

<sup>1</sup>For a detailed account, see P. M. Joseph, thesis, Harvard University, June, 1967 (unpublished).

<sup>2</sup>W. A. Blanpied, J. S. Greenberg, V. W. Hughes, P. Kitching, D. C. Lu, and R. C. Minehart, Phys. Rev. Letters 14, 741 (1965).

<sup>3</sup>R. L. Anderson, E. Gabathuler, D. Jones, B. D. McDaniel, and A. M. Sadoff, Phys. Rev. Letters 9, 131 (1962).

<sup>4</sup>This is borne out by measurements at somewhat lower energies: V. B. Elings, K. J. Cohen, D. A. Garelick, S. Homma, R. A. Lewis, W. Lobar, P. D. Luckey, and L. S. Osborne, Phys. Rev. 156, 1433 (1967); G. Buschhorn, J. Carroll, R. D. Eandi, P. Heide, R. Hübner, W. Kern, U. Kötz, P. Schmüser, and H. J. Skronn, Phys. Rev. Letters 17, 1027 (1966), and 18, 571 (1967); Z. Bar-Yam, J. de Pagter, M. M. Hoenig, W. Kern, D. Luckey, and L. S. Osborne, Phys. Rev. Letters 19, 40 (1967).

<sup>5</sup>S. D. Drell, Phys. Rev. Letters 5, 278 (1960); Rev. Mod. Phys. 33, 458 (1961). We modified Drell's formula to include exact recoil kinematics and the proper be-

havior at the threshold  $M(X) \rightarrow M_p + m_\pi$ .

<sup>6</sup>J. P. Dowd, D. O. Caldwell, K. Heinloth, and T. R. Sherwood, Phys. Rev. Letters 18, 414 (1967). This group has also analyzed pion photoproduction in terms of Reggeized pion exchange. They deduced the Regge trajectory by an analysis of the  $t$  dependence at fixed  $s$ .

<sup>7</sup>G. Zweig, Nuovo Cimento 32, 659 (1964).

<sup>8</sup>G. Kramer and P. Stichel, Z. Physik 178, 519 (1964).

<sup>9</sup>This form is a slight modification of the results given in this reference and Ref. 10. It differs from the expression used in Ref. 8, in that we approximate  $1/\Gamma(1+\alpha) \approx 1$ . This extra factor introduces a zero at  $\alpha = 1$ . However, because our data show no sign of a dip at finite  $t$ , and also because the actual pion trajectory may never reach  $\alpha = -1$  if the conjecture of Pignotti is correct, we choose to ignore this factor.

<sup>10</sup>M. Kawaguchi and M. J. Moravcsik, Phys. Rev. 107, 563 (1957). The answer given by these authors in Eq. (3.6) is correct, although there is a factor of  $\frac{1}{4}$  missing from Eq. (3.3).

#### PHASE-SHIFT ANALYSIS OF HIGH-PRECISION $(p,p)$ POLARIZATION DATA AT 19.7 MeV\*

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A recent high-precision  $(p,p)$  polarization measurement at 19.7 MeV from Berkeley has been phase-shift analyzed. It is not consistent with other  $(p,p)$  measurements at nearby energies.

Slobodrian and co-workers at Berkeley have recently published<sup>1</sup> the results of new high-precision measurements of the  $(p,p)$  polarization asymmetry at 9.6, 15.6, and 19.7 MeV. These measurements are of interest for two reasons: (1) They, together with recent<sup>2</sup> and forthcoming experiments from Saclay, constitute the first of a new generation of very precise low-energy nucleon-nucleon measurements which feature the use of polarized beams and/or polarized targets. (2) The new Berkeley data are in flat contradiction with other types of  $(p,p)$  data at neighboring energies.

As background information for the present paper, we should comment that we have just finished an analysis of all of the available  $(p,p)$  data from 1 to 400 MeV.<sup>3</sup> In the process of this work we have selected a self-consistent set of some 839 data points, including nine different types of experiments, that span the en-

ergy region. The only solution type that gives a fit to these data is the Type-I solution of Stapp<sup>4</sup> and of MacGregor.<sup>5</sup> When, at the conclusion of this analysis, we became aware of the new Berkeley data,<sup>1</sup> it was of interest to see how they would fit in. As we describe below, they do not fit in.

