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NONLINEAR INSTABILITIES IN BEAM-PLASMA SYSTEMS

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The possibility of nonlinear interactions inducing instability in plasma devices containing available free energy has been examined with a number of model equations.[1,2] The difficulty in predicting instability with such models is that stability or instability is determined by finding the sign of a sum of large numbers with different signs and, hence, is critically subject to the model chosen. This method has led to a number of misleading results. In this Letter we derive a necessary and sufficient condition that the usual three-wave resonant scattering equations[3] possess unbounded solutions for arbitrarily small perturbations which does not rely on such models. For a number of interactions this condition is easy to evaluate, and in particular, it follows that the nonlinear resonant interactions of the usual beam-plasma modes in homogeneous plasmas generate large amplitude fluctuations even for wavelengths which are linearly stable.

The standard weak turbulence equations[1-3] describing the three-wave resonant interaction of longitudinal modes in a homogeneous plasma can be written as

$$\frac{\partial N_K^{\alpha}}{\partial t}=S_K^{\alpha}\sum_{\vec{k}',\vec{k}'',\beta,\gamma}|V_{kk'k''}^{\alpha\beta\gamma}|^2\delta(\omega_k^{\alpha}+\omega_{k'}^{\beta}+\omega_{k''}^{\gamma})\delta(\vec{k}+\vec{k}'+\vec{k}'')N_k^{\alpha}N_{k'}^{\beta}N_{k''}^{\gamma}\left[\frac{S_k^{\alpha}}{N_k^{\alpha}}+\frac{S_{k'}^{\beta}}{N_{k'}^{\beta}}+\frac{S_{k''}^{\gamma}}{N_{k''}^{\gamma}}\right].\quad(1)$$

Here $N_k^{\alpha}\equiv(8\pi)^{-1}|\partial\epsilon/\partial\omega_k^{\alpha}||E_{\vec{k}}^{\alpha}|^2$ is the number density of plasmons with wave number $\vec{k}$ and of the type "α," where "α" labels the different eigenfrequencies $\omega_{\vec{k}}^{\alpha}$ which exist for a given k; $|E_{\vec{k}}^{\alpha}|^2$ is the square of the electric field amplitude of the "α" mode; $|V_{kk'k''}^{\alpha\beta\gamma}|^2$ is a matrix element calculated from stationary plasma functions which is symmetric under interchange of the indices (k,α), (k',β), (k'',γ); $\epsilon(\vec{k},\omega)$ is the linear dielectric function which is zero for $\omega=\omega_k^{\alpha}$; and S_k^{α} is the sign of $(\partial\epsilon/\partial\omega)_{\omega=\omega_k^{\alpha}}$. To find a stability criterion one needs only the integral conditions which follow from (1):

$$\frac{\partial}{\partial t}\sum_{k,\alpha}\ln\left[\frac{N_k^{\alpha}(t)}{N_k^{\alpha}(0)}\right]$$

$$=\sum_{k,k',k''}|V_{kk'k''}^{\alpha\beta\gamma}|^2\delta(\vec{k}+\vec{k}'+\vec{k}'')\delta(\omega_k^{\alpha}+\omega_{k'}^{\beta}+\omega_{k''}^{\gamma})N_k^{\alpha}N_{k'}^{\beta}N_{k''}^{\gamma}\left[\frac{S_k^{\alpha}}{N_k^{\alpha}}+\frac{S_{k'}^{\beta}}{N_{k'}^{\beta}}+\frac{S_{k''}^{\gamma}}{N_{k''}^{\gamma}}\right]^2\geqslant 0\quad(2)$$

and for the time-independent functions b_k^{α},

$$\frac{\partial}{\partial t}\sum_{k,\alpha}b_k^{\alpha}N_k^{\alpha}=0\quad(3a)$$

for all time if

$$S_k^{\alpha} b_k^{\alpha} + S_{k'}^{\beta} b_{k'}^{\beta} + S_{k''}^{\gamma} b_{k''}^{\gamma} = 0 \qquad (3b)$$

for all $(\vec{k}, \alpha)$, $(\vec{k}', \beta)$, $(\vec{k}'', \gamma)$ which satisfy

$$\omega_k^{\alpha} + \omega_{k'}^{b} + \omega_{k''}^{\gamma} = 0,$$
$$\vec{k} + \vec{k}' + \vec{k}'' = 0. \qquad (3c)$$

