monicity. Details and complete results will be communicated in a future publication.

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†On leave from Case Western Reserve University.

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BAND MIXING IN ^{152}Sm AND ^{154}Gd

L. L. Riedinger* and Noah R. Johnson
Chemistry Division, Oak Ridge National Laboratory,† Oak Ridge, Tennessee

and

J. H. Hamilton
Vanderbilt University,‡ Nashville, Tennessee

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Alaga et al.[1] have used the collective model of Bohr and Mottelson[2] (BM) to predict the ratio of reduced transition probabilities from a member of an excited vibrational band to two different members of the ground-state band in deformed nuclei. This value is simply the ratio of the squares of the appropriate Clebsch-Gordan coefficients. However, it is known that these simple intensity predictions based on an adiabatic approximation break down. This results from the fact that the wave functions of the two bands become mixed and cause significant corrections to the matrix elements describing the transitions. The best-studied cases demonstrating this behavior involve the $E2$ transitions between the members of β- and γ-vibrational bands and the ground-state band in even-even deformed nuclei. Here the deviations between the experimental branching ratios and the simple predictions have been analyzed in terms of a band-mixing parameter Z_K, which is a measure of the rotation-vibration interaction. The subscript K refers to the K quantum number of the vibrational band considered. For these transitions from the β- and γ-vibrational bands, Bohr and Mottelson[3] have included the effects of band mixing to define the reduced $E2$ transition probability, $B(E2)$. This expression involves the unmixed transition probability, the mixing parameter, and spin-dependent terms.

We have experimentally determined the ratios of reduced $E2$ transition probabilities from members of the β- and γ-vibrational bands in ^{152}Sm and ^{154}Gd from the corresponding ratios of gamma-ray intensities. The mixing parameters needed to bring these experimental ratios into agreement with the predicted ratios of Alaga et al.[1] have then been calculated and compared with some microscopic-model predictions.[4-6]

A 6-cm^3 Ge(Li) detector, with resolution of 3.1-keV full width at half-maximum for 661.6 keV, was used to measure the gamma-ray intensities in the decay of ^{152}Eu (12.4 y) and ^{154}Eu (16 y). Extensive NaI-Ge(Li) and NaI-NaI coincidence measurements have also been performed on these nuclei. In ^{152}Sm, the coincidence data have been used to assign the intensity of the $2^{+\prime} \rightarrow 4^+$ transition (note that the prime refers to a member of the β band) since another gamma ray of this same energy exists elsewhere in the decay scheme. All other gamma-ray intensities are results of the Ge(Li) singles experiments and are in good agreement with the coincidence data.

First, we will summarize the results for the γ band. In each of the nuclei ^{152}Sm and ^{154}Gd, we see the seven expected $E2$ transitions between the 2^+, 3^+, and 4^+ states of the γ band and the 0^+, 2^+, and 4^+ members of the ground-state rotational band. Values of the band-mixing parameter, Z_2, calculated from the various $B(E2)$ ratios are internally consistent and lead to weighted averages of $-(7.7\pm0.9)\times10^{-2}$ for ^{152}Sm and $-(6.6\pm0.7)\times10^{-2}$ for ^{154}Gd. When corrected by these Z_2 parameters, the BM-predicted $B(E2)$ ratios are in excellent agreement with experiment. There is equally good agreement between the experimental $B(E2)$ ratios and the ratios predicted by Davidson,[7] who uses an asymmetric rotor model with deformation vibrations but no asymmetry vibrations.