If one considers only  $(p,p)$  differential cross-section data, then Clementel and Villi<sup>6</sup> showed many years ago that four different sets of  $P$  waves combined with one  $S$ - $D$  wave combination all give equivalent precision fits to the differential cross section. These solutions were catalogued in detail by MacGregor.<sup>5</sup> Of these solutions, Types I and III predict a positive polarization, and Types II and IV predict a negative polarization.<sup>5</sup> The Type-I solution is the low-energy continuation of the Stapp No. 1 solution.<sup>4</sup>

The recent measurements of Slobodrian<sup>1</sup> give

a polarization of about  $-1.7 \pm 0.3\%$  at 19.7 MeV and  $45^\circ$  scattering angle. These workers carried out a simple phase-shift analysis of these data combined with nearby differential cross-section data<sup>1</sup> and found, not surprisingly, a Type-II solution. We repeated their analysis and confirmed the Type-II solution, and we also found a Type-IV solution giving an equivalent fit to the data. The Type-I solution predicts a small polarization at 20 MeV (roughly 0.3%), and one that furthermore goes through zero at  $45^\circ$ , right where the Slobodrian data<sup>1</sup> have their maximum (absolute) values. Thus it is apparent that no Type-I solution will fit these data.

In order to obtain a quantitative evaluation of this situation, we selected the  $(p, p)$  data in the energy range 18.2 to 30 MeV from our self-consistent data table<sup>3</sup> and combined it with the Slobodrian polarization data at 19.7 MeV.<sup>1</sup> (The Slobodrian data at 9.6 and 15.6 MeV are essentially consistent with a zero value for the polarization and thus present no problem.) We made about 30 different computer runs in analyzing these data. The six runs shown in Table I summarize the results that we obtained.

Solution 1 of Table I is the energy-dependent 30-parameter solution that gives a good fit to the 839 data points from 9 to 400 MeV.<sup>3</sup> When the Slobodrian data<sup>1</sup> were added to the search problem, the code simply ignored them in the search. (In this problem the entire 9- to 400-MeV data set was searched. In Table I we show only the data in the region of interest.) Since our energy-dependent forms<sup>3</sup> are too "stiff" to produce the local wiggles required to fit the Slobodrian data, this does not constitute a definite test. The energy-dependent analysis is clearly form-limited over any small region of energies. Thus the remainder of our studies included only energy-independent analyses (solutions 2-6 of Table I) in which the phases were free to adopt any values as dictated by the  $\chi^2$  minimization.

Solution 2 of Table I is the standard Type-I solution obtained by fitting to the data listed in Table I. All of the data are fit reasonably accurately with the exception of the Slobodrian data. (If the Slobodrian data are deleted, a precision fit can be made to the 19.2-MeV  $C_{NN}$  datum point.)

Solution 3 is the Type-II solution that we obtain by fitting  $S$ ,  $P$ , and  $D$  waves to the data

shown. Since the  $A_{XX}$  and  $C_{NN}$  measurements<sup>2</sup> were made only at  $90^\circ$ , they are not sensitive to the (antisymmetric)  $P$  waves, and solution 3 is essentially just a fit to  $\sigma$  and  $P$  data. Thus we get a good Type-II solution. This is the solution found by Slobodrian and coworkers.<sup>1</sup>

Solution 4 is obtained by adding the data above 25.63 MeV to the problem, as shown, and carrying out a search. The Type-II solution is in violent contradiction with the  $R$  and  $A$  measurements. When higher phases are added in the one-pion-exchange approximation, then solution 4 goes over into solution 5, which is the Type-I solution again. Adding  $F$  waves as free parameters also causes the same effect when the  $R$  and  $A$  data are included.

Solution 6 of Table I is the Type-IV solution. While it can give a good fit to the  $\sigma$  and  $P$  data, it also is in violent disagreement with the  $R$  and  $A$  data.

There is one question that arises in this kind of analysis. If we include data from 18 to 30 MeV, the analysis cannot really be classified as a "single-energy" analysis. Certainly the changes in the phase shifts are appreciable over this wide an energy span. We handle this by assigning a (fixed) energy derivative to the phase shift while allowing the phase shift to vary freely. The energy derivatives are conventionally taken from our energy-dependent analyses. But the choices for the energy derivatives can affect the analysis. Accordingly, for the present study, we tried a number of different choices for the phase-shift slopes. Some of these are indicated in Table I. The slopes labeled NORMAL are from the standard Type-I energy-dependent analysis. Solutions 2 and 5 of Table I have different slopes for  ${}^3P_1$  and  ${}^3P_2$ , and the effect can be seen to be very small. We also tried slopes chosen to favor the Type-II or Type-IV solution, and we freed as many as nine phases at one time. However none of our efforts altered the results shown in Table I in any appreciable manner. We conclude that the energy spread of the data in Table I has been adequately dealt with.

