RADIATIVE PION ABSORPTION AND MUON CAPTURE

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The hypotheses of partially conserved axial-vector currents and of current commutators are used to prove a direct relationship between the radiative absorption of soft pions and muon capture in nuclei without recourse to the impulse approximation or assumptions concerning nuclear structure.

Recently, it has been noted[1-4] that the processes of radiative pion absorption and muon capture in complex nuclei are dynamically related. In particular, the transition operator involved in the former process is essentially identical to the axial-vector part of the operator appearing in muon capture. Both these operators excite spin-isospin modes in nuclei, and radiative pion absorption has the further virtue that one can detect the outgoing photon[5] and thus establish the distribution of nuclear excitation strength for the spin-isospin oscillations.

Thus far, proofs of the connection between the two processes under discussion here have been based[1-4] on the impulse approximation for the radiative-pion-absorption amplitude. For very low-energy pions, the leading term in that amplitude is seen by inspection to be the same as the nonrelativistic-muon-capture amplitude in the nuclear space. It has been suggested,[6] however, that in the spirit of treating the nucleus as an elementary particle in muon capture[7] one may be able to use the hypothesis[8,9] of partially conserved axial-vector currents (PCAC) to prove this connection without appeal to the impulse approximation. The present note accomplishes that goal for soft pions; in doing so, it also makes use of a current commutator hypothesis.[10-12]

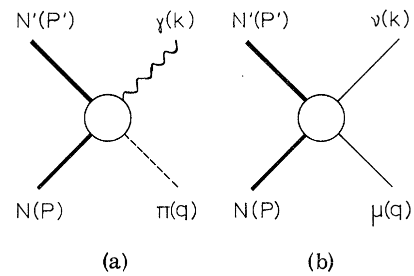

FIG. 1. The two processes under consideration. The quantities in parentheses are the four-momenta of the associated particles.

We consider the processes

$$\pi^- + N \to N' + \gamma, \tag{1a}$$

and

$$\mu^- + N \to N' + \nu_\mu, \tag{1b}$$

where N and N' are the initial and final nuclear states, and the pion or muon is initially bound in an atomic orbit. These processes are illustrated in Fig. 1, where the definitions of the kinematical variables we shall use here are also given. For the sake of convenience we consider very slow pions or muons, $q \approx 0$ and $q_0 \gtrsim m_{\pi(\mu)}$; the effects of the weak atomic binding must then be incorporated by inserting the appropriate hydrogenic wave function. The transition rate for muon capture is[13,14]

$$W_\mu = \tfrac{1}{2}G^2(2\pi)^4 \int d\vec{P}'\, d\vec{k}\, \delta^{(4)}(q+P-k-P') \times \sum_{\substack{\text{nuclear}\\ \text{spins}}} \{2\,\mathrm{Re}[M_0^*(M_0-\hat{k}\cdot\vec{M})] + (\vec{M}^*\cdot\vec{M}-M_0^*M_0) + \mathrm{Im}[\hat{k}\cdot(\mathrm{M}\times\mathrm{M}^*)]\}, \tag{2}$$

where $G \cong 10^{-5}/(938\ \mathrm{MeV})^2$ and $M_\mu = \langle N'|J_\mu^\dagger(0) + J_\mu^{5\dagger}(0)|N\rangle$ is the matrix element of the sum of the nuclear weak current and axial-vector current. In what follows, we concern ourselves only with the contributions of the axial-vector current and assume that we may rely on nuclear-structure considerations[4] to identify these contributions.

For the pion-absorption process (1a), the transition rate is

$$W_\pi = \frac{1}{4q_0P_0}\frac{1}{(2\pi)^2}\int\frac{d\vec{P}'}{2P_0'}\frac{d\vec{k}}{2k}\delta^{(4)}(q+P-k-P')\sum_{\text{spin}}|T_{fi}|^2, \tag{3}$$

where the T-matrix element is defined through

$$\langle N'\gamma_{\text{out}}|N\pi_{\text{in}}^-\rangle = i(2\pi)^{-2}\delta^{(4)}(q+P-k-P')[16q_0P_0P_0'k]^{-1/2}T_{fi}. \tag{4}$$

The electromagnetic field may be eliminated by writing

$$\langle N'\gamma_{\text{out}}|N\pi_{\text{in}}^-\rangle = ie\int d^4x\,(2\pi)^{-\frac{3}{2}}(2k)^{-\frac{1}{2}}e^{ikx}\epsilon_\mu\langle N'|j_\mu(x)|N\pi_{\text{in}}^-\rangle, \tag{5}$$

where ϵ_μ is the photon polarization vector and we have introduced the isovector current $j_\mu(x)$ acting in the nuclear space. Taking this current to be translationally invariant, we get

$$T_{fi} = e(2\pi)^{9/2}[8q_0P_0P_0']^{1/2}\epsilon_\mu\langle N'|j_\mu(0)|N\pi_{\text{in}}^-\rangle. \tag{6}$$

