taken very seriously, but it is worth keeping in mind that the standard calculation[8] of the electron-neutrino cross section may well be wrong.

Is this model renormalizable? We usually do not expect non-Abelian gauge theories to be renormalizable if the vector-meson mass is not zero, but our Z_μ and W_μ mesons get their mass from the spontaneous breaking of the symmetry, not from a mass term put in at the beginning. Indeed, the model Lagrangian we start from is probably renormalizable, so the question is whether this renormalizability is lost in the reordering of the perturbation theory implied by our redefinition of the fields. And if this model is renormalizable, then what happens when we extend it to include the couplings of $\vec{A}_\mu$ and B_μ to the hadrons?

I am grateful to the Physics Department of MIT for their hospitality, and to K. A. Johnson for a valuable discussion.

*This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(30-1)2098).

†On leave from the University of California, Berkeley, California.

[1]The history of attempts to unify weak and electromagnetic interactions is very long, and will not be reviewed here. Possibly the earliest reference is E. Fermi, Z. Physik 88, 161 (1934). A model similar to ours was discussed by S. Glashow, Nucl. Phys. 22, 579 (1961); the chief difference is that Glashow introduces symmetry-breaking terms into the Lagrangian, and therefore gets less definite predictions.

[2]J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[3]P. W. Higgs, Phys. Letters 12, 132 (1964), Phys. Rev. Letters 13, 508 (1964), and Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Letters 13, 585 (1964).

[4]See particularly T. W. B. Kibble, Phys. Rev. 155, 1554 (1967). A similar phenomenon occurs in the strong interactions; the ρ-meson mass in zeroth-order perturbation theory is just the bare mass, while the A_1 meson picks up an extra contribution from the spontaneous breaking of chiral symmetry. See S. Weinberg, Phys. Rev. Letters 18, 507 (1967), especially footnote 7; J. Schwinger, Phys. Letters 24B, 473 (1967); S. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967), Eq. (13) et seq.

[5]T. D. Lee and C. N. Yang, Phys. Rev. 98, 101 (1955).

[6]This is the same sort of transformation as that which eliminates the nonderivative $\vec{\pi}$ couplings in the σ model; see S. Weinberg, Phys. Rev. Letters 18, 188 (1967). The $\vec{\pi}$ reappears with derivative coupling because the strong-interaction Lagrangian is not invariant under chiral gauge transformation.

[7]For a similar argument applied to the σ meson, see Weinberg, Ref. 6.

[8]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1957).

SPECTRAL-FUNCTION SUM RULES, ω-φ MIXING, AND LEPTON-PAIR DECAYS OF VECTOR MESONS*

R. J. Oakes†
Brookhaven National Laboratory, Upton, New York

and

J. J. Sakurai
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received 18 October 1967)

Within the framework of vector-meson dominance, the current-mixing model is shown to be the only theory of ω-φ mixing consistent with Weinberg's first sum rule as applied to the vector-current spectral functions. Relations among the leptonic decay rates of ρ^0, ω, and φ are derived, and other related processes are discussed.

We begin by considering Weinberg's first sum rule[1] extended to the $(1+8)$ vector currents of the eightfold way[2]:

$$\int dm^2[m^{-2}\rho_{\alpha\beta}^{(1)}(m^2)+\rho_{\alpha\beta}^{(0)}(m^2)]=S\delta_{\alpha\beta}+S'\delta_{\alpha 0}\delta_{\beta 0}, \tag{1}$$

where S and S' are constants independent of α and β $(=0, 1, \cdots, 8)$ and the spectral functions are defined as usual via

$$\Delta_{\mu\nu}{}^{\alpha\beta}(q) = -i\int d^4x\, e^{-iq\cdot x}\langle 0|T(J_\mu{}^\alpha(x)J_\nu{}^\beta(0))|0\rangle$$

$$= \int dm^2 \rho_{\alpha\beta}{}^{(1)}(m^2)\,(\delta_{\mu\nu} + m^{-2}q_\mu q_\nu)/(q^2+m^2-i\epsilon)$$

$$+\int dm^2\rho_{\alpha\beta}{}^{(0)}(m^2)q_\mu q_\nu/(q^2+m^2-i\epsilon) + \text{Schwinger terms.} \tag{2}$$