Equation (2) states that the plasmon entropy is nondecreasing, and in addition (if $\vec{k}$ space is assumed of finite extent) it shows that if all N's remain bounded and are initially nonzero, then N_k^{α} remains nonzero except possibly for a set of measure zero in the limit of continuous $\vec{k}$. Equation (2) also proves that all N's remain bounded only if the system reaches a state such that

$$N_k^{\alpha} N_{k'}^{\beta} N_{k''}^{\gamma} \left[\frac{S_k^{\alpha}}{N_k^{\alpha}} + \frac{S_{k'}^{\beta}}{N_{k'}^{\beta}} + \frac{S_{k''}^{\gamma}}{N_{k''}^{\gamma}} \right] = 0 \qquad (4)$$

for all resonating triplets. By the previous statement one now concludes that all N's remain bounded only if the N's evolve to a state $N_k^{\alpha} = M_k^{\alpha}$, where M_k^{α} is defined as any positive (or zero on a set of measure zero) bounded solution of

$$\frac{S_k^{\alpha}}{M_k^{\alpha}} + \frac{S_{k'}^{\beta}}{M_{k'}^{\beta}} + \frac{S_{k''}^{\gamma}}{M_{k''}^{\gamma}} = 0 \qquad (5)$$

for all $(\vec{k}, \alpha)$, $(\vec{k}', \beta)$, $(\vec{k}'', \gamma)$ satisfying (3c). By (1) it is evident that if the M_k^{α} exist, they are equilibrium plasmon densities.

Equation (3a) implies the usual laws of conservation of energy and momentum if b_k^{α} is chosen equal to $s_k^{\alpha}\omega_k^{\alpha}$ and $s_k^{\alpha}\vec{k}$, respectively. These particular forms identically satisfy (3b) by (3c). More generally, if (3b) possesses any positive bounded solution, then (3a) restricts the N_k^{α} to be bounded for all time. However, the M's are the most general positive bounded solutions to (3b), and therefore the existence of the M_k^{α} implies that (1) has only bounded or stable solutions.

From this discussion it follows that a necessary and sufficient condition that (1) have only bounded solutions is that an M_k^{α} exists. It is apparent that a sufficient condition that (1) have unbounded solutions is that for any (k, α), (k', β), (k'', γ) satisfying (3c), the S_k^{α}, $S_{k'}^{\beta}$, $S_{k''}^{\gamma}$ have the same sign. On the other hand, a sufficient condition for (1) to have only stable solutions is that a frame of reference exist where all $S_k^{\alpha}\omega_k^{\alpha}$ or $k_i S_k^{\alpha}$ are of the same sign. Said another way, if a frame of reference exists in which all waves have positive (or negative) energy, or a particular component of momentum is positive (or negative) for all interacting waves, then the system is stable.

For homogeneous plasmas with cold beams, the dispersion relations are such that the simplified sufficiency conditions determine the stability properties of the system. For example, for electron beams in a plasma without a magnetic field,

$$\epsilon(\vec{k}, \omega) = 1 - \tfrac{1}{2}\omega_{pe}^2[(\omega - ku)^{-2} + (\omega + ku)^{-2}], \qquad (6)$$

where ω_{pe} is the total electron plasma frequency, and k is the wave-vector component along the direction of the beams. For $k^2 > k_c^2 = \omega_{pe}^2/u^2$, the modes of (6) are linearly stable and the methods of weak turbulence are applicable. For such wave vectors the four solutions of (6) are

$$\omega_k^1 = -\omega_k^4 = (k/|k|)[\tfrac{1}{2}\omega_{pe}^2 + (ku)^2 + \tfrac{1}{2}(\omega_{pe}^4 + 8k^2u^2\omega_{pe}^2)^{1/2}]^{1/2},$$
$$\omega_k^2 = -\omega_k^3 = (k/|k|)[\tfrac{1}{2}\omega_{pe}^2 + (ku)^2 - \tfrac{1}{2}(\omega_{pe}^4 + 8k^7u^7\omega_{pe}^2)^{1/2}]^{1/2}. \qquad (7a)$$

Solving the resonance condition (3c), it can be shown that many interactions can take place where all the s's are positive. For example, the interaction

$$\omega_k^1 + \omega_{k'}^3 + \omega_{k''}^2 = 0 \qquad (7b)$$

is possible for $k > k_c$, $k' > k_c$, $k'' < -k_c$ and here $S_k^1 > 0$, $S_{k'}^2 > 0$, $S_{k''}^3 > 0$. Hence (1) has unbounded solutions. If the system is shortened to exclude the longer wavelength modes, all resonances with all s's of the same sign cease for $k^2 \gtrsim 1.3 k_c^2$. For shorter systems only the modes ω^1 and ω^2 and, separately, ω^3 and ω^4 resonantly interact. As energy is separately conserved for each set and since, in the frame of reference moving with velocity $+u$, ω^1 and ω^2 are both positive energy while ω^3 and ω^4 are both negative energy in a frame of reference moving with velocity $-u$, it follows that the nonlinear set of equations has only bounded solutions for these short systems.

