of Scientific Research, U. S. Air Force, under AFOSR Grant No. AF-AFOSR-869-65, through The Ohio State University Research Foundation.

[1]L. Shen, N. M. Senozan, and N. E. Phillips, Phys. Rev. Letters 14, 1025 (1965).

[2]H. Suhl, B. J. Matthias, and L. R. Walker, Phys. Rev. Letters 3, 552 (1959).

[3]C. C. Sung and L. Y. L. Shen, Phys. Letters 19, 101 (1965).

[4]A. Griffin and K. Maki, in Proceedings of the Tenth International Conference on Low Temperature Physics, Moscow, 1966 (to be published).

[5]C. C. Sung and Victor K. Wong, to be published.

[6]E. S. Rosenblum, S. H. Autler, and K. H. Gooen, Rev. Mod. Phys. 36, 74 (1964); T. McConville and B. Serin, Phys. Rev. 140, A1169 (1965).

[7]W. A. Fietz and W. W. Webb, Phys. Rev. 161, 423 (1967).

[8]An excellent review of Type-II superconductivity is given by A. L. Fetter and P. C. Hohenberg, A Treatise on Superconductivity, edited by R. D. Parks (to be published), Chap. 15.

[9]D. R. Tilley, Proc. Phys. Soc. (London) 84, 573 (1964).

[10]V. A. Moskalenko, Zh. Eksperim. i Teor. Fiz. 51, 1162 (1966) [translation: Soviet Phys.–JETP 24, 780 (1967)].

[11]The mistake in Ref. 10 can be traced to the steps going from Eq. (36) to Eq. (39). Eq. (39) as it stands is incorrect, and the factor R_n^2/B_n should be inserted within the summation sign to render it correct. As Equation (39) is the basic equation in discussing the mixed states, the conclusions following Eq. (39), e.g., the conclusion that the free energy is not a monotonic function of the structure of the struction constant σ, are not correct.

[12]Our attention is drawn to the paper by I. Peschel, Solid State Commun. 4, 495 (1966), in which the mixed states near T_c are correctly analyzed.

[13]E. Helfand and N. R. Werthamer, Phys. Rev. Letters 13, 686 (1964); Phys. Rev. 147, 288 (1966).

[14]K. Maki and T. Tsuzuki, Phys. Rev. 139, A868 (1965).

[15]G. Eilenberger, Phys. Rev. 153, 584 (1967).

[16]A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963).

[17]We have chosen units such that $\hbar = c = k_B = 1$.

[18]A more complete discussion of this point can be found in Ref. 8.

[19]The value of K_1 in the free-electron BCS model, denoted by K_{1f}, is ≈ 0.1. In the two-band model, $K_1 \approx K_{1f}\alpha(N_s/N_d)^{1/2} \approx 2$, where the factor $(N_s/N_d)^{1/2}$ arises from $H_c \sim (N_d)^{1/2}$.

STATIC QUADRUPOLE MOMENT OF VIBRATIONAL, EVEN NUCLEI AND THE COUPLING SCHEME FOR ODD NUCLEI*

Leonard S. Kisslinger† and Krishna Kumar
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 31 August 1967)

It is observed that an anharmonic treatment of the low-lying collective modes in vibrational and transition regions is consistent with detailed theoretical calculation and with experimental static quadrupole moments and other systematic experimental data in even-even nuclei. The resulting effect on even-odd nuclei is to provide a significant improvement in the coupling scheme.

The low-energy systematics of even-even nuclei and odd-even nuclei are usefully described[1] in terms of pure collective modes and an interplay of single-particle and collective modes, respectively. The phenomenological vibrational model[1] and its microscopic description[2] in terms of the pairing-plus-quadrupole model and the quasi random-phase approximation (QRPA) have been quite successful in describing many of the systematics of "spherical" nuclei in terms of these modes and their interactions. However, the harmonic-vibrational picture needs to be modified in order to account for the static quadrupole moment of the first 2^+ state, Q_{22}, which has been found to have approximately the same magnitude as the transition moment responsible for the excitation of the 2^+ state, Q_{20}, for many "vibrational" nuclei for which experimental information is available (instead of being zero, as predicted by the vibrational model).[3] Other evidence in even nuclei which precludes a harmonic-vibrational picture includes the energy systematics of the two phonon states and the cross-over transition rates which are small but larger than one obtains from the first-order corrections to the QRPA.[2]

Recently, it has been shown[4] that the pairing-plus-quadrupole model is capable of explaining the changes in the even-nuclei systematics from well-developed rotational spectra based on deformed shapes to vibrational-type spectra based on spherical (or weakly deformed) shapes, as well as many deviations from the two kinds of spectra. In this Letter we shall discuss a modification of the vibrational model suggested[5] by the work of BK and its application to the KS coupling scheme for odd-A nuclei.

