mation, multipole expansion, and nuclear-model discussion.[16]

We note that in making the modification for absorption from bound states, the above results will be valid only if the pion and muon are captured out of the same atomic orbit, i.e., the $1S$ orbit.[17] Furthermore, in identifying the matrix element $\langle N'|J_\mu^{5\dagger}(0)|N\rangle$ of Eqs. (12) and (13) with that of Eq. (2), we are neglecting the relatively mild dependence[4,16] of the nuclear matrix element on differing momentum transfers arising because $m_\pi > m_\mu$. Of course, gauge invariance is satisfied in Eqs. (12) and (13) only in the limit $m_\pi \to 0$.

Lastly, it should be pointed out that a technique similar to the one presented here can be applied to a variety of other problems involving nuclear interactions with two currents, provided that no resonances are encountered which would invalidate the soft-pion assumption.

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CURRENT ALGEBRA AND THE DECAY MODES $\eta \to \pi^+\pi^-\pi^0\gamma$ AND $\eta \to \pi^0\gamma\gamma$

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Recently Price and Crawford[1] searched for the decay mode

$$\eta \to \pi^+\pi^-\pi^0\gamma, \tag{A}$$

and found the experimental upper limit

$$R \equiv \frac{\Gamma(\pi^+\pi^-\pi^0\gamma)}{\Gamma(\pi^+\pi^-\pi^0)} < 0.9\,\%. \tag{1}$$

This is usually compared with the theoretical prediction[1,2] based on the power of α and a phase space factor, a, of order $a \simeq 100$ for the three- to four-body final states:

$$R \simeq (\alpha a)^{-1} \simeq 1, \tag{2}$$

which is in contradiction with Eq. (1).

The above comparison is, however, invalid because of the fact that two pions in (A) are in a relative P state, and hence the decay $\eta \to \pi^+\pi^-\pi^0\gamma$ is inhibited due to the angular momentum barrier.[3]

If the decay modes $\eta \to \pi^+\pi^-\pi^0\gamma$ and $\eta \to \pi^0\gamma\gamma$ are now compared, one obtains

$$R' \equiv \frac{\Gamma(\pi^+\pi^-\pi^0\gamma)}{\Gamma(\pi^0\gamma\gamma)} \simeq (\alpha a)^{-1} \simeq 1, \tag{3}$$

which is to be compared with the recent experimental data,[4,5] $R' < 0.6\,\%$.

Further, an independent prediction for the decay $\eta \to \pi^+\pi^-\pi^0\gamma$ can also be made from the consideration of A invariance of Bronzan and Low.[6] The decay $\eta \to \pi^+\pi^-\pi^0\gamma$ is A allowed, while the decay $\eta \to \pi^+\pi^-\gamma$ is A forbidden. Assuming that the A-forbidden processes are inhibited by a factor $\epsilon \simeq 0.01$ (which gives the observed equal frequencies for the decays $\eta \to \gamma+\gamma$ and $\eta \to \pi^0\gamma\gamma$, after taking into account the two- to three-body phase-space factor), it is predicted that

$$R'' \equiv \frac{\Gamma(\pi^+\pi^-\pi^0\gamma)}{\Gamma(\pi^+\pi^-\gamma)} \simeq (\epsilon a)^{-1} \simeq 1, \tag{3'}$$

which is again in contradiction with the observed[5] ratio $R'' < 4\,\%$.

The purpose of the present Letter is to relate the decay rate of $\eta \to \pi^+\pi^-\pi^0\gamma$ to that of $\eta \to \pi^0\gamma\gamma$ by using the technique of the current algebra, the partial conservation of axial-vector currents (PCAC), and the equal-time commutation relation between the axial-vector currents.

From the considerations of C invariance and the lowest order of electromagnetic interaction, it is easy to see that the three pions in (A) have $C=-1$, and hence belong to the totally antisymmetric $T=0$ state. Since π^0 has $C=+1$, $\pi^+\pi^-$ are then in the $C=-1$, $T=1$, P state. If the transition (A) occurs through $E1$, the total angular momentum and parity of the three pions in (A) will be $J^P=1^+$; hence π^0 will be in an S state relative to $\pi^+\pi^-$. On the other hand, if it is an $M1$ transition, the three-pion final state will have $J^P=1^-$, and the, and the third pion will again be in a P state relative to $\pi^+\pi^-$ pair.[7] In the following considerations it is the $E1$ transition which is relevant and from the symmetry consideration, it is easy to see that any two of the three pions in (A) can be in a relative P state.