For a weak ion beam system in a strong magnetic field and for $\omega/\omega_{pe} \ll k_{\parallel}\lambda_{De}$,

$$\epsilon(\vec{k}, \omega) = 1 + \frac{1}{k^2\lambda_{De}^2} - \frac{k_{\parallel}^2\omega_{pi}^2}{k^2}\left[\frac{1}{\omega^2} + \frac{\eta}{(\omega - k_{\parallel}u)^2}\right]. \qquad (8a)$$

Here $\lambda_{De} = \bar{v}_e/\omega_{pe}$ is the electron Debye length, ω_{pi} is the ion plasma frequency, $\eta \ll 1$ is the ratio of the ion-beam density to the total density, $k_{\parallel}$ is the wave vector along the field, and $k^2 = k_{\parallel}^2 + k_{\perp}^2$. All modes of (8a) are linearly stable if $\bar{v}_e^2 \gg u^2 > (m/M)\bar{v}_e^2$. However, again many resonances induce nonlinear growth. For beam velocities in the linearly stable regime, the four solutions are

$$\omega_k^{\ 1} = k_{\parallel}u + k_{\parallel}\omega_{pi}\eta^{1/2}(k^2 + 1/\lambda_{De}^2 - \omega_{pi}^2/u^2)^{-1/2},$$

$$\omega_k^{\ 2} = k_{\parallel}u - k_{\parallel}\omega_{pi}\eta^{1/2}(k^2 + 1/\lambda_{De}^2 - \omega_{pi}^2/u^2)^{-1/2},$$

$$\omega_k^{\ 3} = -\omega_k^{\ 4} = k_{\parallel}(m/M)^{1/2}\bar{v}_e(1 + k^2\lambda_{De}^2)^{-1/2}. \qquad (8b)$$

The interaction $\omega_k^{\ 1} + \omega_{k'}^{\ 3} + \omega_{k''}^{\ 2} = 0$ is nonlinearly unstable but, as the system is shortened, stops resonating for $k_{\parallel} \gtrsim \omega_{pi}/u$. The coupling $\omega_k^{\ 3} + \omega_{k'}^{\ 4} + \omega_{k''}^{\ 2} = 0$ is also unstable but has a critical wave number of $k_{\parallel} \gtrsim 2\omega_{pi}/u$. For shorter systems only modes ω^1 and ω^2 interact, but they are both positive energy in a frame moving with velocity u, and hence in a short system the interactions of the low-phase-velocity mode are completely stable. However, in addition to these low-phase-velocity modes, (8b), this system has linearly stable electron plasma oscillations

$$\omega_k^{\ \pm} \cong \pm(k_{\parallel}\omega_{pe}/k)[1 + \tfrac{3}{2}(k\lambda_{De})^2 + \cdots].$$

A resonance of the form $\omega_k^{\ 3} + \omega_{k'}^{\ +} + \omega_{k''}^{\ 2} = 0$ can take place and is nonlinearly unstable. This interaction is only stopped when the $\omega_k^{\ +}$ mode becomes ill defined, i.e., $k\lambda_{De} \approx 1$; however, the matrix element $|V_{kk'k''}^{\ 3+2}|^2$ is extremely small for this resonance and associated nonlinear growth rates are initially slow.

Similar extensions of the instability regions by resonant scattering occur for other beam-plasma systems. It is apparent that these extensions are important for the understanding of basic experiments and in the theory of the structure of shock waves in collisionless plasmas.

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ELECTROLUMINESCENCE OF KI AT 77°K*

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The majority of electroluminescence studies undertaken have been on semiconductors.[1] The theories put forth to explain the excitation of this phenomenon involve (1) the injection of charges into the crystal by the electrodes,[2] (2) direct field ionization of impurities[3] or of the valence band itself,[4] and (3) the acceleration of charged carriers to optical energies.[5,6] We make use of the first and last of these mechanisms to explain the intrinsic electroluminescence of pure KI at 77°K.

A voltage of 2500 V rms at 500 cps was applied across a crystal 0.5 mm thick using one Ag and one In electrode. The emitted light was observed perpendicular to the field at 1000 cps by using a phase-sensitive amplifier. The signal was found to be 90° out of phase with the reference frequency. The resulting spec-