Our conclusions from all of this work are the following: (1) Type-II and Type-IV solutions exist that give a good fit to differential cross-section data and to the Slobodrian polarization data<sup>1</sup>; however, these solutions are in total disagreement (by 9 or 10 standard deviations) with  $R$  and  $A$  data points at 27.6 MeV. They are, as we know from other analyses,<sup>7</sup>

Table I. Phase-shift analysis of  $(p,p)$  data near 19.7 MeV. The solutions are described in the text. The data breakdown shows the contribution to the least-squares sum  $\chi^2$  from each type of data included in the analysis.  $M$  is the  $\chi^2$  sum divided by the number of data points.

Solution	1	2	3	4	5	6
Phases-Type	I - 30A	I	II	II	II+I	IV
$^1S_0$	50.9	51.5	52.0	52.3	51.2	51.2
$^1D_2$	0.5	0.8	0.7	0.7	0.7	0.5
$^3P_0$	7.1	5.4	4.5	7.5	8.6	-6.7
$^3P_1$	-4.3	-2.2	4.7	3.2	-4.7	2.2
$^3P_2$	2.0	2.3	-3.0	-2.7	2.3	-0.6
$\epsilon_2$	-0.6	-1.0	0	0	(-0.7)	0.5
$^3F_2$	0.1	-0.2	0	0	(0.1)	1.4
Slopes	NORMAL	NORMAL	S, D, P <sub>0</sub> P <sub>1</sub> , P <sub>2</sub> ZERO	S, D, P <sub>0</sub> P <sub>1</sub> , P <sub>2</sub> ZERO	S, D, P <sub>0</sub> P <sub>1</sub> , P <sub>2</sub> ZERO	S NORMAL D ZERO P(= NORMAL)
Data Searched (MeV)	49	49	32	41	41	49
Type				$\chi^2$ Per Data Set		
18.2	8 $\sigma^a$	5.4	8.1			12.1
19.2	1C <sub>NN</sub> <sup>b</sup>	0.3	5.9	5.3	3.0	8.8
19.2	1A <sub>XX</sub> <sup>b</sup>	0.1	0.1	1.2	11.5	1.5
19.7	5P <sup>c</sup>	37.3	40.2	12.8	10.2	40.3
23.5	1C <sub>NN</sub> <sup>b</sup>	0.0	1.3	1.8	3.5	0.0
23.5	1A <sub>XX</sub> <sup>b</sup>	0.5	0.1	0.2	2.2	0.0
25.63	23 $\sigma^d$	13.6	10.1	5.9	6.4	9.1
26.5	1C <sub>NN</sub> <sup>b</sup>	0.1	0.2		39.5	8.0
26.5	1A <sub>XX</sub> <sup>b</sup>	2.8	0.1		1.8	0.7
27.0	1C <sub>NN</sub> <sup>e</sup>	0.1	0.0		2.8	0.9
27.6	3A <sup>f</sup>	4.3	3.8		72.1	6.4
27.6	2R <sup>f</sup>	1.1	0.9		110.4	0.2
28.16	1 $\sigma^g$	1.4	5.8		0.1	0.1
30.0	1P <sup>h</sup>	5.2	3.2		8.2	0.4
M <sub>TOTAL</sub>	1.5	1.6	0.9	6.6	1.9	5.9
M <sub>19.7</sub>	7.5	8.0	2.6	2.0	8.1	2.7
M <sub>TOTAL-19.7</sub>	0.8	0.9	0.5	7.3	1.0	6.2

<sup>a</sup>J. L. Yntema and M. G. White, Phys. Rev. 95, 1226 (1954).

<sup>b</sup>Ref. 2.

<sup>c</sup>Ref. 1.

<sup>d</sup>J. H. Jeong, L. H. Johnston, D. E. Young, and C. N. Waddell, Phys. Rev. 118, 1080 (1960).

<sup>e</sup>N. Jarmie, J. E. Brolley, H. Kruse, H. C. Bryant, and R. Smythe, Phys. Rev. 155, 1438 (1967).

<sup>f</sup>A. Ashmore, B. W. Davies, M. Devine, S. J. Hoey, J. Litt, and M. E. Shepherd, Nucl. Phys. 73, 256 (1965).

<sup>g</sup>L. H. Johnston and Y. S. Tsai, Phys. Rev. 115, 1293 (1959).

<sup>h</sup>C. J. Batty, R. S. Gilmore, and G. H. Stafford, Nucl. Phys. 45, 481 (1963).

also in disagreement with the complete  $(p,p)$  data sets at 142 and 210 MeV. (2) The Type-I solution, which is required to fit our 839-

piece data set,<sup>3</sup> cannot be made to fit the new Berkeley data.<sup>1</sup> In particular, two of the Slobodrian data points at 19.7 MeV are 4 and 5

standard deviations away from the Type-I prediction. This result holds true for all of our attempts to modify the Type-I solution as described above. The statistical chance of a 5-standard-deviation fluctuation is 1 in  $2 \times 10^6$ . The value for  $M$ , listed at the bottom of Table I, should be of order unity for a good fit to the data.