For the β band, the situation seems to be more complex, however. The 0^+, 2^+, and 4^+ members of the β bands in the nuclei under consideration lie at fairly low energies (respectively, 685, 811, and 1023 keV for ^{152}Sm and 682, 815, and 1048 keV for ^{154}Gd). Ratios of reduced $E2$ transition probabilities from the 2^+ and 4^+ members of the β bands are calculated from experiment and given in column 3 of Table I. We have determined the mixing parameter Z_0 for each ratio and these values are listed in column 4. For each nucleus, there is a wide variation in the individual Z_0 values calculated, contrary to what was found for the γ bands. Only by reducing the $E2$ intensity of the $2^{+\prime} \rightarrow 2^+$ transition to one-half the observed photon value is internal consistency in these Z_0 values obtained for both ^{152}Sm and ^{154}Gd. Since we had performed exhaustive Ge(Li)-NaI coincidence measurements to verify the intensities assigned to the $2^{+\prime} \rightarrow 2^+$ transitions, we were inclined to attribute the variations in Z_0 to possible $M1$ radiation in these transitions. Very recently, Liu et al.[8] reached a similar conclusion in their work on these same nuclei. However, Hamilton et al.[9] have performed measurements of the angular correlation for the $2^{+\prime} \rightarrow 2^+ \rightarrow 0^+$ cascade in ^{154}Gd and their results indicate that the $2^{+\prime} \rightarrow 2^+$ transition is essentially pure $E2$. Also, McGowan, Sayer, and Stelson[10] have performed angular-correlation measurements on ^{152}Sm during the Coulomb excitation of the $2^{+\prime}$ level. Their preliminary results indicate that the $2^{+\prime} \rightarrow 2^+$ transition in this nucleus is either about 80% $M1$ or pure $E2$. Since the ^{152}Sm and ^{154}Gd nuclei are so similar (both have 90 neutrons, which is just at the onset of the region of deformation), one is inclined to assume that the $2^{+\prime} \rightarrow 2^+$ transition in ^{152}Sm is also probably pure $E2$.

The results of these coincidence and angular correlation experiments present us with a very mysterious situation. In both ^{152}Sm and ^{154}Gd, there appears to be no way to bring into agreement the three mixing parameters calculated from the branchings of the $2^{+\prime}$ level.

For the $4^{+\prime}$ level of ^{154}Gd we are able to get but a single value of Z_0 since only the weak $4^{+\prime} \rightarrow 2^+$ and $4^{+\prime} \rightarrow 4^+$ transitions were observed. The mixing parameter calculated from the intensities of these transitions is $-(4.9\pm1.1)\times10^{-2}$. This Z_0 value agrees with that calculated from the $(2^{+\prime} \rightarrow 4^+)/(2^{+\prime} \rightarrow 0^+)$ ratio and therefore suggests that the mixing parameters from the $4^{+\prime}$ level may be internally consistent. However, Meyer[11] has measured the ^{154}Gd gamma-ray intensity of the very weak $4^{+\prime} \rightarrow 6^+$ transition, which was not observed in our work. Using this intensity along with our intensities for the other two transitions from the $4^{+\prime}$ level, we obtain Z_0 values which are internally consistent only when the $E2$ intensity of the $4^{+\prime} \rightarrow 4^+$ transition is about 60% of the total gamma-ray intensity. Even then, the average value is considerably smaller than the average obtained when only the decays of the $2^{+\prime}$ level are considered. Since our coincidence

Table I. Experimental and theoretical ratios of reduced $E2$ transition probabilities from members of the β bands in ^{152}Sm and ^{154}Gd.

Nucleus	$\frac{I' \to I_1}{I' \to I_2}$	$\frac{B(E2;\, I' \to I_1)}{B(E2;\, I' \to I_2)}$ Experiment	$-Z_0 \times 10^2$	$\frac{B(E2;\, I' \to I_1)}{B(E2;\, I' \to I_2)}$ (Theory) B-M with Mixing[a]	Davidson[b]
^{152}Sm	$\frac{2+' \to 4+}{2+' \to 2+}$	2.47 ± 0.88	1.2 ± 1.5	5.36 ± 0.61	4.28
	$\frac{2+' \to 0+}{2+' \to 2+}$	0.19 ± 0.05	8.0 ± 1.1	0.33 ± 0.04	0.38
	$\frac{2+' \to 4+}{2+' \to 0+}$	13.0 ± 5.2	4.5 ± 1.2	16.2 ± 3.8	11.3
^{154}Gd	$\frac{2+' \to 4+}{2+' \to 2+}$	2.97 ± 0.75	2.0 ± 1.2	6.75 ± 0.49	4.34
	$\frac{2+' \to 0+}{2+' \to 2+}$	0.12 ± 0.03	9.8 ± 0.8	0.25 ± 0.03	0.37
	$\frac{2+' \to 4+}{2+' \to 0+}$	25.4 ± 8.0	6.5 ± 0.9	27.0 ± 4.7	11.7
	$\frac{4+' \to 2+}{4+' \to 4+}$	0.11 ± 0.11	4.9 ± 1.1	0.004 ± 0.01	0.24

[a]The predictions of the Bohr-Mottelson model are corrected for band mixing through the use of weighted averages of the Z_0 values in column 4. For ^{152}Sm this average is $-(5.2 \pm 0.7) \times 10^{-2}$; for ^{154}Gd, $-(6.7 \pm 0.5) \times 10^{-2}$.