BK calculate the potential energy and the six kinetic-energy functions of Bohr's collective Hamiltonian[1] by applying time-dependent Hartree-Bogolyubov techniques to the pairing-plus-quadrupole model. The energy levels, transition probabilities, and static moments are then computed by using a completely numerical method of solving the Bohr Hamiltonian. For example, this calculation predicts for Pt^{196} a "vibrational" spectrum ($E_{2'+}=E_{4+}=2.3\ E_{2+}$), a large quadrupole moment $Q_{22}/Q_{02}=-0.65$ (this ratio would be +1.0 for a prolate rotator), and a small branching ratio $B(E2;2'\rightarrow 0)/B(E2;2'\rightarrow 2)=0.014$ [this ratio is 0.0 (0.7) in the vibrational (rotational) model]. The calculated potential energy of deformation is quite different from that required by the vibrational model (a parabola around a spherical minimum): It has a maximum at $\beta=0$, and the lowest minimum occurs for a deformed, oblate shape with $\beta=-0.15$, $\gamma=0°$. Measured from this minimum, the spherical maximum is 1.5 MeV high and there is a secondary deformed (prolate) minimum at 0.6 MeV. However, the energy of zero-point motion is large, and the nuclear ground state lies 1.5 MeV above the lowest potential minimum. Thus, the nucleus is transitional as expected.

One main purpose of this note is to point out that even though the numerical wave functions obtained by BK look rather complicated, the convergence of their expansion in the basis of spherical phonons is quite rapid for nuclei at the edge of the vibrational region. By a spherical phonon, we mean a quadrupole phonon of angular momentum 2 and energy $\hbar\omega$, corresponding to an intrinsic potential energy

$$V(\langle Q\rangle)=\tfrac{1}{2}C\langle Q\rangle^2, \qquad (1)$$

where $\langle Q\rangle$ is the expectation value of the quadrupole moment and C is a constant determined from the force parameters and the microscopic properties of the particular nucleus. The phonon model Hamiltonian of an even nucleus in terms of the phonon creation operator B^+ is

$$H=\hbar\omega\sum_\mu(-)^\mu B_\mu B_{-\mu}, \qquad (2)$$

and the lowest few states are $|0\rangle, B^+|0\rangle$, $(B^+B^+)_{I=0,2,4}|0\rangle, \cdots$, where $|0\rangle$ is the phonon vacuum. The "true" Hamiltonian has additional, anharmonic terms (the exact nature of these terms requires further study and is beyond the scope of this short note) which cause mixing between the phonon-model states. The nuclear ground state is written as $|0^+\rangle=[a_0+a_1(B^+B^+)_0+a_2(B^+B^+B^+)_0+\cdots]|0\rangle$, the first excited state as $|2^+\rangle=[b_0B^++b_1(B^+B^+)_2+b_2(B^+B^+B^+)_2+\cdots]|0\rangle$, the second excited 2^+ state as $|2'^+\rangle=[c_0(B^+B^+)_2+c_1B^++c_2(B^+B^+B^+)_2+\cdots]|0\rangle$, and so forth.

By calculating the overlaps of the numerical wave functions of Pt^{196} with those of the phonon model (in the β,γ representation), BK find that $a_0=0.924$, $a_1=-0.323$, $a_2=-0.173$; $b_0=0.923$, $b_1=-0.289$, $b_2=-0.174$; and $c_0=0.942$, $c_1=0.283$, $c_2=0.038$. (The remaining 1% of $|0^+\rangle$ and 3% of $|2^+\rangle$ and $|2'^+\rangle$ each are distributed over phonon wave functions containing more than three phonons.) It is easily checked that these mixtures do give the large quadrupole-moment ratio and the small branching ratio of Pt^{196}, discussed above. These predicted ratios have not been tested experimentally, but the situation is strongly reminiscent of Cd^{114} and, in fact, only slight changes in these mixtures of the phonon wave functions give $Q_{22}/Q_{02}=+1.0$ and $B(E2;2'\rightarrow 0)/B(E2;2'\rightarrow 2)=0.013$ in good agreement with experiment.[6]

