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PHONON DISPERSION MEASUREMENTS ON A hcp He^4 SINGLE CRYSTAL AT 27 atm[*]

F. P. Lipschultz,[†] V. J. Minkiewicz, T. A. Kitchens, G. Shirane, and R. Nathans
Brookhaven National Laboratory, Upton, New York

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Phonon dispersion relations along $[10\bar{1}0]$ in the hcp phase of He^4 have been determined by triple-axis neutron spectrometry. Optical as well as acoustic branches were observed at 27 atm and 0.99°K in a single crystal grown from the superfluid phase.

The lattice dynamics of solid helium is fundamentally important because it is only for this solid that the zero-point motion cannot be considered as a small perturbation. Only recently, however, have neutron fluxes become sufficiently intense and techniques been developed for growing sufficiently large helium crystals to allow an inelastic neutron-scattering determination of the phonon dispersion relations. In this Letter we report the first measurements of such a study.

The single crystals used in this experiment were grown at a constant temperature of 1.30°K by supplying helium in the superfluid phase through an open capillary. Heat was removed at the bottom of the 2.4-cm-diameter, 5.0-cm-long cylindrical sample container in such a way that the solidification rate was 1.0-1.5 mm/min. When the sample was completely solid, the pressure was slowly increased to 27.0 atm. This method is similar to those used by several groups[1] to produce good crystals of helium. The specimen was located in a He^3 cryostat designed for long-term temperature and pressure stability. The sample was retained in the experiment described here for about 10 days at 0.99 ± 0.05°K and 27.0 ± 0.1 atm in this apparatus.

The crystals grown during this experiment were unseeded and their orientations were determined by elastic neutron scattering. Because of the complexity of the apparatus, the crystal could be tilted only ±5°, thereby limiting the usable zones. The data reported here were taken in the $[01\bar{1}]$ zone of a crystal of high quality, having a smooth rocking curve with a full-width at half-maximum of 9′. It was found by beam masking techniques to be approximately $12 \times 12 \times 10$ mm. This is perhaps the largest single crystal of helium ever reported. The lattice constants for this crystal are $a = 3.671 \pm 0.002$ Å and $c/a = 1.638 \pm 0.001$, which agree reasonably with extrapolated neutron and x-ray results.[2]

The experimental neutron-group profiles shown in Fig. 1 were taken on a triple-axis spectrometer at the Brookhaven high-flux beam reactor using the constant-Q scan[3] and 13-meV incident neutrons. In helium the large Debye-Waller factor, $B \cong 29$ Å^2, necessitates that data be taken around low-order reflections, namely $(10\bar{1}0)$ and $(1\bar{1}01)$ in this zone, rather than the more customary higher-order reflections. The phonon dispersion curves along the $[10\bar{1}0]$ are plotted in Fig. 2. No significant shift in transverse phonon energies was found by changing the sample temperature by ±0.5°K. Because the $[01\bar{1}]$ axis is 31.6° from the c ax-

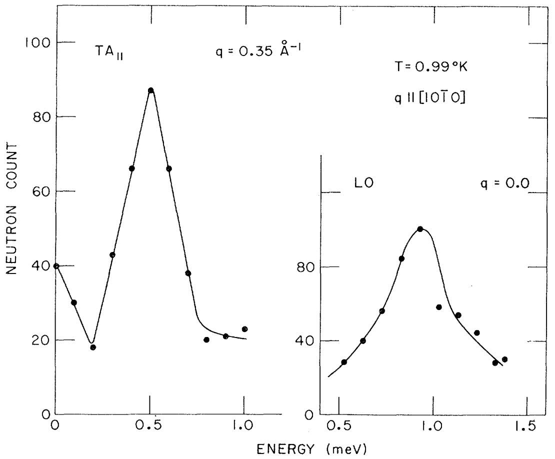

FIG. 1. Typical neutron-group profiles in hcp He^4 at 27.0 atm. Each datum required about 20 min.

is, the transverse phonon branches observed may contain a minor contribution from vibrations perpendicular to the plane. Of course the longitudinal branches are not perturbed by using this zone.

The data, as illustrated in Fig. 1, have well-resolved peaks characteristic of the very high resolution of the spectrometer and long lifetimes of the phonons of all branches except for the longitudinal acoustic (LA) at high momentum transfer. The error bars in Fig. 2 indicate the statistics and experimental linewidths, which are primarily due to instrumental resolution.[4] The LA branch shows an anomalously rapid decrease in peak intensity at $q = 0.5$ Å^{-1} and $\epsilon = 1.5$ meV. This anomaly appears to be accompanied with some line broadening. We have, in fact failed to observe well-defined phonon groups in the search areas indicated by the shaded regions of Fig. 2. The decrease of the intensity of the LA branch may be explained in terms of the structure and Debye-Waller factors. Line broadening from three-phonon processes may also contribute to this decrease.[5]

The usual approximations in the theory of lattice dynamics of noble gas solids fail for solid helium because the zero-point kinetic energy is approximately equal to the potential energy.[6] Using the time-dependent Hartree approximation and the results of variational calculations of the ground-state wave functions, Nosanow and Werthamer[7] have overcome some of the difficulties in the theory. They have calculated the velocity of compressional and shear sounds and, therefore, the initial slopes of the dispersion relations. These theoretical slopes correspond to 0.58 km/sec for the longitude branch and 0.31 and 0.245 km/sec for the transverse branches along [10$\bar{1}$0]. The agreement is seen to be good. [The conversion factor is $\hbar = 6.582$ (meV/Å^{-1})/(km/sec)].