[b]Predictions result from asymmetric rotor model with $\gamma = 10.73°$, $\mu = 0.40$ for ^{152}Sm and $\gamma = 11.62°$, $\mu = 0.40$ for ^{154}Gd.

results for this very weak $4^{+\prime} \to 4^+$ transition are inconclusive and since no angular-correlation measurements have been performed on it, it is not possible to know if this apparent discrepancy is real.

A possible explanation for the existence of $M1$ radiation in the $2^{+\prime} \to 2^+$ and $4^{+\prime} \to 4^+$ transitions is the presence of an admixed $K = 1^+$ state in the β and ground bands. In a consideration of such admixtures, Mikhailov[12] has derived an expression for the spin-dependent amplitude of the admixed K state and has found that the $M1$ component in the $4^{+\prime} \to 4^+$ transition should be greater than that in the $2^{+\prime} \to 2^+$ transition by a factor 3.3. Since the $2^{+\prime} \to 2^+$ transition of ^{154}Gd has been found by Hamilton et al.[9] to be essentially pure $E2$ (the experimental errors allow for only a small $M1$ contribution), any possible $M1$ component in the $4^{+\prime} \to 4^+$ transition, arising from a $K = 1^+$ admixture, would be expected to be too small to explain the variation in Z_0 values from the $4^{+\prime}$ level. It therefore seems that whatever is perturbing the branching ratios from the $2^{+\prime}$ level in ^{154}Gd is having a similar effect on the branching ratios from its $4^{+\prime}$ level. In ^{152}Sm, we observe only a single transition from the $4^{+\prime}$ level ($4^{+\prime} \to 4^+$). Consequently, no Z_0 value from this level is obtained.

Although the implications of these varying Z_0 values are not understood, we take an average of each set of values in Table I, in order to carry out additional calculations and make comparisons with the model predictions. For ^{152}Sm, the weighted average is $-(5.2 \pm 0.7) \times 10^{-2}$; for ^{154}Gd, $-(6.7 \pm 0.5) \times 10^{-2}$. In each case, all three of the Z_0 values obtained from the branching ratios of the $2^{+\prime}$ level have been included in the average. This is improper, since only two unique branching ratios can be obtained from three intensities. However, the variations in the three Z_0 values make it impossible to choose any two values as characteristic of the level. The resulting Z-corrected $B(E2)$ ratios are shown in column 5 of Table I. Note that agreement between these values and the original ratios in column 3 can be achieved only if the $E2$ intensity of the $2^{+\prime} \to 2^+$ transition in both ^{152}Sm and ^{154}Gd is assumed to be one-half of the total gamma-ray intensity. In column 6, the corresponding asymmetric rotor predictions of Davidson[7] show a similar degree of variance with experiment.

There have been several theoretical predictions[4-6] of band-mixing parameters. These along with our experimental weighted-average values of Z_0 and Z_2 are shown in Table II. The predicted values of Z_0 and Z_2 result from microscopic models of particles interacting through the quadrupole and pairing forces. Both Marshalek[4] and Bes et al.[5] take into account mixing of the bands by including a Coriolis interaction. Marshalek[4] assumes an adiabatic nucleus and a small-amplitude perturbation with three different considerations: Case 1 is a phenomenological treatment where the moment of inertia is assumed to be proportional to the square of the deformation. Case 2 is a pure microscopic treatment where the pairing gap parameters are found empirically from other pairing energies. Case 3 is also a microscopic treatment where the pairing gap parameters are those which reproduce the experimental moments of inertia using the cranking model. Bes et al.[5] perform calculations in the nonadiabatic limit of the random-phase approximation. Pavlichenkov[6] uses a microscopic model including a rotation-vibration interaction. These model predictions are compared with weighted averages of the Z_0 and Z_2 values in each nucleus. The agreement between experiment and theory is poor in the case of ^{152}Sm but considerably better for ^{154}Gd. In the latter case, Z_2 from experiment lies closer to the microscopic-model predictions than to the value predicted by the phenomenological approach.

