ic Energy Commission, Army Research Office (Durham), National Aeronautics and Space Administration, and Advanced Research Projects Agency.

†Alfred P. Sloan Fellow.

[1]M. H. Cohen and D. H. Douglass, Jr., preceding letter [Phys. Rev. Letters 19, 118 (1967)].

[2]L. Holland, The Vacuum Deposition of Thin Films (John Wiley & Sons, Inc., New York, 1956).

[3]For a review of early work on low-temperature deposition, see R. Hilsch, in Non-Crystalline Solids, edited by V. D. Freschette (John Wiley & Sons, Inc., New York, 1958).

[4]W. Rühl, Z. Physik 186, 190 (1965).

[5]In a previous experiment an Al film less than 100 Å was prepared at room temperature. The resistance measured at 10 μA increased markedly as the temperature was lowered. This indicated that the film probably had the structure suggested by C. A. Neugebauer and P. H. Wilson, in Basic Problems in Thin Film Physics: Proceedings of the International Symposium on Thin Film Physics, Göttingen, 1965, edited by R. Niedermayer and H. Mayer (Vanderhoeck and Ruprecht, Göttingen, 1966), p. 579, namely islands of metal separated by electron barriers. This structure has previously been reported by investigators at Brookhaven National Laboratory and RCA. When the resistance was measured at 2 μA, the curve started to break away from the 10-μA curve at a temperature estimated to be between 10 and 16°K, and the film became totally superconducting at 3°K. In the terms of this paper we interpret this result as arising from -*SBS*- combinations formed in the plane of the film, where the *B* material is probably Al_2O_3. We also note that some part of the sample must have gone superconducting between 10 and 16°K.

[6]V. L. Ginzburg, Phys. Letters 13, 101 (1964).

[7]G. J. Thompson and J. M. Blatt, Phys. Rev. Letters 5, 6 (1963); A. Paskin and A. D. Singh, Phys. Rev. 140, A1965 (1965); V. Z. Kresin and B. A. Tavger, Zh. Eksperim. i Teor. Fiz. 50, 1689 (1966) [translation: Soviet Phys.—JETP 23, 1124 (1966)]; R. Parmenter, to be published.

[8]Electron micrographs have been taken of both quartz and glass surfaces. The quartz showed a surface roughness ~100 Å. The glass slides were smooth on a 30-Å scale.

PARALLEL FIELD DEPENDENCE OF VORTEX PINNING IN SUPERCONDUCTING FILMS*

R. Deltour† and M. Tinkham
Harvard University, Cambridge, Massachusetts
(Received 10 May 1967)

The experimentally measured influence of the parallel component of the magnetic field on the depinning threshold of vortices in superconducting films is related directly to the reduction of the square of the Ginzburg-Landau order parameter.

Numerous authors have shown that a type of Lorentz force tends to move quantum flux vortices in current-carrying type-II superconductors in a direction perpendicular to both magnetic field and current, leading to a resistive voltage.[1] The maximum pinning force resisting this driving force can be determined by measuring for a given current I the magnetic field H_{cr} corresponding to the onset of a measurable voltage along the superconductor.[2] Systematic study of the pinning force is complicated by the fact that it depends on both H and I, which cannot be varied independently in a given geometry while maintaining the threshold condition.

Thin films offer a convenient system for overcoming this difficultly because they allow the problem to be treated essentially in only two dimensions. The vortex structure is determined by $H_\perp$, the component of the field perpendicular to the surface, while the parallel component $H_\parallel$ acts essentially independently to reduce the superconducting order parameter Δ and hence the pinning strength. The driving force acting on the vortices is completely determined once the current passing along the film and the perpendicular component of the field are fixed, as has been shown experimentally.[3] The relation between the driving force and the magnetic field is

$$F_{\text{driving}} = kIH_\perp = kIH\sin\theta, \qquad (1)$$

where k may have a current dependence because of redistribution of current density, but this does not concern us since the experiments were done at constant current. θ is the angle between the field and the surface of the film, the azimuthal angle having been shown[3] to be unimportant in thin films.

