ticular attention being paid to the longitudinal groups at the higher q values. In spite of our low residual background, the low scattered intensities make the problem particularly difficult. For this reason, new growth techniques which hopefully will allow the production of significantly larger and more suitable oriened crystals are being investigated.

The authors gratefully acknowledge informative discussions with M. Collins, P. Craig, V. Emery, P. Klemens, J. A. Leake, and R. Werthamer. The invaluable assistance of T. Oversluizen and M. Rosso in designing, testing, and operating the experimental equipment is also gratefully acknowledged.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Present address: Physics Department, University of Connecticut, Storrs, Connecticut.

[1]A. I. Shal'nikov, Zh. Eksperim. i Teor. Fiz. 41, 1056 (1961) [translation: Soviet Phys.–JETP 14, 753 (1962)]; L. P. Mezhov-Deglin, Zh. Eksperim. i Teor. Fiz. 49, 66 (1965) [translation: Soviet Phys.–JETP 22, 47 (1966)]; F. P. Lipschultz and D. M. Lee, Phys. Rev. Letters 14, 1017 (1965); C. C. Ackerman, et al., Phys. Rev. Letters 16, 789 (1966); J. E. Vos et al. Phys. Letters 24A, 738 (1967).

[2]W. H. Keesom and K. W. Taconis, Physica 5, 161 (1938); D. G. Henshaw, Phys. Rev. 109, 328 (1958); A. Schuch and R. Mills, Phys. Rev. Letters 8, 469 (1962).

[3]B. N. Brockhouse, Inelastic Scattering of Neutrons in Solids and Liquids (International Atomic Energy Agency, Vienna, Austria, 1961) p. 113.

[4]M. J. Cooper and R. Nathans, Acta Cryst. 23, 357 (1967).

[5]P. G. Klemens, J. Appl. Phys. 38, 4573 (1967). See also C. C. Ackerman and R. A. Guyer, Solid State Commun. 5, 671 (1967).

[6]L. H. Nosanow and G. L. Shaw, Phys. Rev. 119, 968 (1962); F. W. de Wette and B. R. A. Nijboer, Phys. Letters 18, 19 (1965); H. Horner, Z. Physik 205, 72 (1967).

[7]L. H. Nosanow and N. R. Werthamer, Phys. Rev. Letters 15, 618, 997 (1965).

[8]J. H. Vignos and H. A. Fairbank, Phys. Rev. 147, 185 (1966); F. P. Lipschultz and D. M. Lee, Phys. Rev. Letters 14, 1017 (1965).

STABILIZATION OF DENSITY-GRADIENT INSTABILITIES BY THE DISPERSION OF CURVATURE DRIFT VELOCITY

G. Laval, E. K. Maschke, R. Pellat, and M. Vuillemin
Groupe de Recherches de l'Association Euratom–Commissariat à l'Energie Atomique sur la Fusion, Fontenay-aux-Roses, France
(Received 25 September 1967)

In plasma stability calculations, one usually neglects the dispersion of the velocities of particle drifts in a curved magnetic field. If the dispersion of curvature drift velocities is taken into account, the low-frequency electrostatic drift instabilities driven by a density gradient are stabilized by ion Landau damping in the direction of the curvature drift. Examples of marginal stability curves demonstrate the importance of this effect.

In calculations on the low-frequency electrostatic drift instability driven by a density gradient of the plasma, the effect of magnetic field curvature is usually simulated by introducing fictitious gravitational forces which produce particle drifts equal to the average drifts caused by the curvature.[1-3] The use of a gravity means that all particles have the same drift velocity, and therefore there will be no Landau effect in the direction of this drift. This approximation is justified as long as the drift velocity of the particles due to such an equivalent gravity is very much smaller than the phase velocity of the unstable mode in the direction of this drift. However, this condition is not fulfilled when the component of the wave vector $\vec{K}$ in the drift direction is sufficiently large. In this case there will be a Landau effect for the ions in the direction of their drift motion, in addition to the usual Landau effect in the direction parallel to the magnetic field. In the present work, we show that even with weak favorable curvature, the dispersion of curvature drift velocities provides a strongly stabilizing effect.

We consider a plane plasma slab with a density gradient in the x direction and a magnetic field in the z direction. We compare the case where a gravitational acceleration g exists in the x direction with the case where the magnetic lines have a curvature $1/R$ in the x-z plane. The gravitational and curvature drift velocities are both directed in the y direction. The equilibrium distribution function for each species of particles is taken to be

$$f_0=\frac{n_0}{(\pi)^{3/2}(V_{\mathrm{th}})^3}\left[1-\epsilon\left(\frac{V_y}{\omega_c}+x\right)\right]\exp\left\{-\left(\frac{\vec{V}}{V_{\mathrm{th}}}\right)^2+\varphi\right\}, \quad (1)$$