To this point, we have assumed that the low-lying 0^+ state and its rotational sequence in each nucleus result from symmetric vibrations in the quadrupole field of the nucleus. However, the unexplained behavior of the branching ratios from these levels raises serious questions about the nature of the 0^+ states in ^{152}Sm and ^{154}Gd. Very recently Mikoshiba, Sheline, Udagawa, and Yoshida[13] calculated some of the properties of the ten lowest excited 0^+ states in several deformed nuclei through the use of a microscopic treatment which includes the effects of both pairing and quadrupole fields along with their possible couplings. Their results indicate that only near the beginning and end of the rare-earth deformed region is the β-vibrational character concentrated into the lowest excited 0^+ state and that even here there is a strong admixture of pairing vibrational character in these states. One might speculate that admixtures of pairing vibrational states into these "β-vibrational" levels may affect the $E2$ branchings and thus may explain the inconsistency in the mixing parameters for the β bands in ^{152}Sm and ^{154}Gd. However, Udagawa[14] has pointed out that the predictions of branching ratios from their treatment are essentially the same as those of Bohr and Mottelson. It seems that a new treatment of these vibrations in transitional deformed nuclei is needed.

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Table II. Experimental values and model predictions of band-mixing parameters.

Nucleus	$-Z_0\times10^2$ Exptl[a]	$-Z_2\times10^2$ Exptl[a]	$-(Z_0$ or $Z_2)\times10^2$ Theory: Marshalek[b] Case 1	Marshalek[b] Case 2	Marshalek[b] Case 3	Bes et al.[c]	Pavlichenkov[d]
^{152}Sm	5.2 ± 0.7		6.2	6.7	7.0		
		7.7 ± 0.9	8.9	4.7	4.4	3.5	6.4
^{154}Gd	6.7 ± 0.5		6.3	6.3	6.3		
		6.6 ± 0.7	9.9	5.6	5.3	6.0	7.9

[a]Weighted averages of the various Z_0 and Z_2 values are used here.
[b]Reference 4.
[c]Reference 5.
[d]Reference 6.

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GAMMA RAYS FOLLOWING ^{40}Ar-INDUCED REACTIONS*

David Ward, F. S. Stephens, and J. O. Newton†
Lawrence Radiation Laboratory, University of California, Berkeley, California
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We have bombarded separated Sn and Te isotopes with ^{40}Ar projectiles in order to study the (^{40}Ar,xn) reactions and evaluate them as a means to produce excited nuclei for spectroscopic studies. This proves to be an excellent method for populating ground-band collective levels, and such levels have been identified in the 88-, 90-, and 92-neutron Er and Yb isotopes.

When medium and heavy nuclei are bombarded with heavy ions of moderate energy, the dominant reaction has been found to be complete fusion followed by the evaporation of neutrons, (HI,xn), and a correct choice of bombarding energy can often lead to an almost unique product. A number of studies[1-4] have recently been made of the γ-ray cascade which occurs as the last step in the de-excitation of a (HI,xn)-reaction product, and this technique promises to become an important one in nuclear spectroscopy. Thus far the "heavy ion" used in these studies has ranged from protons to ^{19}F. The purpose of this Letter is to report our results using ^{40}Ar as the projectile in such studies.

The interest in heavier ions for these studies lies in (a) the considerably greater linear and angular momentum given to the compound system; (b) the accessibility to regions of the periodic table that cannot easily be reached with lighter ions; and (c) the production of very neutron-deficient compound systems with lower excitation energy. The minimum excitation energy of a compound system increases with projectile mass up to around 20 and then decreases slowly because the larger negative Q value for the reaction with heavier projectiles more than offsets the increased bombarding energy necessary to exceed the Coulomb barrier. This is of considerable importance for spectroscopic studies because a lower excitation energy, in general, permits the (HI,xn) product to be made more specifically, resulting in cleaner spectra.

We have studied γ-ray spectra from ^{40}Ar reactions using a lithium-drifted germanium counter that measured 6 cm^2 by 0.8 cm deep and operated at 2.0-keV resolution for γ rays around 600 keV. In all cases this counter was at 90° to the beam direction and about 2 cm from the target. The targets generally used were prepared by evaporating about 700 μg cm^{-2} of separated isotope onto a 0.003-cm thick