The experiments were done on evaporated films of tin, 1.5 cm long, 2 mm wide, and approximately 1500 Å thick, deposited on a flat glass substrate at liquid-nitrogen temperature. The films were immersed in a pressure-regulated liquid-He^4 bath, and the longitudinal voltage, measured using a four-terminal technique sensitive to ~0.2 μV, was fed to an XY recorder whose other channel was driven by a signal proportional to the applied magnetic field.

We have measured the value of the magnetic field $H_{cr}(\theta)$, at the depinning threshold, with a given constant current I passing along the film, with emphasis on orientations close to parallelism.

Since our experiments were performed near T_c, the Ginzburg-Landau (GL) theory should form a valid basis for their interpretation. The leading terms in the GL expansion of the free energy density in a superconductor are proportional to $|\Delta|^2$ and $\xi_T^2|(i\nabla-2\pi\vec{A}/\varphi_0)\Delta|^2$, where ξ_T is the usual temperature-dependent coherence length, $\vec{A}$ is the magnetic vector potential, and φ_0 is the flux quantum $hc/2e$. Spatially averaged, these terms should be of equal order of magnitude and dominate over the Δ^4 term in the free energy, provided Δ is depressed from its thermal equilibrium value by a magnetic field as in our experiments. Thus, we can take the free energy density to scale with Δ^2. Inhomogeneities in the material lead to spatial variations of the scaling coefficient. The pinning energy arises because some positions of the vortex array with respect to the inhomogeneities lead to a lower total free energy than do others. The pinning force will be a derivative of this energy with respect to relative position of the vortex array and the inhomogeneities. Two characteristic lengths enter such a derivative: the scale of the defect structure and the coherence length, dependent on temperature but not on field, which gives the scale of length for the variation of Δ within a vortex. At constant temperature both these lengths are constant, and thus we expect that we can write the pinning force in the form

$$F_{\text{pinning}}=k'\Delta_m^{\;2}, \tag{2}$$

where Δ_m is the maximum value of Δ between vortex centers and k' is independent of field.

In the thin-film limit, it can be shown[4] that the GL solutions for arbitrary field orientations can be written in terms of those for the perpendicular field case by simple scaling procedures. The length scale in the film plane is determined by $H_\perp$, which fixes the spacing of the vortices. The amplitude scale of Δ^2 depends explicitly on the parallel component of the field through the same factor found in the purely parallel field case, namely $[1-(H_\parallel/H_{c\parallel})^2]$. The amplitude scale is also a unique decreasing function g of the perpendicular field normalized to $H_{c\perp}[1-(H_\parallel/H_{c\parallel})^2]$, the critical perpendicular field component in the presence of the parallel field.[5] Thus

$$\Delta_m^{\;2}=\Delta_0^{\;2}(T)[1-(H_\parallel/H_{c\parallel})^2]g\{(H_\perp/H_{c\perp})[1-(H_\parallel/H_{c\parallel})^2]^{-1}\}. \tag{3}$$

In this, $H_{c\parallel}$ and $H_{c\perp}$ are the usual parallel and perpendicular critical fields of the film, and $\Delta_0(T)$ is the zero-field value of the order parameter Δ. At the depinning threshold, for the vortices which are on the verge of moving, we have

$$F_{\text{driving}}=F_{\text{pinning}},$$

or taking into account (1), (2), and (3),

$$kIH_\perp=k'\Delta_0^{\;2}[1-(H_\parallel/H_{c\parallel})^2]g\{(H_\perp/H_{c\perp})[1-(H_\parallel/H_{c\parallel})^2]^{-1}\}. \tag{4}$$

From this we see that

$$kI=g_1\{(H_\perp/H_{c\perp})[1-(H_\parallel/H_{c\parallel})^2]^{-1}\}, \tag{5}$$

