ing to us the probable relevance of the temperature derivatives of transport quantities, and M. Blume, V. Emery, M. E. Fisher, P. C. Martin, and J. Langer for a number of comments.

Note added in proof.—Dr. M. E. Fisher and Dr. J. S. Langer have independently pointed out to us that while the Ornstein-Zernike model is a reasonable one for large ranges, at short ranges one may expect the resistivity to be controlled by factors involving the system energy. Should this occur, then $\alpha(T)$ is expected to vary approximately with the specific heat, as is qualitatively observed.

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TEMPERATURE AND FREQUENCY DEPENDENCE OF THE AMPLITUDE OF THE RADIO-FREQUENCY SIZE EFFECT IN GALLIUM*

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In recent years a number of workers have used the radio-frequency size effect (RFSE) to determine the dependence of the mean collision time on temperature for well-defined groups of carriers on the Fermi surface.[1-4] The underlying assumption used in the analysis of the experimental data has been that the strength of a resonance is proportional to $\exp(-1/\omega_c\tau)$, where ω_c is the cyclotron frequency and τ the mean collision time. This Letter reports measurements of the strength of the RFSE in very pure gallium which cannot be explained in such simple terms. We have found resonances whose strength S increases monotonically with tem-

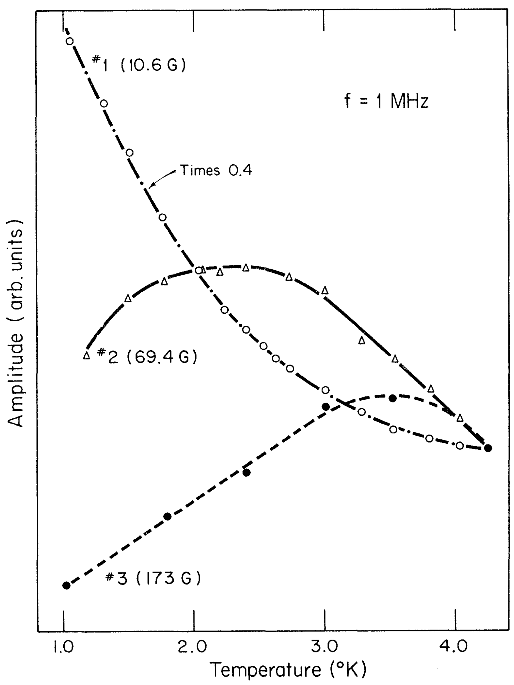

FIG. 1. Temperature dependences of three RFSE resonances measured in a gallium single crystal with the c axis normal to the surface. The ordinate is given on a linear scale with all three resonances normalized at 4.2°K. Note that the amplitude of resonance No. 1 has been multiplied by 0.4.

perature in the range from 1 to 4°K, others in which S has a maximum near 3°K, and others which closely follow the simple exponential rule described above. We show that the temperature dependence of S is strongly affected by the frequency of the rf field. The dependence of strength on both frequency and temperature can be explained by a simple model which takes into account the phase shift of the rf field between successive oribts of the resonating electrons. The model shows that it is possible to extract from the RFSE data not only the "diameters" of extremal orbits on the Fermi surface but also the temperature dependence of τ and of the cyclotron frequency ω_c.

We measure changes in the reactive skin depth δ, using two-sided asymmetric rf excitation of the specimen. A tunnel-diode oscillator circuit placed in the helium bath is used to supply the rf field. A complete description of the apparatus and technique will be given elsewhere.

Figure 1 shows examples of the different types of behavior we observe at an operating frequency of 1 MHz. The curve labeled No. 1 is for a resonance at 10.6 G in a crystal 0.168 mm thick, with the c axis normal to the surface and the magnetic field H parallel to the b axis. Curves Nos. 2 and 3 are for resonances in the same specimen with $H = 69.4$ G aligned 45° from the b axis, and $H = 173$ G aligned 12° from the a axis, respectively. The identification of these resonances with sheets of the Fermi surface has been described elsewhere.[5] The resonance amplitude is the difference between values of $d\delta/dH$ taken at two prominent extrema in the resonance line which preserve their general shape as the frequency and temperature are changed.