Evidently, the phonons provide a good representation for the low-energy properties of even, spherical nuclei. This does not mean that a perturbation treatment based on the vibrational model would be adequate. As discussed above, the anharmonicities of the Hamiltonian can be quite large. Also, the actual signs and values of a's, b's, and c's would depend on the details of the microscopic structure, especially in those regions where nuclear deformation sets in. However, one can make use of certain characteristic features of the anharmonicity which appears systematically in the entire vibrational region. For example, the quadrupole moment of the first 2^+ state tends to be of the order of the $0^+\rightarrow 2^+$ transition moment, while the $2'^+\rightarrow 0^+$ cross-over transitions are always of single-particle magnitude. It can easily be shown that these features require the ratios a_1/a_0, b_1/b_0, and c_1/c_0 to be of the same magnitude (about ±0.3). The sign of b_1/b_0 is opposite to that of the quadrupole moment of the 2^+ level, and the relative sign of b_1 and c_1 is determined from orthogonality. Thus, even the scanty data on the static quadrupole moments of the 2^+ states along with the BK calculation allow a good deal of systematics to be attempted. We shall make use of this modified vibrational picture for a study of coupling schemes in odd-mass nuclei as will now be described.

A survey of odd-mass nuclei in regions where the even spectra resemble a quadrupole vibrator rather than a rotator show that the quantum numbers for the lowest levels are generally those of quasiparticles or quasiparticles plus phonons in a spherical representation, rather than those corresponding to a strongly coupled system in a deformed representation. The basic assumptions of this work for the odd-mass nuclei are that a spherical Hartree-Fock representation can be used and that the particle modes (quasiparticles) are independent of (commute with) the collective modes. Thus, having determined the states in the even nuclei, one can calculate the energies and other properties of the states in the odd-mass nuclei if the particle-phonon interaction is known. We use the phonon-quasiparticle interaction of KS:

$$H_{\text{int}} = \sum_{ij} X_{ij}(B^+ + B)(\alpha_j^+ \alpha_i), \tag{3}$$

where α_j^+ is a quasiparticle creation operator, and the force parameters X_{ij} are taken from the BCS and QRPA calculation. In this model the entire effect of the static quadrupole moment is assumed to be given by the anharmonicity of the even-even system.[7]

The results for the odd-proton and odd-neutron nuclei near the 28 closed shell are given in Figs. 1 and 2. For Cu^{63} the present work resembles the calculation of Thankappan and True,[8] who introduced a diagonal quadrupole matrix element as a parameter. One can see from Fig. 1 that the main new effect for Cu^{63}