where ω_c is the cyclotron frequency; and for gravity,

$$\epsilon=-\frac{1}{n}\frac{dn}{dx}+2\frac{g}{V_{\mathrm{th}}^{2}},$$

$$\varphi=2\frac{g}{V_{\mathrm{th}}^{2}}x=2\frac{x}{R}; \quad (2)$$

while for curvature,

$$\epsilon=-(1/n)(dn/dx),$$

$$\varphi=0. \quad (3)$$

The quantity φ in the gravity case is due to the fact that only the sum of kinetic and potential energy is constant of motion. For the drift velocity in the unperturbed motion of the particles, we have used

$$v_{\mathrm{grav}}=-g/\omega_c=-V_{\mathrm{th}}^{2}/\omega_c R, \quad (4)$$

$$v_{\mathrm{curv}}=-\frac{1}{\omega_c}\left(\frac{V_{\parallel}^{2}}{R}+\frac{1}{2}\frac{V_{\perp}^{2}}{R}\right). \quad (5)$$

In the case of curvature, the low-frequency dispersion relation obtained in the localized approximation, assuming quasineutrality and neglecting the electron Larmor radius, has the form

$$1+\frac{T_e}{T_i}=(\omega+k_yV_{di})\frac{T_e}{T_i}\pi^{-1/2}\int_{-\infty}^{+\infty}du_{\parallel}\int_0^{\infty}d(u_{\perp}^{2})\frac{J_0^{2}(k_ya_iu_{\perp})\exp[-(u_{\parallel}^{2}+u_{\perp}^{2})]}{\omega+k_{\parallel}V_{\mathrm{th}i}u_{\parallel}+k_yV_{ci}(u_{\parallel}^{2}+\frac{1}{2}u_{\perp}^{2})}$$

$$+(\omega+k_yV_{de})\pi^{-1/2}\int_{-\infty}^{+\infty}du_{\parallel}\int_0^{\infty}d(u_{\perp}^{2})\frac{\exp[-(u_{\parallel}^{2}+u_{\perp}^{2})]}{\omega+k_{\parallel}V_{\mathrm{th}e}u_{\parallel}+k_yV_{ce}(u_{\parallel}^{2}+\frac{1}{2}u_{\perp}^{2})}. \quad (6)$$

In this equation,

$$V_{dj}=\tfrac{1}{2}(a_j/r)V_{\mathrm{th},j},$$

$$V_{cj}=-(a_j/R)V_{\mathrm{th},j} \quad (j=i,e), \quad (7)$$

where

$$1/r=(1/n)dn/dx,$$

and a_j is the Larmor radius of the species j.

The corresponding dispersion relation for the case of gravity is obtained from Eq. (6) by replacing V_{dj} by $V_{dj}+V_{cj}$, and in the denominators of the integrals $u_{\parallel}^{2}+\frac{1}{2}u_{\perp}^{2}$ by 1.

In the following we shall assume $|V_{cj}|\ll|V_{dj}|$, and we restrict our discussion to favorable curvature.

The stabilizing effect of the dispersion of the curvature drift velocity can qualitatively be seen as follows. In the case of gravity, the denominators of the ion and electron integrals always have a zero in the domain of integration of $u_{\parallel}$. Hence, both integrals have a nonvanishing imaginary part which for small k_y is proportional to $\exp\{-(\omega/k_{\parallel}V_{\mathrm{th}})^2\}$. Let us recall that the imaginary part of the electron integral destabilizes the drift instability, whereas the imaginary part of the ion integral is stabilizing.

In the case of curvature we may, for the purpose of this discussion, ignore the integration

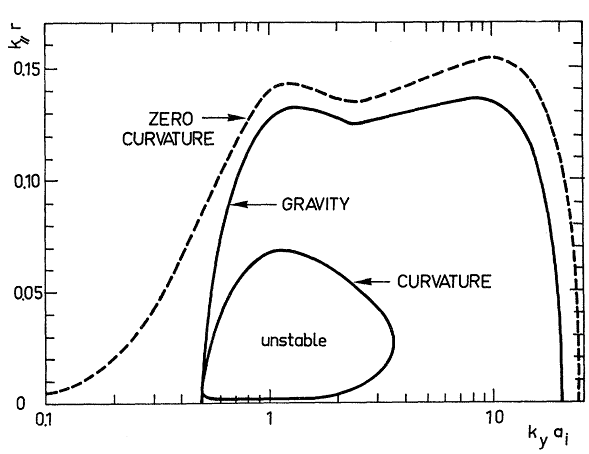

FIG. 1. Marginal stability curves for $(m_i/m_e)^{1/2}=43$; $T_e/T_i=1$; $r/R=0.05$.