In this last matrix element we now use standard reduction techniques[14] to take out the pion field $\varphi^\dagger$; then

$$\langle N'|j_\mu(0)|N\pi_{\text{in}}^-\rangle = i\int d^4y(2\pi)^{-\frac{3}{2}}(2q_0)^{-\frac{1}{2}}e^{-iqy}(\Box+m_\pi^2)\theta(-y_0)\langle N'|[j_\mu(0),\varphi^\dagger(y)]|N\rangle, \tag{7}$$

and the PCAC hypothesis allows us to replace this field operator with the divergence of the weak axial-vector current by using

$$\partial_\lambda J_\lambda^{5\dagger}(y) = if_\pi m_\pi^2\varphi^\dagger(y), \tag{8}$$

with $f_\pi = 0.94m_\pi$. In the limit of a soft-pion assumption we have

$$\lim_{q\to 0}(2q_0)^{1/2}\langle N'|j_\mu(0)|N\pi_{\text{in}}^-\rangle = \frac{1}{(2\pi)^{3/2}f_\pi}\int d^4y\,\theta(-y_0)\left\langle N'\left|\left[j_\mu(0),\frac{\partial}{\partial t}J_0^{5\dagger}(y)\right]\right|N\right\rangle$$

$$= \frac{+1}{(2\pi)^{3/2}f_\pi}\langle N'|[j_\mu(0),Q_a^\dagger(0)]|N\rangle, \tag{9}$$

where we have introduced the pseudoscalar

$$Q_a^\dagger(t) = \int d\vec{y}\,J_0^{5\dagger}(y), \tag{10}$$

and integrated by parts with the neglect of surface terms.[15] Under the current-commutator hypothesis, we take[10]

$$[j_\mu(0),Q_a^\dagger(0)] = J_\mu^{5\dagger}(0). \tag{11}$$

Then

$$T_{fi} = (+e/f_\pi)(2\pi)^3[4P_0P_0']^{\frac{1}{2}}\epsilon_\mu\langle N'|J_\mu^{5\dagger}(0)|N\rangle, \tag{12}$$

and

$$W_\pi = \frac{e^2}{f_\pi^2}\frac{(2\pi)^4}{2q_0}\int d\vec{P}'\frac{d\vec{k}}{k}\delta^{(4)}(q+P-k-P')\sum_{\substack{\text{nuclear}\\ \text{spin}}}[|\langle N'|\vec{J}^{5\dagger}(0)|N\rangle|^2 - |\langle N'|\hat{k}\cdot\vec{J}^{5\dagger}(0)|N\rangle|^2], \tag{13}$$

which involves precisely the same nuclear axial-vector–current matrix element as we had in Eq. (2). At this point, one may proceed with standard techniques to evaluate this matrix element, using for instance the "elementary-particle" approach[7] or the more conventional Foldy-Wouthuysen transformation, multipole expansion, and nuclear-model discussion.[16]

We note that in making the modification for absorption from bound states, the above results will be valid only if the pion and muon are captured out of the same atomic orbit, i.e., the 1S orbit.[17] Furthermore, in identifying the matrix element $\langle N'|J_\mu^{5\dagger}(0)|N\rangle$ of Eqs. (12) and (13) with that of Eq. (2), we are neglecting the relatively mild dependence[4,16] of the nuclear matrix element on differing momentum transfers arising because $m_\pi > m_\mu$. Of course, gauge invariance is satisfied in Eqs. (12) and (13) only in the limit $m_\pi \to 0$.

Lastly, it should be pointed out that a technique similar to the one presented here can be applied to a variety of other problems involving nuclear interactions with two currents, provided that no resonances are encountered which would invalidate the soft-pion assumption.

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[17]For nuclei lighter than ^{6}Li, the pion absorption will most likely meet this condition automatically [A. M. L. Messiah and R. E. Marshak, Phys. Rev. 88, 678 (1952)]; for $A > 6$ absorption from the $2P$ atomic level becomes increasingly competitive and must be separated out experimentally if comparison with present theory is to be made. Note also that in light nuclei the 1S atomic wave function is essentially a constant in the nuclear volume as is the free wave function in the limit $q \to 0$.

CURRENT ALGEBRA AND THE DECAY MODES $\eta \to \pi^+\pi^-\pi^0\gamma$ AND $\eta \to \pi^0\gamma\gamma$

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Recently Price and Crawford[1] searched for the decay mode

$$\eta \to \pi^+\pi^-\pi^0\gamma, \tag{A}$$

and found the experimental upper limit

$$R \equiv \frac{\Gamma(\pi^+\pi^-\pi^0\gamma)}{\Gamma(\pi^+\pi^-\pi^0)} < 0.9\,\%. \tag{1}$$

This is usually compared with the theoretical prediction[1,2] based on the power of α and a phase space factor, a, of order $a \simeq 100$ for the three- to four-body final states:

$$R \simeq (\alpha a)^{-1} \simeq 1, \tag{2}$$

which is in contradiction with Eq. (1).

The above comparison is, however, invalid because of the fact that two pions in (A) are in a relative P state, and hence the decay $\eta \to \pi^+\pi^-\pi^0\gamma$ is inhibited due to the angular momentum barrier.[3]