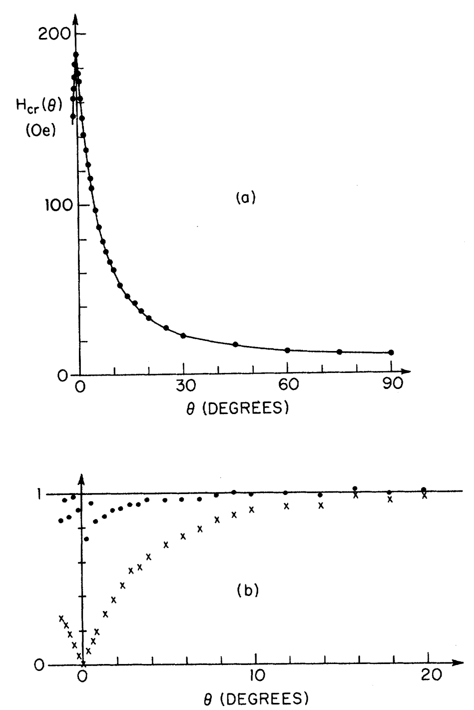

FIG. 1. (a) Angular dependence of the depinning critical field for a film of tin carrying a current of 1 mA. Dots are experimental points; solid curve is theory. (b) X's: experimental values of $H_{cr}(\theta)\sin\theta/H_{cr}(90°)$. Dots: the same data plotted according to Eq. (6), showing that the parallel-field-component correction accounts for the reduction of the depinning threshold at small angles. [Film characteristics: thickness 1400 Å; $T/T_c = 0.97$; $H_{c\perp} = 16.5$ Oe, $H_{c\|} = 186.5$ Oe, and $H_{cr}(90°) = 11.7$ Oe.]

where $g_1(x) \sim x^{-1}g(x)$. Thus, at constant current, we expect the threshold value of $H_\perp[1-(H_\|/H_{c\|})^2]^{-1}$ to be constant, or equivalently, that

$$\frac{H_{cr}(\theta)\sin\theta}{H_{cr}(90°)[1-(H_{cr}(\theta)\cos\theta/H_{c\|})^2]} = 1. \qquad (6)$$

The dots in Fig. 1(a) represent the experimental values of $H_{cr}(\theta)$ for a film of tin 1400 Å thick at $T = 0.9T_c$ for which κ was determined to be 0.7. [The film thickness was determined from $H_{c\|}$ and $H_{c\perp}$, and confirmed by optical interferometry; κ was determined using the relation $H_{c\perp} = (2)^{1/2}\kappa H_{cb}$.] The continuous line has been calculated by using Eq. (6) and the experimental results for $H_{cr}(90°)$ and $H_{c\|}$, determined using the same value of the current ($I = 1$ mA) as for the depinning experiments at intermediate angles.

A more critical way to present the results was used in Fig. 1(b), where we have plotted as dots the left-hand side of Eq. (6). For comparison, the crosses represent the results obtained from the same experimental data but without applying the parallel-field-component correction in (6). The type of plot used is very sensitive to any experimental error at small angles since it is the ratio of two quantities, each of which is approaching zero. This may partially account for the scatter of the dots in that region.

The agreement between the experimental results and expression (6) is surprisingly good, since no correction was made to take into account the higher order terms in the expansion of the free energy density in the pinning expression. Also, no correction was made for any redistribution of current density with change in magnetic field, and it was assumed that the current densities were low enough that expression (3) could be used, although it was derived for the case of zero current. It is possible that the trend toward a lowering of the points in Fig. 1(b) at the very small angles (<3°) could be due to these factors.

*Research supported in part by National Science Foundation, U. S. Office of Naval Research, Advanced Research Projects Agency, and National Atlantic Treaty Organization.

†Chargé de Recherches du Fond National de Recherche Scientifique.

[1]See, for example, C. F. Hempstead and Y. B. Kim, Phys. Rev. Letters 12, 145 (1964).

[2]Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. 139, A1163 (1965).

[3]R. Deltour and M. Tinkham, Phys. Letters 23, 183 (1966).

[4]P. Wyder, to be published.

[5]M. Tinkham, Phys. Rev. 129, 2413 (1963); in Basic Problems in Thin Film Physics: Proceedings of the International Symposium on Thin Film Physics, Göttingen, 1965, edited by R. Niedermayer and H. Mayer (Vandenhoeck and Ruprecht, Göttingen, 1966), p. 499.