Assuming that the vector spectral functions are dominated by the known vector mesons,[3] Eq. (1) yields

$$m_\rho{}^2/f_\rho{}^2 = (\tfrac{3}{4}f_Y{}^2)(m_\varphi{}^2\cos^2\theta_Y + m_\omega{}^2\sin^2\theta_Y) = m_{K^*}{}^2/f_{K^*}{}^2, \tag{3}$$

$$m_\omega{}^2\tan\theta_Y = m_\varphi{}^2\tan\theta_B, \tag{4}$$

where the coupling constants and mixing angles[4] are defined by the following covariant matrix elements of the usual octet currents [isopin (I), strangeness-changing (ΔS), and hypercharge (Y)] and singlet current [baryon (B)]:

$$\langle 0|J_\mu{}^{(I)}(0)|\rho\rangle = \langle 0|J_\mu{}^3(0)|\rho^0\rangle = m_\rho{}^2 f_\rho{}^{-1}\epsilon_\mu,$$

$$\langle 0|J_\mu{}^{(\Delta S)}(0)|K^*\rangle = \langle 0|J_\mu{}^6|(K^{*0}+\bar{K}^{*0})/\sqrt{2}\rangle = m_{K^*}{}^2 f_{K^*}{}^{-1}\epsilon_\mu,$$

$$\langle 0|J_\mu{}^{(Y)}(0)|\varphi\rangle = (2/\sqrt{3})\langle 0|J_\mu{}^8(0)|\varphi\rangle = m_\varphi{}^2 f_Y{}^{-1}\epsilon_\mu\cos\theta_Y,$$

$$\langle 0|J_\mu{}^{(Y)}(0)|\omega\rangle = (2/\sqrt{3})\langle 0|J_\mu{}^8(0)|\omega\rangle - m_\omega{}^2 f_Y{}^{-1}\epsilon_\mu\sin\theta_Y,$$

$$\langle 0|J_\mu{}^{(B)}(0)|\varphi\rangle = (\tfrac{2}{3})^{\frac{1}{2}}\langle 0|J_\mu{}^0(0)|\varphi\rangle = m_\varphi{}^2 f_B{}^{-1}\epsilon_\mu\sin\theta_B,$$

$$\langle 0|J_\mu{}^{(B)}(0)|\omega\rangle = (\tfrac{2}{3})^{\frac{1}{2}}\langle 0|J_\mu{}^0(0)|\omega\rangle = m_\omega{}^2 f_B{}^{-1}\epsilon_\mu\cos\theta_B. \tag{5}$$

We note that Eq. (4) is precisely the relation between θ_Y and θ_B expected on the basis of current-mixing model[4,5] and is inconsistent with the mass-mixing model[6] (that requires $\theta_Y = \theta_B$). It is easy to show that the converse statement is also true: In the vector-meson-dominance approximation, the current-mixing model (as defined by Kroll, Lee, and Zumino,[4] and Coleman and Schnitzer[5]) is the only theory of $\omega-\varphi$ mixing compatible with the generalized first sum rule of Weinberg. Note that Eq. (4) implies that the transformation that relates $\omega^{(Y)}$ (pure octet) and $\omega^{(B)}$ (pure singlet), with ω and φ is not orthogonal.[7]

To estimate θ_Y and θ_B we must introduce SU(3) violations of the octet type. But let us first observe that when the spin-zero excitations are ignored ($\rho_{\alpha\beta}{}^{(0)} \equiv 0$), the first sum rule states that SU(3) becomes exact at $q=0$ (as well as at $q\to\infty$[8]) as far as $\Delta_{\mu\nu}{}^{\alpha\beta}$ is concerned. This means that when we write the propagator matrix $\Delta_{\mu\nu}{}^{\alpha\beta}$ (suppressing the matrix indices) as

$$\Delta_{\mu\nu} = \Delta\delta_{\mu\nu} + \Delta' q_\mu q_\nu, \tag{6}$$

the matrix Δ^{-1} (which can be shown to be linear in q^2 whenever the pole approximation holds for Δ[5]) must have the form

$$\Delta^{-1} = M_0{}^2 + q^2\Pi_0 + q^2\delta, \tag{7}$$

where $M_0{}^2$ and Π_0 are SU(3) symmetric. It is now natural to introduce first-order octet symmetry breaking in the matrix δ as follows:

$$\delta = \begin{pmatrix} -2\epsilon & 0 & 0 & 0 \\ 0 & \epsilon & 0 & 0 \\ 0 & 0 & 2\epsilon & \beta \\ 0 & 0 & \beta & 0 \end{pmatrix}. \tag{8}$$