The model for the resonance strength is based on a path-integral calculation[6] of the change in the total current in each skin layer due to the electrons which transverse both skin layers at least once. (Details of the calculation will be presented elsewhere.) The derivation has been carried through only to the extent that all frequency and collision-time dependent factors are extracted, leaving the final expression for the strength in terms of these factors and unknown dimensionless expressions which depend on the field distribution in the metal. As in Chambers's theory of the cyclotron-resonance line shape,[7] the field distribution is assumed to be unaffected by the (relatively small) resonant contribution to the current. The basic idea is that an electron making a pass through the skin layer at time t changes its velocity by $eE(t)\Delta t/m^*$, where Δt is the transit time. This electron has a probability $\exp(-n\pi/\omega_c\tau)$ of having made n such passes, the nth pass being in a field $E(t)\exp(-in\pi\omega/\omega_c)$. Summing over n shows that the total contribution to the current is proportional to the quantity $A = A_1 + iA_2$, with

$$A_1 = \frac{1-\alpha\cos\theta}{1-2\alpha\cos\theta+\alpha^2}, \quad A_2 = \frac{-\alpha\sin\theta}{1-2\alpha\cos\theta+\alpha^2}, \qquad (1)$$

where $\alpha = e^{-x}$, $x = \pi/\omega_c\tau$, and $\theta = 2\pi\omega/\omega_c$. The effect of this current on the skin depth depends on the phase of the nonresonant current with respect to the applied electric field. One can easily show that if the total current in the skin layer changes by $\Delta J = \Delta J_1 + i\Delta J_2$, then the change in δ is given by

$$\frac{\Delta\delta}{\delta} = -\frac{4\pi\omega\delta}{c^2E(0)}[f_1(\varphi)\Delta J_1 + f_2(\varphi)\Delta J_2],$$

where $f_1 = 2\cos\varphi$, $f_2 = 2-\csc^2\varphi$, and φ is the argument of the surface impedance Z. To calculate ΔJ we assume that the field distribution in the metal is $E(z)=E(0)F(z/\delta)/F(0)$, where the z axis is normal to the surface and extends into the metal. Contributions to the current density are calculated as outlined above, and after some manipulation we obtain an expression for $d\delta/dH$ which is valid insofar as the ω and τ dependences are concerned, provided that we restrict ourselves to the principal extrema of the resonance. We find

$$\frac{d\delta}{dH}=\frac{8\pi D^{1/2}}{H\sqrt{2}}\frac{\rho e^2}{p_F c^2}\omega\delta^{7/2} \times[f_1(\varphi)\,\mathrm{Re}\{F(0)^{-1}AG((D-d)/\delta)\} +f_2(\varphi)\,\mathrm{Im}\{F(0)^{-1}AG((D-d)/\delta)\}], \quad (2)$$

where D is the "diameter" of the extremal orbit in the field H, d is the specimen thickness, and ρ the electron density. A is defined by Eq. (1) and the other symbols have their usual meaning. The function G is given by

$$G(u)=\int_{-\infty}^{u}\frac{F(\xi)}{(u-\xi)^{1/2}}d\xi,$$

assuming that the electron trajectory is elliptical while in the skin layer. [Because of the singularity in the integrand, $G(u)$ is a measure of the rf electric field strength at $u=z/\delta$.] The functions F and, hence, G are not known a priori, nor is the angle φ, except in zero field. However, the difference in $d\delta/dH$ between one extremum and another can be written as

$$(d\delta/dH)_1-(d\delta/dH)_2\equiv S=\mathrm{const} \times\omega\delta^{7/2}[f_1(\varphi_{\mathrm{eff}})A_1+f_2(\varphi_{\mathrm{eff}})A_2],$$

where φ_{eff} is regarded as an undetermined empirical parameter. In fact it turns out that φ_{eff} must be taken quite close to 60°, which is the known value for φ at $H=0$, in order to get even qualitative agreement with experiment. Finally, we need to know the frequency dependence of δ in the presence of the dc field H. We have measured this at 1°K and at $H\simeq 11$ and 70 G and find the same $\omega^{-1/3}$ dependence one gets for δ in the anomalous limit in zero field. Therefore, the experimental values of $\omega^{1/6}S$ should be proportional to $f_1A_1(\alpha,\theta)+f_2A_2(\alpha,\theta)$. Using log-log paper it is possible to fit theory and experiment by shifting the experimental data expressed as a function of frequency over a family of theoretical curves (corresponding to different values of α) expressed as functions of θ.

In Figs. 2(a) and 2(b) we show the result of this fitting procedure for the resonances labeled No. 1 and No. 2 in Fig. 1, respectively. The ordinate labeled "amplitude" is $\omega^{1/6}S$. The abscissa is the variable θ, and separate scales for the frequency are included. The range of measurement is from 85 kHz to 65 MHz. The solid line is the predicted behavior, with $\varphi_{\mathrm{eff}}=60°$ in both cases. In Fig. 2(a) we find $x\equiv\pi/\omega_c\tau=0.004$ and $\omega_c=19\times10^3$ MHz, while in Fig. 2(b) $x=0.223$ and $\omega_c=2\times10^3$ MHz. For resonance No. 1 these values correspond to $m^*=0.01m_0$ and $\tau=4.1\times10^{-8}$ sec. For resonance No. 2 we get $m^*=0.6m_0$ and $\tau=0.7\times10^{-8}$ sec. We estimate that the uncertainty in m^* and τ arising from scatter and other errors in the data is of the order of 30 to 50%, depending on the magnitude of $\omega_c\tau$.

