in anthracene has been reported, through pumping with unpolarized light[11]; this comes about because of selective de-excitation of higher bands to the magnetic states of an excited triplet, and is a different mechanism from that considered here.

(E) Liquids.—The basic ideas of the above ENPOPS schemes can be readily extended to liquids containing paramagnetic ions or other magnetic species, provided that U_1 and U_2 can be made sufficiently different (this usually requires large spin-orbit coupling), and that the oscillator strength and available light intensity combine to give $U \sim T_{1e}^{-1}$, required for optical saturation. The nuclei of interest are those in the abundant diamagnetic solvent molecules, which have a rapidly fluctuating interaction with the ion, either of the hfs form $\vec{I} \cdot A \cdot \vec{S}$ or of dipole-dipole form. The first case is similar to (A) above except that the hfs is averaged out. However, $w_2 \gg w_3$ if the hfs fluctuation is nearly isotropic, and we conclude that if nuclear-spin memory exists, then pumping the liquid with circularly polarized light will yield the nuclear polarization of Eq. (3). For the dipolar case in liquids[8] $w_2:w_3:w_4 = 2:12:3$ and one should find a reversed nuclear polarization.

It can also be shown that ENPOPS should apply to magnetically concentrated substances which display an Overhauser effect.

Experiments to test these various cases are underway at the University of California, Berkeley, California. It is a pleasure to acknowledge a stimulating discussion with Professor P. L. Scott, leading to Case (B) above.

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THICKNESS OF A ROTATING LIQUID-HELIUM FILM*

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The thickness of a rotating liquid-helium film has been measured at various angular velocities and temperatures. The results are consistent with hydrodynamical calculations assuming that the superfluid component remains at rest and that the normal component alone rotates. The failure to induce rotation in the superfluid component is interpreted as evidence that vortex lines with their axes perpendicular to the film are difficult to create.

Rotation experiments fall into several classes. If a bulk sample of helium is used, the critical velocity is very small and its effects are difficult to observe. Experiments in which flow takes place in packed powders or their equivalent obtain large critical velocities but have complex geometries. In such experiments rotation probably takes place without the presence of Onsager-Feynman vortex lines. A rotating helium film has the advantage that it combines the simple geometry of bulk-liquid experiments with critical velocities of the order of magnitude of 50 cm/sec.[1]

The interesting question is whether or not the film rotates with the surface on which it is formed. If the film is brought into motion, then the surface of the film should curve for the same reason that a classical liquid in a rotating bucket has a parabolic shape. For the film, however, the change in the surface is of the order of Angstroms instead of centimeters because the force field is not gravity but the van der Waals forces between the helium atoms and the wall. We have used a variation of Jackson's[2] optical technique to measure the change in film thickness caused by rotation. The film thickness is measured near the center of a rotating cylinder where the effect is expected to be a maximum. The rotating cylinder, which is partially immersed in the helium bath, forms the rotor of a superconducting motor similar to that developed by Schoch.[3] The use of a superconducting motor eliminates the heat which would be produced by bearing friction and motor control currents. The surface on which the film thickness is measured is a gold film which has been evaporated onto Pyrex glass.

The expected magnitude of the change in film thickness can be calculated by solving the two-fluid hydrodynamical equations using methods similar to those of Kontorovich[4] and Meservey.[5] In solving the hydrodynamical equations we have assumed a particular class of velocity patterns for the superfluid and normal components of the film:

$$v_n = R\omega, \quad v_s = R\omega'.$$

This includes a motionless superfluid ($\omega' = 0$) and a classically rotating superfluid ($\omega = \omega'$) as a special cases. The solution is

$$gH - \frac{\alpha}{d^N} - \frac{\rho_s}{\rho}\frac{R^2\omega'^2}{2} - \frac{\rho_n}{\rho}\frac{R^2\omega^2}{2} = \text{const}, \qquad (1)$$

where d = film thickness, H = height of the film above bulk liquid, and α = a constant. The term α/d^N represents all the film-thickness–dependent energy terms. Its largest contribution comes from van der Waals forces. When $\omega = \omega' = 0$, Eq. (1) becomes the profile equation of the static film thickness. The van der Waals term α/d^N can then be obtained from measurements of the static film thickness. The constant in Eq. (1) is the chemical potential and may be evaluated by considering the point at which the film joins the bulk-liquid level. At this point $H = 0$, $d = \infty$, and $R = R_c$ = the radius of the cylinder. The geometry of the rotating cylinder and film are illustrated in Fig. 1.