Here the rows and columns are labeled I, ΔS, Y, and B, in that order. From (7) and (8) we not only recover the first sum rule (3) and (4), as we must, but also obtain one additional relation,[9]

$$\tfrac{1}{3}(4m_{K^*}^{-2}-m_\rho^{-2}) = m_\varphi^{-2}\cos^2\theta + m_\omega^{-2}\sin^2\theta, \quad (9)$$

where, for convenience, we have introduced a "third" mixing angle θ such that[4]

$$\tan\theta = (m_\omega/m_\varphi)\tan\theta_Y = (m_\varphi/m_\omega)\tan\theta_B. \quad (10)$$

It is amusing that our proposal amounts to the requirement that $\int dm^2 m^{-4}\rho_{\alpha\beta}^{(1)}(m^2)$ satisfy the Gell-Mann–Okubo formula, which, when combined with the first sum rule, allows one to derive the formula (9) for the inverse square of the mass in the meson-dominance approximation. This is perhaps reasonable because the first sum rule (with $\rho_{\alpha\beta}^{(0)}\equiv 0$) demands exact symmetry at $q=0$, but, when we consider the next order terms in q^2, first-order symmetry breakings must be taken into account. Numerically, Eqs. (9) and (10) give[10]

$$\theta = 28.2°,\quad \theta_Y = 35.0°,\quad \theta_B = 22.5°. \quad (11)$$

Next we comment on the recent work of Das, Mathur, and Okubo[2] (hereafter referred to as DMO) which treats the same problem. Like us, they assume the first sum rule (1), but, in addition, they propose that $\int dm^2\rho_{\alpha\beta}^{(1)}(m^2)$ satisfy the Gell-Mann–Okubo formula. This suggestion, when combined with the first sum rule, leads to

$$\tfrac{1}{3}(4m_{K^*}^{2}-m_\rho^{2}) = m_\varphi^{2}\cos^2\theta + m_\omega^{2}\sin^2\theta \quad (12)$$

in our notation. This mass-squared relation, which corresponds to a very complicated nonoctet symmetry-breaking matrix δ in Eq. (7), gives

$$\theta = 39.8°,\quad \theta_Y = 47.2°,\quad \theta_B = 32.7° \quad (13)$$

in contrast to our result (11).

Recall that these angles are directly measurable from the lepton-pair decays of ω and φ, since[11]

$$\frac{\Gamma(\omega\to l^+l^-)}{\Gamma(\varphi\to l^+l^-)} = \frac{m_\omega}{m_\varphi}\tan^2\theta_Y = \frac{m_\varphi}{m_\omega}\tan^2\theta. \quad (14)$$

In addition, we have[12]

$$\tfrac{1}{3}m_\rho\Gamma(\rho\to l^+l^-) = m_\omega\Gamma(\omega\to l^+l^-) + m_\varphi\Gamma(\varphi\to l^+l^-), \quad (15)$$

which follows from the first sum rule alone. Equations (14) and (15) can be conveniently summarized by constructing a graphical representation, as shown in Fig. 1. The predictions of our model and the DMO model are compared in Table I.[13] Unfortunately, data are not available for comparison; however, the experiments are currently in progress.[14]

Finally, we wish to discuss briefly three other related problems:

(i) Using vector-meson dominance and requiring the vanishing of the $\varphi\pi\gamma$ coupling constant (which may be justified, since experimentally the $\varphi\pi\rho$ coupling is anomalously weak), we can deduce $(f_Y^2/4\pi)\sin^{-2}\theta_Y$ from[15]

$$\frac{\Gamma(\omega\to\pi^0\gamma)}{\Gamma(\pi^0\to 2\gamma)} = \frac{2}{3}\left(\frac{m_\omega^2-m_\pi^2}{m_\omega m_\pi}\right)^3\left(\frac{f_Y}{e}\right)^2\sin^{-2}\theta_Y = \frac{57\ \text{MeV}}{\Gamma(\omega\to l^+l^-)}. \quad (16)$$

