rightarrow \bar{Y}^* Y$, and $\pi N \rightarrow K^* \Lambda$, since the geometrical factor which favors K exchange relative to K^* exchange is much smaller. As a consequence, K^* exchange can be important at low momenta, and should become the dominant process for momenta above ~20 GeV/c.

[9] It is clear from Eqs. (1a) and (3) that the addition to the pion-exchange term of a negative contribution from the conspirator (destructive interference) will have the same general effect as the absorptive corrections in the modified single-pion-exchange amplitude, hence, lead to a forward peak in the np charge-exchange cross section.

[10] I. J. McGee and A. R. Swift, private communication. Because of the low $t=0$ intercept of the daughter Regge trajectory, the interference term which leads to the forward peak decreases too rapidly with increasing s.

[11] F. Arbab and J. W. Dash (to be published).

[12] A conspiracy exists of the coefficient functions M_S and M_V in the usual expansion of the NN scattering amplitude in terms of Dirac covariants vanish at $t=0$, while M_T remains finite. These conditions are satisfied by the graph in question.

[13] K. Huang and I. J. Muzinich, to be published; K. Huang, private communication.