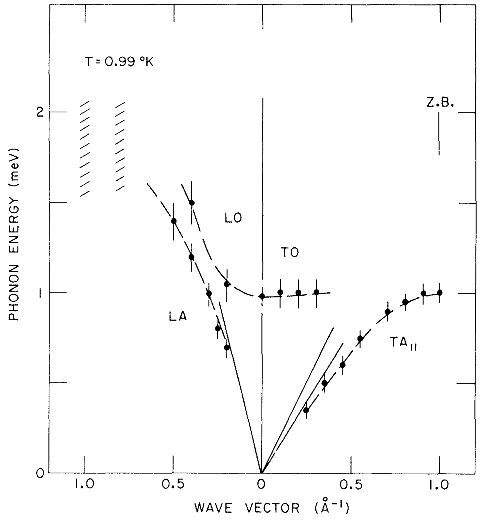

FIG. 2. The phonon dispersion relations for hcp He^4 at 27.0 atm and 0.99°K along the [10$\bar{1}$0] direction. The solid lines correspond to the velocities of sound computed by L. H. Nosanow and N. R. Werthamer, Phys. Rev. Letters 15, 618, 997 (1965). The lower TA line is for motion parallel to the basal plane. (1 meV ≡ 11.6°K ≡ 0.242×10^{12} cps.)

Measurements of the velocity of sound[8] at 10 Mc/sec have been performed on hcp solid He^4 in this pressure region under conditions similar to the present situation, but with crystal orientation unknown. The observed sound velocities, 0.54 and 0.47 km/sec for longitudinal waves and 0.23, 0.31, and possibly 0.245 km/sec for transverse waves, showed little spread from these values and were reproducible. This suggests that samples used to measure 10-Mc/sec sound velocities were very likely single crystals and that the directions of sound propagation were probably close to crystal axes. If these assumptions are correct, then the 0.54-km/sec longitudinal velocity and 0.23- and 0.245-km/sec transverse velocities are consistent with the slopes of the respective acoustic phonon branches at small wave vector.

Our studies of the phonon dispersion relation in solid helium are being continued, with particular attention being paid to the longitudinal groups at the higher q values. In spite of our low residual background, the low scattered intensities make the problem particularly difficult. For this reason, new growth techniques which hopefully will allow the production of significantly larger and more suitable oriened crystals are being investigated.

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†Present address: Physics Department, University of Connecticut, Storrs, Connecticut.

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STABILIZATION OF DENSITY-GRADIENT INSTABILITIES BY THE DISPERSION OF CURVATURE DRIFT VELOCITY

G. Laval, E. K. Maschke, R. Pellat, and M. Vuillemin
Groupe de Recherches de l'Association Euratom–Commissariat à l'Energie Atomique sur la Fusion,
Fontenay-aux-Roses, France

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In plasma stability calculations, one usually neglects the dispersion of the velocities of particle drifts in a curved magnetic field. If the dispersion of curvature drift velocities is taken into account, the low-frequency electrostatic drift instabilities driven by a density gradient are stabilized by ion Landau damping in the direction of the curvature drift. Examples of marginal stability curves demonstrate the importance of this effect.

In calculations on the low-frequency electrostatic drift instability driven by a density gradient of the plasma, the effect of magnetic field curvature is usually simulated by introducing fictitious gravitational forces which produce particle drifts equal to the average drifts caused by the curvature.[1-3] The use of a gravity means that all particles have the same drift velocity, and therefore there will be no Landau effect in the direction of this drift. This approximation is justified as long as the drift velocity of the particles due to such an equivalent gravity is very much smaller than the phase velocity of the unstable mode in the direction of this drift. However, this condition is not fulfilled when the component of the wave vector $\vec{K}$ in the drift direction is sufficiently large. In this case there will be a Landau effect for the ions in the direction of their drift motion, in addition to the usual Landau effect in the direction parallel to the magnetic field. In the present work, we show that even with weak favorable curvature, the dispersion of curvature drift velocities provides a strongly stabilizing effect.

We consider a plane plasma slab with a density gradient in the x direction and a magnetic field in the z direction. We compare the case where a gravitational acceleration g ex-