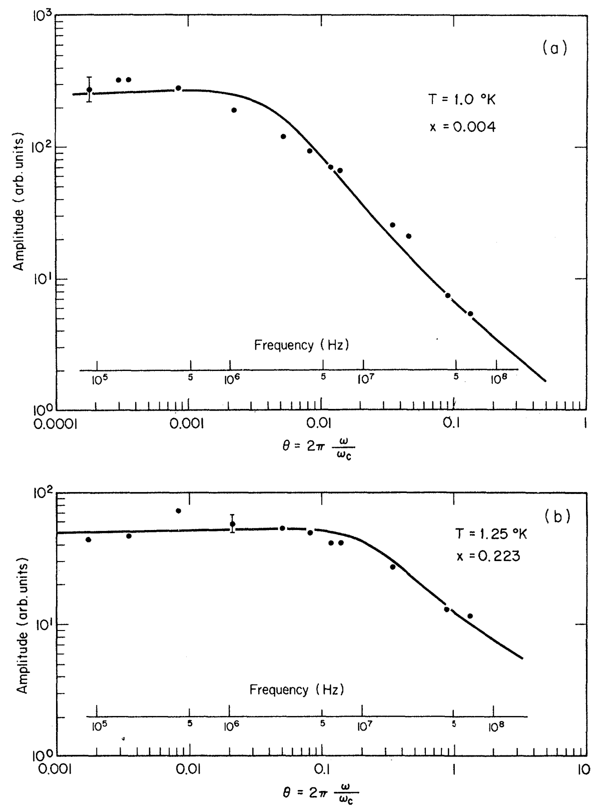

FIG. 2. Frequency dependence of the resonance amplitude near 1°K. The solid curves represent the theoretical expression for $\omega^{1/6}S$ as function θ. The points are the measured data for resonances (a) No. 1 and (b) No. 2.

By repeating this fitting process at various temperatures, the function $\tau(T)$ can be obtained. This is a very laborious procedure. Instead we have calculated the dependence of S on τ at fixed frequency (hence fixed θ) and compared this with the experimental graphs of amplitude versus T^n. Fig. 3 shows the result for resonance No. 1 at three frequencies, with the assumption $n \simeq 2$.[8] (Strictly speaking, we find that n varies slightly with frequency, from 1.8 at low frequencies to 2 near 65 MHz.) The agreement between theory and experiment is very striking, thereby confirming the values of x, θ, and φ_{eff} determined by the curve-fitting procedure for the frequency dependence (Fig. 2). The most important point is that the quasiexponential behavior of resonance No. 1 goes over smoothly to an almost flat dependence on T as the frequency is increased to 43 MHz, which corresponds to $\omega \simeq 0.014\omega_c$. Even at $\omega \simeq 10^{-3}\omega_c$ ($\theta = 0.008$), a pronounced curvature and latent saturation is seen (and predicted) in the temperature dependence of the resonance amplitude.

A similar determination of $\tau(T)$ has been made for resonance No. 2. We find $\tau \propto T^{-2}$ to T^{-3} depending on frequency. All the qualitative features of Fig. 3 are reproduced, except that the frequency scale is shifted. In particular, the model predicts that near $\theta = 0.003$, all traces of the maximum observed at 1 MHz should disappear. The corresponding measurements (at 85 kHz) show a rapidly increasing strength as T is reduced, in nearly perfect agreement with the model if $\tau \propto T^{-2}$. In addition, at 65 MHz S increases monotonically with T, in agreement with the model for $\theta = 1.3$. In general the quantitative agreement is not nearly as good as in the case of resonance No. 1. A clue to the difficulty may be found in the observation that the measured values of the resonance width ΔH do not scale proportionally to the skin depth, whereas the model requires that $\Delta H \propto \delta$, as can be seen from the form of Eq. (2). We also find that the shape of the resonance changes slightly as the frequency decreases, and new extrema appear. Both of these observations indicate that the assumption $E(z) = E(0) \times F(z/\delta)/F(0)$ may not be valid in this case.