The behavior of the superfluid and normal components can be separated experimentally by varying the temperature. At low temperatures most of the film is superfluid, while near the λ point most of the film is normal. The two parameters that affect the results are the bath height (H) and the temperature (T). By holding the bath height constant and varying the temperature we were able to study the relative behavior of the two components. The results are given in Fig. 2. Because we expect the rotating film to be thinner, we have plotted decreases in film thickness in the positive direction. The effect of rotation on the film thickness is a monotonically increasing function of the temperature. At the lowest temperature measured, no change larger than the experimental error occurs. The fact that the results are temperature dependent shows that the superfluid and normal components do not behave in the same way. The null effect at the

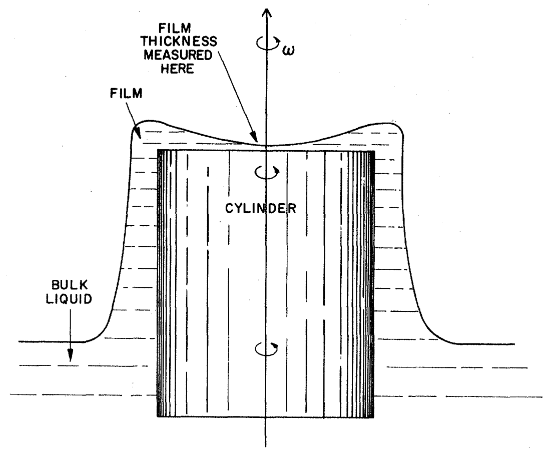

FIG. 1. Curvature of a rotating film.

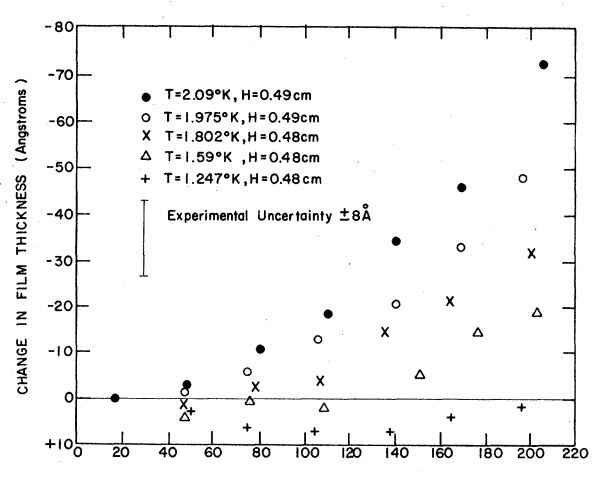

FIG. 2. Change in film thickness versus angular velocity at constant bath height $H = 0.49$ cm for various temperatures.

lowest temperature is consistent with the superfluid remaining at rest ($v_s = 0$). If the superfluid component remains at rest and the normal component rotates classically, Eq. (1) can be rewritten as

$$d = 323 \times 10^{-6}(H + \Delta H)^{-1/4}\ \text{cm}, \qquad (2)$$

where

$$\Delta H = \frac{\rho_n}{\rho}\frac{\omega^2}{2g}(R_c^2 - R^2). \qquad (3)$$

We have used Hemming's static film-thickness measurements on gold[6] to evaluate the van der Waals term α/d^N. For small changes in ΔH, Eq. (2) can be expanded to give the change in film thickness,

$$\frac{\Delta d}{d} = -\frac{1}{4}\frac{\rho_n}{\rho}\frac{\omega^2}{2gH}(R_c^2 - R^2). \qquad (4)$$

The temperature dependence should be approximately that of ρ_n/ρ. This conclusion is independent of the $\frac{1}{4}$ power law because ρ_n/ρ appears only as a multiplicative constant. The change in film thickness at 200 rpm versus ρ_n/ρ is plotted in Fig. 3. The temperature dependence of our results is the same as ρ_n/ρ within the accuracy of the experiment. All of our results are consistent with the superfluid remaining at rest ($v_s = 0$) and the normal component rotating ($v_n = R\omega$).

The result that the normal component rotates is, of course, quite expected. The superfluid results are more surprising. Rotation apparently has no effect on the superfluid despite the fact that the peripheral velocity of the cylinder exceeds the film-flow critical velocity

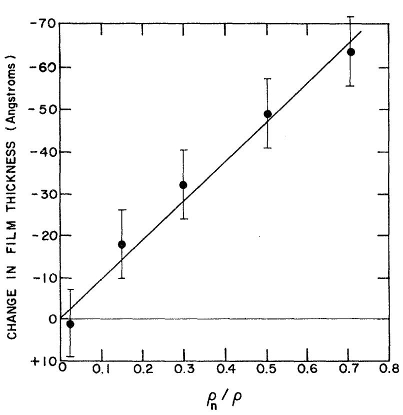

FIG. 3. Change in film thickness versus ρ_n/ρ at constant bath height $H = 0.49$ cm and constant angular velocity $\omega = 200$ rpm.

by more than a factor 2. Also, the film on the cylinder is connected to the bulk liquid so that the critical velocity is always exceeded at a low height, where the film is thick and the critical velocity is small. We believe that the explanation of this result is contained in some remarks by Anderson.[7] He points out that in the critical-velocity expression ($v_c \sim \hbar/mD$), the relevant dimension D for vortex lines with their axes perpendicular to the film is the distance to the nearest wall measured perpendicular to the vortex core, rather than the film thickness. The critical velocity for this type of vortex line should be about 10^5 smaller than that for other types of vortices. Large critical velocities are observed experimentally; so he concludes that this type of vortex line does not exist in the film. Only vortex lines of this type can bring helium into rotation. Other types of vortices will be created when the observed critical velocity is exceeded, but these cannot simulate classical rotation.