A phase-shift analysis is, by itself, devoid of predictive powers. It cannot be used to ascertain that experiment  $A$  is correct and experiment  $B$  is incorrect. But it can be used to say that  $A$  is inconsistent with  $B$ . We have gone through the rather painful discussion of phase-shift analyses given above to convince the reader that there is no way in which a phase shift analysis can be made to reconcile the new Berkeley polarization data<sup>1</sup> with all of the other nearby ( $p, p$ ) data listed in Table I. Of course, it is possible that the Slobodrian data are correct, and that the few nearby points that contradict these data are themselves incorrect. However, if this is true, then the character of the phase shift solution must change drastically in going from 20 to 140 MeV. And there is no other supporting evidence to suggest the need for this change. Also, the dominance of either a Type-II or Type-IV solution at 20 MeV would pose great problems for the potential-modelist, since it is widely accepted that the

nucleon-nucleon potential is dominated in its outer parts by the one-pion-exchange mechanism, and one-pion exchange gives a tensor splitting of the  $P$  waves at low energy that is characteristic of the Type-I solution.

Since the technique used by Slobodrian and coworkers,<sup>1</sup> namely to bombard hydrogen with alpha particles and produce a 100% polarized low-energy proton beam,<sup>8</sup> appears to be a decisive improvement in experimental techniques, and since the Berkeley group have made very convincing checks on sources of systematic and statistical error, it is of great importance to have the discrepancy described here resolved as soon as possible.

<sup>1</sup>R. J. Slobodrian, J. S. C. McKee, H. Bichsel, and W. F. Tivol, *Phys. Rev. Letters* **19**, 595 (1967).

<sup>2</sup>P. Catillon, D. Garreta, and M. Chapellier, *Nucl. Phys.* **B2**, 93 (1967).

<sup>3</sup>M. H. MacGregor, R. A. Arndt, and R. M. Wright, to be published.

<sup>4</sup>H. P. Stapp, T. Ypsilantis, and N. Metropolis, *Phys. Rev.* **105**, 302 (1957).

<sup>5</sup>M. H. MacGregor, *Phys. Rev.* **113**, 1559 (1959).

<sup>6</sup>E. Clementel and C. Villi, *Nuovo Cimento* **2**, 352, 1162 (1955).

<sup>7</sup>See, for example, M. H. MacGregor, R. A. Arndt, and A. A. DuBow, *Phys. Rev.* **135**, B628 (1964).

<sup>8</sup>This technique was used first by L. Rosen and J. E. Brolley, *Phys. Rev.* **107**, 1454 (1957).

#### $\Lambda K$ ENHANCEMENT AT 1.7 GeV PRODUCED IN $\pi p \rightarrow \Lambda K \pi$ INTERACTIONS AT 6 GeV/c\*

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We have observed a marked  $\Lambda K^+$  enhancement centered at 1.7 GeV with a width of 0.17 GeV in the interaction  $\pi^- p \rightarrow \Lambda K^+ \pi^-$  at 6 GeV/c. This effect does not show up strongly in the similar interactions  $\pi^\pm p \rightarrow \Lambda K^+, \pi^\pm, 0$ . We discuss the production mechanism in terms of  $s$  (direct) and  $t$  (crossed) channels, and examine the question of whether this enhancement is associated with the  $N_{1/2}^*(1688)$ .

In an investigation of  $\pi^\pm p$  interactions at 6 GeV/c from exposures consisting of 230 000  $\pi^-$  pictures and 80 000  $\pi^+$  pictures in the Brookhaven National Laboratory (BNL) 80-inch liquid-hydrogen bubble chamber, we report the observation of a  $\Lambda K^+$  enhancement centered at 1.7 GeV in the reaction  $\pi^- p \rightarrow \Lambda K^+ \pi^-$ . This reaction has been extensively investigated from 2 to 4 GeV/c and shows no evidence for this enhancement.<sup>1</sup> Relevant information concern-

ing this investigation in three-body final states is shown in Table I.

The Dalitz plot and  $\Lambda K$  mass squared projection for each of the three final states are shown in Figs. 1(a) through 1(c). The marked  $\Lambda K^+$  enhancement centered at 1.7 GeV is evident in Fig. 1(a), and the shaded area represents the events which do not contain either  $K^*(890)$  or  $K^*(1420)$ . However, corresponding ( $\Lambda K$ ) mass plots in Figs. 1(b) and 1(c) do not show