over $u_\perp$. The denominators in the integrals are now quadratic functions of $u_\parallel$ and have in general two zeros in the domain of integration of $u_\parallel$. The smaller of these two roots gives the main contribution to the imaginary part of the integral. If k_y is sufficiently small, this smaller root will be near to $\omega/k_\parallel V_{\rm th}$ and we obtain approximately the same result as with gravity. However, if k_y is large enough, the situation changes. Since for $|k_y a_i| \gg 1$ the dispersion relation yields $\omega \approx -k_y V_{de}/(2\pi^{1/2}k_y a_i)$, we have $\omega \approx -k_y V_{ci}$ for $|k_y a_i| \approx R/(4\pi^{1/2}r)$. Thus, for all values of $|k_y a_i|$ larger than this, there will be strong ion Landau damping in the y direction. But even for $|k_y a_i| < R/(4\pi^{1/2}r)$, when the perpendicular Landau effect takes place in the exponential tail of the velocity distribution, it will be enhanced with respect to the parallel Landau effect because the drift velocity is proportional to the square of $u_\parallel$. The electron term shows the opposite behavior because ω and $k_y V_{ce}$ have the same sign, so that the imaginary part of the electron integral will be smaller than in the case of gravity. In fact, for small enough $k_\parallel$ there will be no zero of the denominator, and the imaginary part of the electron integral will be strictly zero.

In order to make a quantitative comparison, we have made machine calculations and obtained marginal stability curves for gravity and for true curvature. The curvature calculation has been simplified by replacing the $u_\perp^2$ in the denominators of the integrals in Eq. (6) by 1, keeping only $u_\parallel^2$ variable. This should be a good approximation.

In Figs. 1 and 2, examples of marginal stability curves are represented. The marginal stability curve for zero curvature is also shown for reference. Figure 2 clearly demonstrates the strong stabilizing effect which was to be expected according to the above qualitative discussion. If the ratio r/R is raised to 0.11, the unstable region calculated with the true curvature disappears entirely, whereas in the calculation with gravity an important unstable domain subsists. Only for $r/R = 0.25$ does the gravity calculation yield over-all stability. In Fig. 3 the dependence of the marginal stability curves on T_e/T_i is illustrated.

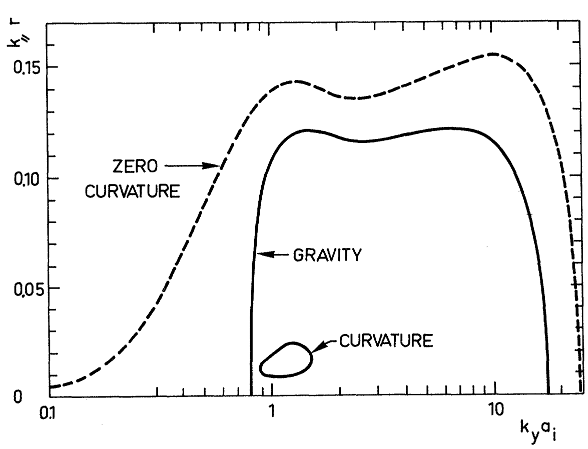

FIG. 2. Marginal stability curves for $(m_i/m_e)^{1/2} = 43$; $T_e/T_i = 1$; $r/R = 0.1$.

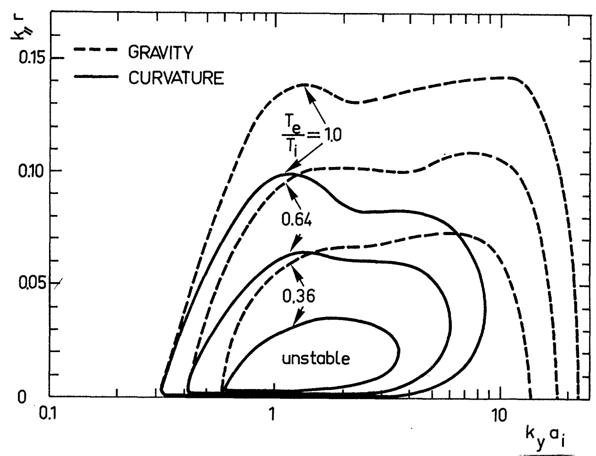

FIG. 3. Marginal stability curves for $(m_i/m_e)^{1/2} = 43$; $r/R = 0.025$; $T_e/T_i = 1.0$, 0.64, 0.36.

For unfavorable curvature, the situation is obviously reversed. Preliminary numerical results show, indeed, that the calculation with true curvature yields a larger unstable domain of wave numbers than the calculation with gravity.

In conclusion, this work has shown that in regions of favorable curvature, the low-frequency drift instability is less dangerous than the gravity calculation would indicate. In particular, the short perpendicular wavelengths are easily stabilized, in contrast to what has been predicted by Krall and Rosenbluth.[1] It should, however, be emphasized that the results obtained are of course quite sensitive to the details of the distribution function.

[1]N. A. Krall and M. N. Rosenbluth, Phys. Fluids 8, 1488 (1965).

[2]J. D. Jukes, Plasma Physics and Controlled Nuclear Fusion Research (International Atomic Energy Agency, Vienna, Austria, 1966), p. 643.

[3]B. Coppi, G. Laval, R. Pellat, and M. N. Rosenbluth, International Centre for Theoretical Physics, Trieste, Report No. IC/66/55, 1966 (to be published).