This gives $(f_Y^2/4\pi)\sin^{-2}\theta_Y = 10\pm 2$. If $f_\rho^2/4\pi$ is 2.6 (corresponding to $\Gamma_\rho = 128$ MeV in the ρ-dominance limit), we get, from (3), $\theta_Y = 33°\pm 5°$ $(\theta = 27°\pm 5°)$ in remarkable agreement

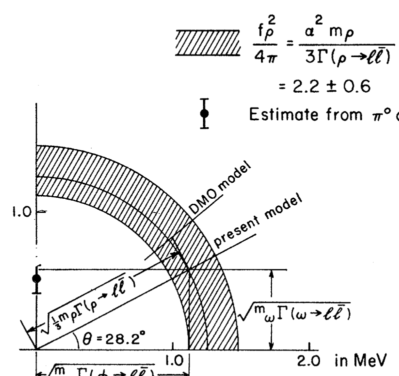

FIG. 1. Graphical representation of ω-φ mixing and the lepton-pair decays of ρ^0, ω, and φ. The ρ-meson coupling constant used here is consistent with most experimental data on $\rho\to l^+l^-$ [see, e.g., S. C. C. Ting, in Proceedings of the 1967 International Symposium on Electrons and Photons at High Energies, Stanford, California, September, 1967 (to be published)] and also with the ρ dominance of the pion form factor.

Table I. ω-φ mixing angles and lepton-pair decays of ρ^0, ω, and φ. Note that both models satisfy sum rule (15).

	θ (deg)	θ_Y (deg)	θ_B (deg)	$\frac{\Gamma(\omega\to l^+l^-)}{\Gamma(\rho\to l^+l^-)}$	$\frac{\Gamma(\varphi\to l^+l^-)}{\Gamma(\rho\to l^+l^-)}$
Present model	28.2	35.0	22.5	0.074	0.195
DMO model	39.8	47.2	32.7	0.134	0.149

with our estimates (11). (See also Fig. 1.)

(ii) If we assume that the electromagnetic form factors of K and π are dominated by ρ, ω, and φ, we obtain with the aid of (3) and the formalism of Ref. 4[16]

$$\frac{\Gamma(\varphi\to K\bar{K})}{\Gamma(\rho\to\pi\pi)}=\frac{3(m_\varphi^{\,2}\cos^2\theta_Y+m_\omega^{\,2}\sin^2\theta_Y)}{2m_\varphi^{\,2}}\left[\frac{\cos\theta_B}{\cos(\theta_Y-\theta_B)}\right]^2\left(\frac{p_{K\bar{K}}}{p_{\pi\pi}}\right)^3=\frac{3}{2}\left(\frac{p_{K\bar{K}}}{p_{\pi\pi}}\right)^3\cos^2\theta, \tag{17}$$

which leads to $\Gamma(\varphi\to K\bar{K})=5.4$ MeV for $\Gamma(\rho\to\pi\pi)=128$ MeV. This value appears somewhat larger than the currently accepted value[10] $\Gamma(\varphi\to K\bar{K})=3.6\pm1$ MeV.

(iii) The diffraction model for the photoproduction of ρ^0, ω, and φ leads to[17]

$$\sigma(\gamma p\to\rho^0p):\sigma(\gamma p\to\omega p):\sigma(\gamma p\to\varphi p)$$
$$=f_\rho^{\,-2}\sigma_{tot}^{\,2}(\rho^0p):\tfrac{1}{4}f_Y^{\,-2}\sin^2\theta_Y\sigma_{tot}^{\,2}(\omega p):\tfrac{1}{4}f_Y^{\,-2}\cos^2\theta_Y\sigma_{tot}^{\,2}(\varphi p). \tag{18}$$

According to our theory, the famous 9:1:2 ratio for the dimensionless photon-vector-meson coupling constants must be modified as follows:

$$f_\rho^{\,-2}:\tfrac{1}{4}f_Y^{\,-2}\sin^2\theta_Y:\tfrac{1}{4}f_Y^{\,-2}\cos^2\theta_Y$$
$$=9.00:0.65:1.33. \tag{19}$$

It therefore appears that the suppression of the photoproduction of the φ meson[18] is no longer a mystery provided that $\sigma_{tot}(\varphi p)/\sigma_{tot}(\rho^0p)\approx1/2.7$, as suggested by the simple-minded quark model.[19]

One of us (R.J.O.) wishes to thank Dr. Maurice Goldhaber and the Physics Department of Brookhaven National Laboratory for their hospitality.