Now using PCAC in the form[8]

$$\partial_\mu A_a^{\ \mu}(x) = (\mu^2 F_\pi/\sqrt{2})\varphi_a(x) \tag{4}$$

and the equal-time commutator between the axial-vector currents,

$$[A_a^{\ 0}(x), A_b^{\ \nu}(y)]\delta(x^0-y^0) = i\epsilon_{abc}V_c^{\ \nu}(x)\delta(x-y), \tag{5}$$

where $V_c^{\ \nu}(x)$ is the vector current, one can easily obtain[8]

$$\langle\gamma(k), \pi_a(q_a), \pi_b(q_b), \pi_c(q_c); \mathrm{out}\,|\,\eta(p); \mathrm{in}\rangle$$
$$= 2\pi F_\pi^{\ -2}(4q_a^{\ 0}q_b^{\ 0})^{-\frac{1}{2}}\delta(p-k-q_a-q_b-q_c)(q_a-q_b)_\mu\epsilon_{abd}\langle\gamma(k),\pi_c(q_c)\,|\,V_d^{\ \mu}(q_a+q_b)\,|\,\eta(p)\rangle \tag{6}$$

in the limit $q_a^{\ 2}=q_b^{\ 2}=0$, retaining only the first-order terms in q_a and q_b.

The invariant amplitudes (also gauge invariant) for the processes $\eta \to \pi^0\gamma\gamma$ and $\eta \to \pi^+\pi^-\pi^0\gamma$ are[9]

$$\langle\gamma(k), \gamma(k'), \pi^0(q); \mathrm{out}\,|\,\eta(p); \mathrm{in}\rangle$$
$$= i(2\pi)^{-\frac{9}{2}}(16k^0k'^0q^0p^0)^{-\frac{1}{2}}(2\pi)^4\delta(p-k-k'-q)(e^2/m_\eta^{\ 2})(M/\sqrt{2})\{(k\cdot k')\delta_{\mu\nu}-k_\mu k_\nu'\}\epsilon'^\mu\epsilon^\nu, \tag{7}$$

$$\langle\gamma(k), \pi_a(q_a), \pi_b(q_b), \pi_c(q_c); \mathrm{out}\,|\,\eta(p); \mathrm{in}\rangle$$
$$= i(2\pi)^{-6}(32k^0q_a^{\ 0}q_b^{\ 0}q_c^{\ 0}p^0)^{-\frac{1}{2}}(2\pi)^4\delta(p-k-q_a-q_b-q_c)(e/m_\eta^{\ 3})F_\eta\epsilon^\mu$$
$$\times\{k\cdot(q_a-q_b)(q_a+q_b)_\mu - k\cdot(q_a+q_b)(q_a-q_b)_\mu + \text{terms}[(a\to c, b\to a) \text{ and } (a\to b, b\to c)]\}, \tag{8}$$

where ϵ and ϵ' are the polarization vectors of the photons.

From (6), (7), and (8) we obtain

$$F_\eta = eF_\pi^{\ -2}m\eta M. \tag{9}$$

The four-body phase space integrations have been evaluated using the technique given by Dalitz,[10] and we obtain the following expressions[11] for the decay rates:

$$\Gamma(\eta\to\pi^0\gamma\gamma)=\frac{2\alpha^2}{3\pi m_\eta^{\ 3}}|M|^2\int_0^{k_{\max}}dk\,\frac{k^3(k_{\max}-k)^3}{(m_\eta-2k)^3}, \tag{10}$$

where $k_{\max}=\frac{1}{2}m_\eta-\mu^2/2m_\eta$, and

$$\Gamma(\eta\to\pi^+\pi^-\pi^0\gamma)=\frac{3\alpha}{8\pi^5 m_\eta^{\ 7}}|F_\eta|^2\int_0^{k_{\max}'}dk\,\frac{k^3m_\eta^{\ 2}}{E_k^{\ 2}}\int_\mu^{\omega_{\max}}d\omega\,x(k,\omega)q\{(Q^2-2\mu^2)(\omega E_k-2\omega^2-\tfrac{2}{3}q^2)$$
$$+(Q^2-4\mu^2)(\tfrac{1}{12}Q^2-\tfrac{1}{4}[(E_k-\omega)^2+\tfrac{1}{3}q^2])-\mu^2(\omega E_k-\omega^2-\tfrac{1}{3}q^2)\}, \tag{11}$$

where

$$E_k=(m_\eta^{\ 2}-2m_\eta k)^{1/2},$$

$$Q^2=E_k^{\ 2}-2\omega E_k+\mu^2,$$

$$x(k,\omega)=\{(E_k^{\ 2}-2\omega E_k-3\mu^2)/Q^2\}^{1/2},$$

$$k_{\max}'=m_\eta-3\mu,$$

and

$$\omega_{\max}=(E_k^{\ 2}-3\mu^2)/2E_k.$$

The decay rate is predicted to be

$$R'\equiv\Gamma(\pi^+\pi^-\pi^0\gamma)/\Gamma(\pi^0\gamma\gamma)\simeq0.42\,\%. \tag{12}$$