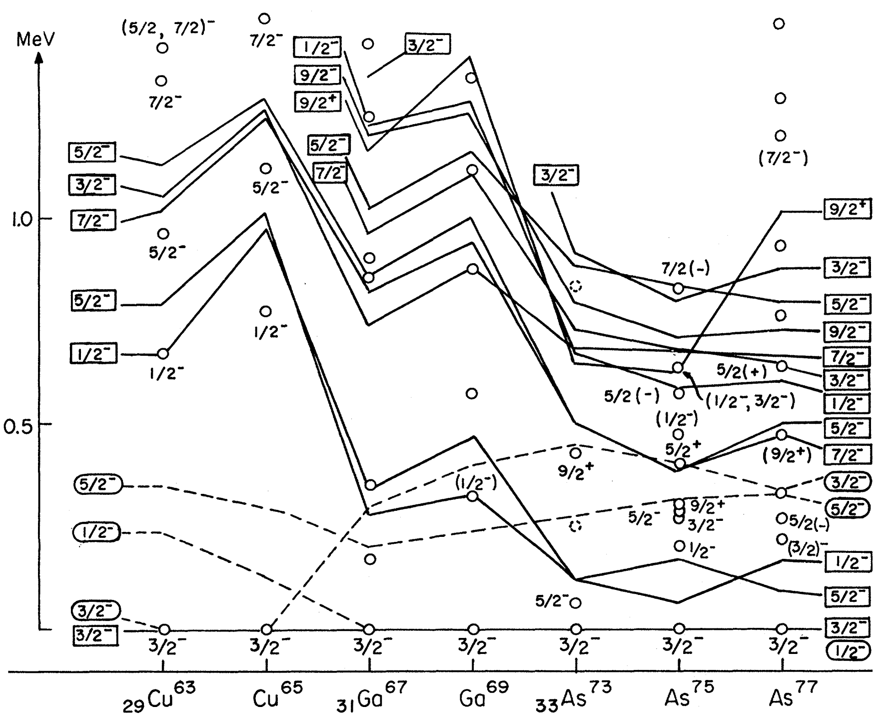

FIG. 1. Energy levels of the odd-mass Cu, Ga, and As isotopes (odd-proton). The calculated levels of the same spin are connected by straight lines labeled with spins in square boxes. The experimental [C. M. Lederer, J. M. Hollander, and I. Perlman, Table of Isotopes (John Wiley & Sons, Inc., New York, 1967), 6th ed. Nuclear Data Sheets, compiled by K. Way et al. (Printing and Publishing Office, National Academy of Sciences-National Research Council, Washington, D. C.). Nucl. Data 1, passim (1967)] values are indicated by open circles. The results of Ref. 2 are given by dashed lines labeled with spins encircled. Values of a_1/a_0, b_1/b_0, and c_1/c_0 (defined in the text) are −0.33, +0.30, and −0.33, respectively. Since the level density becomes quite high above 1 MeV, not all of the levels have been shown.

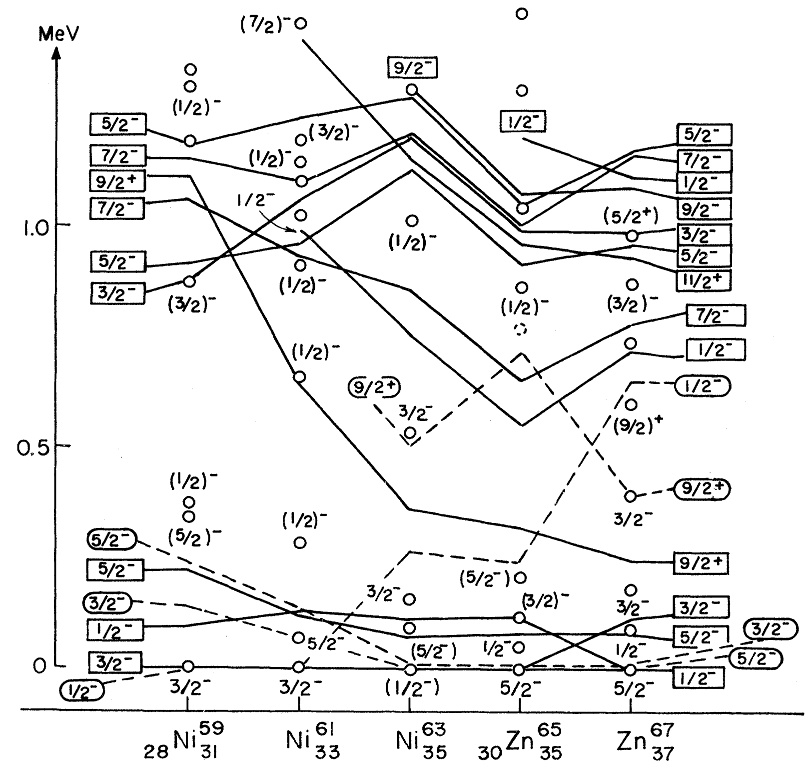

FIG. 2. Energy levels of the odd-mass Ni and Zn isotopes (odd-neutron). Parameters of the calculation, including the a, b, c ratios, are the same as those used for Fig. 1. Not all of the levels have been shown for the reason given in the caption of Fig. 1.

is the reduction of the strong depression of the $\frac{1}{2}^-$ state which follows from the off-diagonal coupling of the phonon and the quasiparticle. The spectrum thus tends to resemble somewhat the simple picture of the ground ($\frac{3}{2}^-$) state coupled to a quadrupole phonon. In fact, the states are much more complicated than this and involve various quasiparticles and phonons. As can be seen in Figs. 1 and 2, with the same choice of parameters, there are striking improvements in the theoretical spectra compared with experiment for both the odd-proton and odd-neutron isotopes in this region when compared with the calculation of Ref. 2. (Some of the differences are due to a better choice of single-particle levels based on recent one-particle transfer experiments. The results of Figs. 1 and 2 probably do not represent the best possible fits, but the purpose of this communication is to discuss the modified coupling scheme which is not expected to be affected significantly by minor changes in the single-particle levels.)