In conclusion we note that the values of m^* and τ are in reasonable agreement with values determined by other techniques.[9,10] Similarly, the dependence of τ on T is compatible with other measurements.[9,11] For example, Moore[9] has shown that $\tau \propto T^{-3}$ for a cyclotron resonance with H parallel to the b axis and $m^* = 0.9m_0$, using the Chambers theory of the linewidth.[7] Our experimental inaccuracies make the determination of m^* and τ rather crude. In spite

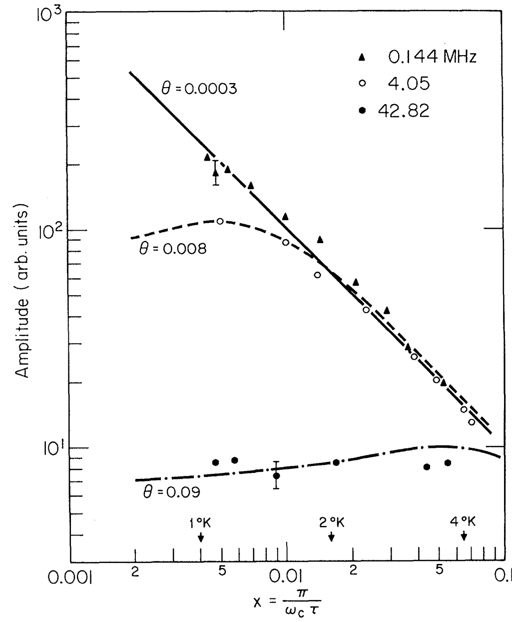

FIG. 3. Theoretical and experimental dependence of amplitude on $x = \pi/\omega_c\tau$. The experimental points are plotted on the assumption that $\tau \propto T^{-n}$, where $n = 1.8$ for $\theta = 0.0003$, $n = 2$ for $\theta = 0.008$ and also for $\theta = 0.09$.

of this, two important results can be deduced from our experiments. First, there is no question that the mean collision time can be very different on different sheets of the Fermi surface, i.e., τ(No. 1) $\simeq 6\tau$(No. 2) near 1°K. Secondly, the mass determinations are accurate enough to enable one to correlate values of m^* determined in cyclotron resonance or de Haas-van Alphen experiments with orbits on the Fermi surface whose shape is known from RFSE caliper measurements.

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J DEPENDENCE FOR $l = 1$ NUCLEON TRANSFER*

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The J dependence for $l = 1$ nucleon-transfer reactions at an incident-deuteron energy of 23 MeV is qualitatively reproduced by distorted-wave claculations. Considerably greater damping of contributions from the interior of the nucleus is required than previously thought necessary in (d,p) reactions. This may imply that the energy dependence of the real potential for protons is stronger than the dependence previously obtained for neutrons.

The experimentally observed angular distributions for the (d,p) reaction at 23 MeV show the J dependence qualitatively predicted by distorted-wave theory. This is in contrast with the large discrepancy between experiment and theory observed at lower energies.

Lee and Schiffer[1] have observed a very pronounced J dependence in (d,p) reactions for p-wave neutron transfer at backward angles. These experiments in the region of incident-deuteron energies around 12 MeV show a very sharp minimum for $p_{1/2}$ neutron transfers near 120° and a relatively flat angular distribution for $p_{3/2}$ neutrons in that angular range. Such an effect is not predicted by the distorted-wave theory, and attempts to reproduce the experimental angular distribution have been unsuccessful. Recently Johnson and Santos[2] have considered the effect of the D component of the deuteron internal wave function and have shown that for $l = 3$ the corrections resulting from the inclusion of the D component are important. However, inclusion of the D component does not produce an appreciable effect for $l = 1$ transfers at backward angles.

In an investigation of the (d, He^3) reaction on the molybdenum isotopes,[3] we have observed a significant experimental difference between the angular distributions corresponding to $p_{1/2}$ proton pickup and those corresponding to $p_{3/2}$ proton pickup. Representative angular distributions are shown in Fig. 1. The J dependence consists in the difference in the depth of the minimum near 18°. That this difference cannot be assigned to Q dependence is established by its observation over a sufficiently large range of Q values in the Zr and Mo isotopes. The distorted-wave calculations[4] reproduce the effect satisfactorily. The calculations show that the main contribution to the J-dependence effect is due to the spin-orbit term of the deuteron potential. Furthermore, if one uses sharp cutoffs of the radial integrals, the effect persists up to a cutoff radius of 6 F. On the other hand, the (d,t) reactions on these same isotopes do not show a significant J dependence in the angular distributions corresponding to $p_{1/2}$ and $p_{3/2}$ neutron pickup, and none is predicted by the distorted-wave calculations.

Since the J dependence for $l = 1$ pickup is correctly given by the distorted-wave calculations for the (d, He^3) and (d,t) experiments, which were done at an incident-deuteron energy of 23 MeV, it seemed of interest to investigate the (d,p) reaction at this same energy. A pronounced J dependence has been found[5] for the