If we now introduce a correction[12] for the X^0-η^0 mixing (the mixing angle being determined from the mass values of X^0 and η^0), then Eq. (12) is reduced by a factor 0.68 (or enhanced by 2.41 depending on the sign of the angle) giving

$$R'\simeq0.28\,\%. \tag{13}$$

The predictions (12) and (13) are to be compared with the experimental upper limits $R'<0.9\,\%$[5] and $R'<0.6\,\%$.[4,13,14]

The decay $\eta\to\pi^+\pi^-\pi^0\gamma$ was expected to be suppressed because of the fact that the transition rate depends on the fourth power of the pion momentum; so the high-frequency part of the photon is suppressed, and the two pions being in relative a P state, there is an angular momentum barrier.

Previous calculations[2] of the branching ratio $\Gamma(\eta\to\pi^+\pi^-\pi^0\gamma)/\Gamma(\eta\to\pi^0\gamma\gamma)$ are based upon the hypothesis of ρ-dominance model and certain other parameters. In the present calculation the only other assumption, except those of the current algebra, PCAC, and the soft-pion limits, is the dominance of the $E1$ transition. Otherwise the calculation is model independent and is free from any parameters. The calculated ratios [Eqs. (12) and (13)], being well within the experimental upper limits, indicate that because of the kinematic dependence of the matrix element on the pion momenta one may perhaps ignore the final-state interactions between the pions in the decay process $\eta\to\pi^+\pi^-\pi^0\gamma$.

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A MODEL OF LEPTONS*

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Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum. However, this raises the specter of unwanted massless Goldstone bosons.[2] This note will describe a model in which the symmetry between the electromagnetic and weak interactions is spontaneously broken, but in which the Goldstone bosons are avoided by introducing the photon and the intermediate-boson fields as gauge fields.[3] The model may be renormalizable.

We will restrict our attention to symmetry groups that connect the observed electron-type leptons only with each other, i.e., not with muon-type leptons or other unobserved leptons or hadrons. The symmetries then act on a left-handed doublet

$$L\equiv\left[\tfrac{1}{2}(1+\gamma_5)\right]\begin{pmatrix}\nu_e\\ e\end{pmatrix}\tag{1}$$

and on a right-handed singlet

$$R\equiv\left[\tfrac{1}{2}(1-\gamma_5)\right]e.\tag{2}$$

The largest group that leaves invariant the kinematic terms $-\bar{L}\gamma^\mu\partial_\mu L-\bar{R}\gamma^\mu\partial_\mu R$ of the Lagrangian consists of the electronic isospin $\vec{T}$ acting on L, plus the numbers N_L, N_R of left- and right-handed electron-type leptons. As far as we know, two of these symmetries are entirely unbroken: the charge $Q=T_3-N_R-\frac{1}{2}N_L$, and the electron number $N=N_R+N_L$. But the gauge field corresponding to an unbroken symmetry will have zero mass,[4] and there is no massless particle coupled to N,[5] so we must form our gauge group out of the electronic isospin $\vec{T}$ and the electronic hyperchange $Y\equiv N_R+\frac{1}{2}N_L$.

Therefore, we shall construct our Lagrangian out of L and R, plus gauge fields $\vec{A}_\mu$ and B_μ coupled to $\vec{T}$ and Y, plus a spin-zero doublet

$$\varphi=\begin{pmatrix}\varphi^0\\ \varphi^-\end{pmatrix}\tag{3}$$

whose vacuum expectation value will break $\vec{T}$ and Y and give the electron its mass. The only renormalizable Lagrangian which is invariant under $\vec{T}$ and Y gauge transformations is

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu+g\vec{A}_\mu\times\vec{A}_\nu)^2-\tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2-\bar{R}\gamma^\mu(\partial_\mu-ig'B_\mu)R-L\gamma^\mu(\partial_\mu ig\vec{t}\cdot\vec{A}_\mu-i\tfrac{1}{2}g'B_\mu)L\\&-\tfrac{1}{2}|\partial_\mu\varphi-ig\vec{A}_\mu\cdot\vec{t}\varphi+i\tfrac{1}{2}g'B_\mu\varphi|^2-G_e(\bar{L}\varphi R+\bar{R}\varphi^\dagger L)-M_1{}^2\varphi^\dagger\varphi+h(\varphi^\dagger\varphi)^2.\end{aligned}\tag{4}$$

We have chosen the phase of the R field to make G_e real, and can also adjust the phase of the L and Q fields to make the vacuum expectation value $\lambda\equiv\langle\varphi^0\rangle$ real. The "physical" φ fields are then φ^-