We have carried out systematic calculations for all of the vibrational nuclei. In the cases in which the static quadrupole moment plays an important role, we find that all of the isotopes in a region behave in a consistent manner. That is, the same choice of parameters gives the best results for both odd-proton and odd-neutron nuclei, sometimes predicting a change in sign of the 2^+ quadrupole moment as one goes through a region of isotopes, and the results are consistent with the known experimental measurements of static quadrupole moments. As one approaches the strongly coupled regions, the method breaks down although it is possible that a more elaborate calculation keeping more phonons in the expansion might take one somewhat farther. There are still some systematic discrepancies, such as the inability to account for the low-lying opposite-parity states of spin $j-1$ without three quasiparticle amplitudes,[2,9] and in some cases it is not easy to disentangle effects due to the changes in the underlying single-particle levels as a function of particle number. A major theoretical uncertainty which has not been dealt with here is the possible modification of the quasiparticle-phonon coupling due to the anharmonicity. Details and complete results will be communicated in a future publication.

We would like to thank Miss C. Berg and Miss B. Colby for their help with the numerical computations and Dr. J. Weneser for helpful discussions. We thank Dr. F. K. McGowan for making his compilation of static quadrupole moments available to us. One of us (KK) would like to thank Professor H. Feshbach for the warm hospitality of the Laboratory of Nuclear Science at MIT.

---

*This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(30-1) 2098.

†On leave from Case Western Reserve University.

[1]A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 26, No. 14 (1952); A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 27, No. 16 (1953); G. Scharff-Goldhaber and J. Weneser, Phys. Rev. 98, 212 (1955).

[2]L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. 35, 853 (1963) (to be called KS); references to earlier work on microscopic description of the modes referred to in the present paper are found in here.

[3]J. deBoer, R. G. Stokstad, G. D. Symons, and A. Winther, Phys. Rev. Letters 14, 564 (1965); see D. A. Bromley and J. Weneser, Comments Nucl. Particle Phys. 1, 75 (1967) for other references; F. K. McGowan and P. H. Stelson, private compilation.

[4]M. Baranger and K. Kumar, in Perspectives in Modern Physics, edited by R. E. Marshak (John Wiley & Sons, Inc., New York, 1966); Phys. Rev. Letters 17, 1146 (1966); Nucl. Phys. A92, 608 (1967); and to be published (to be called BK).

[5]Similar modifications have been suggested in phenomenological calculations by A. K. Kerman and C. M. Shakin, Phys. Letters 1, 151 (1962); and by T. Tamura and T. Udagawa, Phys. Rev. Letters 15, 765 (1965). Similar effects are also contained in the works of A. K. Kerman and A. Klein, Phys. Rev. 138, B1323 (1965); G. Do Dang et al., Phys. Rev. Letters 17, 709 (1966); B. Sorensen, Nucl. Phys. A97, 1 (1967); J. Sawicki, Nucl. Phys. 23, 285 (1961).

[6]F. K. McGowan, R. L. Robinson, P. H. Stelson, and J. L. C. Ford, Jr., Nucl. Phys. 66, 97 (1965).

[7]The calculations for the odd-mass nuclei are similar to those of Ref. 2 except for modification of the various matrix elements due to the anharmonicity. The wave functions will be made available for calculation of various nuclear properties.

[8]V. K. Thankappan and W. W. True, Phys. Rev. 137, B793 (1965).

[9]L. S. Kisslinger, Nucl. Phys. 78, 341 (1966).

BAND MIXING IN ^{152}Sm AND ^{154}Gd

L. L. Riedinger* and Noah R. Johnson
Chemistry Division, Oak Ridge National Laboratory,† Oak Ridge, Tennessee

and

J. H. Hamilton
Vanderbilt University,‡ Nashville, Tennessee
(Received 2 October 1967)

Alaga et al.[1] have used the collective model of Bohr and Mottelson[2] (BM) to predict the ratio of reduced transition probabilities from a member of an excited vibrational band to two different members of the ground-state band in deformed nuclei. This value is simply the ratio of the squares of the appropriate Clebsch-Gordan coefficients. However, it is known that these simple intensity predictions based on an adiabatic approximation break down. This results from the fact that the wave functions of the two bands become mixed and cause significant corrections to the matrix elements describing the transitions. The best-studied cases demonstrating this behavior involve the $E2$ transitions between the members of β- and γ-vibrational bands and the ground-state band in even-even deformed nuclei. Here the deviations between the experimental branching ratios and the simple predictions have been analyzed in terms of a band-mixing parameter Z_K, which is a measure of the rotation-vibration interaction. The subscript K refers to the K quantum number of the vibrational band considered. For these transitions from the β- and γ-vibrational bands, Bohr and Mottelson[3] have included the effects of band mixing to define the reduced $E2$ transition probability, $B(E2)$. This expression involves the unmixed transition probability, the mixing parameter, and spin-dependent terms.

We have experimentally determined the ra-