Another process which might be important in preventing the formation of the equilibrium array of vortex lines needed for rotation is pinning of vortex lines. Vortex lines are well known to have a tendency to stick to walls in bulk helium. The core of a vortex line is still small compared with the film thickness; so we expect the pinning force in the film to be nearly the same as in bulk liquid. The disturbing force which acts to free pinned vortices is due to the relative motion of the rest of the liquid and is proportional to the length of the vortex line on which it acts. Since a vortex line in bulk helium is about 10^5 times longer than a vortex line in the film, it should be much more difficult to free a pinned vortex in the film. We might therefore assume that perpendicular vortex lines are readily formed whereever the velocity of the rotating disk exceeds the critical velocity, but that these lines are firmly pinned and are not able to distribute themselved throughout the film. Although this situation would not lead to solid-body rotation, it can be shown that it would produce changes in film thickness that could have been readily detected at the lowest temperature in our experiment. We conclude that not even pinned perpendicular vortex lines were created in our experiment.

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NONLINEAR INSTABILITIES IN BEAM-PLASMA SYSTEMS

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The possibility of nonlinear interactions inducing instability in plasma devices containing available free energy has been examined with a number of model equations.[1,2] The difficulty in predicting instability with such models is that stability or instability is determined by finding the sign of a sum of large numbers with different signs and, hence, is critically subject to the model chosen. This method has led to a number of misleading results. In this Letter we derive a necessary and sufficient condition that the usual three-wave resonant scattering equations[3] possess unbounded solutions for arbitrarily small perturbations which does not rely on such models. For a number of interactions this condition is easy to evaluate, and in particular, it follows that the nonlinear resonant interactions of the usual beam-plasma modes in homogeneous plasmas generate large amplitude fluctuations even for wavelengths which are linearly stable.

The standard weak turbulence equations[1-3] describing the three-wave resonant interaction of longitudinal modes in a homogeneous plasma can be written as

$$\frac{\partial N_K{}^\alpha}{\partial t}=S_K{}^\alpha\sum_{\vec{k}',\vec{k}'',\beta,\gamma}|V_{kk'k''}{}^{\alpha\beta\gamma}|^2\delta(\omega_k{}^\alpha+\omega_{k'}{}^\beta+\omega_{k''}{}^\gamma)\delta(\vec{k}+\vec{k}'+\vec{k}'')N_k{}^\alpha N_{k'}{}^\beta N_{k''}{}^\gamma\left[\frac{S_k{}^\alpha}{N_k{}^\alpha}+\frac{S_{k'}{}^\beta}{N_{k'}{}^\beta}+\frac{S_{k''}{}^\gamma}{N_{k''}{}^\gamma}\right]. \quad (1)$$

Here $N_k{}^\alpha\equiv(8\pi)^{-1}|\partial\epsilon/\partial\omega_k{}^\alpha||E_{\vec{k}}{}^\alpha|^2$ is the number density of plasmons with wave number $\vec{k}$ and of the type "α," where "α" labels the different eigenfrequencies $\omega_{\vec{k}}{}^\alpha$ which exist for a given k; $|E_{\vec{k}}{}^\alpha|^2$ is the square of the electric field amplitude of the "α" mode; $|V_{kk'k''}{}^{\alpha\beta\gamma}|^2$ is a matrix element calculated from stationary plasma functions which is symmetric under interchange of the indices (k,α), (k',β), (k'',γ); $\epsilon(\vec{k},\omega)$ is the linear dielectric function which is zero for $\omega=\omega_k{}^\alpha$; and $S_k{}^\alpha$ is the sign of $(\partial\epsilon/\partial\omega)_{\omega=\omega_k{}^\alpha}$. To find a stability criterion one needs only the integral conditions which follow from (1):

$$\frac{\partial}{\partial t}\sum_{k,\alpha}\ln\left[\frac{N_k{}^\alpha(t)}{N_k{}^\alpha(0)}\right]$$

$$=\sum_{k,k',k''}|V_{kk'k''}{}^{\alpha\beta\gamma}|^2\delta(\vec{k}+\vec{k}'+\vec{k}'')\delta(\omega_k{}^\alpha+\omega_{k'}{}^\beta+\omega_{k''}{}^\gamma)N_k{}^\alpha N_{k'}{}^\beta N_{k''}{}^\gamma\left[\frac{S_k{}^\alpha}{N_k{}^\alpha}+\frac{S_{k'}{}^\beta}{N_{k'}{}^\beta}+\frac{S_{k''}{}^\gamma}{N_{k''}{}^\gamma}\right]^2\geq 0 \quad (2)$$

and for the time-independent functions $b_k{}^\alpha$,

$$\frac{\partial}{\partial t}\sum_{k,\alpha}b_k{}^\alpha N_k{}^\alpha=0 \quad (3a)$$