*Work supported in part by the U. S. Atomic Energy Commission.

†Present address: The Institute for Advanced Study, Princeton, New Jersey.

[1]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[2]In this note we consider neither the axial-vector currents nor the second sum rule (and its generalizations) of Weinberg. When applied to the vector currents, the second sum rule is incompatible with experiment at least in the vector-meson dominance approximation: S. P. DeAlevis, to be published; T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 470 (1967); J. J. Sakurai, Phys. Rev. Letters 19, 803 (1967).

[3]Although the strangeness-changing current is not conserved, we neglect throughout any contribution from the scalar spectral functions since there is no corresponding meson (κ meson?) known. In any case such a contribution is expected to be of second order in the SU(3) breaking.

[4]Our analysis of ω-φ mixing follows that of N. M. Kroll, T. D. Lee, and B. Zumino [Phys. Rev. 157, 1376 (1967)], who emphasized that we need, in general, two angles to characterize ω-φ mixing.

[5]S. Coleman and H. J. Schnitzer, Phys. Rev. 134, B863 (1964).

[6]S. Okubo, Phys. Rev. Letters 5, 165 (1963); S. L. Glashow, Phys. Rev. Letters 11, 48 (1963); J. J. Sakurai,

Phys. Rev. 132, 434 (1963).

[7]If we abandon vector-meson dominance, it is possible to save the orthogonality of the transformation matrix; cf. K. Dietz and H. Pietschmann, to be published.

[8]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967).

[9]We remark that our procedure leads to exactly the same results as those of Coleman and Schnitzer (Ref. 5), even though they start with the propagator matrix for the vector fields rather than that of the vector currents. Our Eq. (9) can be shown to be equivalent to Eq. (5.78) of Kroll, Lee, and Zumino (Ref. 4).

[10]The following mass values have been used: $m_\rho = 770$ MeV, $m_{K^*} = 892$ MeV, $m_\omega = 783$ MeV, and $m_\varphi = 1019$ MeV [A. H. Rosenfeld, et al., Rev. Mod. Phys. 39, 1 (1967)].

[11]R. F. Dashen and D. H. Sharp, Phys. Rev. 133, B1585 (1964). See also C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Letters 7, 81 (1963).

[12]Das, Mathur, and Okubo (Ref. 2); Sakurai (Ref. 2).

[13]One may argue that the difference between (9) and (12) is second order in the SU(3) breaking, and hence should be insignificant. Yet, as seen from Table I, explicit calculations show that the two models predict rather different results for ω, $\varphi, \to l^+l^-$.

[14]R. Wilson and S. C. C. Ting, private communication.

[15]M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961). The right-hand side of Eq. (5.12) of this reference should be multiplied by 4. M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

[16]The relevance of (3) (or a relation equivalent to it obtained from the soft-π, K techniques) to the φ width has been discussed by many authors, e.g., V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters 16, 947 (1966); P. P. Divakaran and L. K. Pandit, Phys. Rev. Letters 19, 535 (1967). However, these authors use (explicitly or implicitly) the ω-φ mixing angle determined from the mass-mixing model which is now ruled out by the first sum rule.

[17]H. Joos, Phys. Letters 24B, 103 (1967); P. G. O. Freund, Nuovo Cimento 44A, 411 (1966); M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966); K. Kajantie and S. Trefil, Phys. Letters 24B, 106 (1967).

[18]For a summary of the present experimental status see, e.g., F. M. Pipkin, in Proceedings of the 1967 International Symposium on Electrons and Photons at High Energies, Stanford, California, September, 1967 (to be published).

[19]H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966); E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz.—Pis'ma Redakt. 2, 105 (1965) [translation: JETP Letters 2, 65 (1965)].