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THEORY OF SELF-TRAPPED FILAMENTS OF LIGHT

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We present a calculation modeled after the theory of phase transitions to explain the observations on self-trapped filaments of laser light in liquids. The resulting state is shown to be similar to the Abrikosov vortex state in superconductors.

Self-focusing and self-trapping of intense light beams have recently become one of the most important and interesting subjects in nonlinear optics. While self-focusing as a result of intensity-dependent changes of the refractive index is now more or less understood both theoretically[1] and experimentally,[2] the formation of intense filaments arising from self-trapping[3] still remains a mystery. It is believed that the filament formation is also a consequence of the change of refractive index with intensity.[3] However, experimental results indicate that the change in the refractive index of a filament, calculated from the observed intensity in the filament under Kerr effect assumptions, is not sufficient to account for the observed filament size.[4] In addition, a number of other experimental facts have received no satisfactory explanation.

In this paper, we present a calculation which enables us to explain most of the experimental observations on self-trapped filaments. The calculation is based on the assumption of a field-induced phase transition in the medium and is similar to that of vortex formation in Type-II superconductors. Preliminary results of the calculation yield the following predictions: (1) The splitting of an intense beam into small-scale circular filaments is energetically favorable; (2) aside from fluctuations, all filaments have the same size and the same power density; (3) the filament size and the power contained in each filament are characteristics of the medium independent of the input beam intensity. In the calculation, we will assume that a critical field exists and that aside from the intensity-dependent dielectric constant $\epsilon(\omega)=\epsilon_0(\omega)+\epsilon_2(\omega)|E(\omega)|^2$, to produce this field, other nonlinear optical processes can be neglected before the filaments are formed.

Grob and Wagner[5] have also suggested the analog of vortex lines in superconductors to the filaments in this problem. However, they assume that the filament formation is a result of coupling between light fields and density fluctuations in the medium. Their results are essentially the same as those obtained by Chiao, Gamire, and Townes.[3]

Our calculation is modeled after the theory of phase transitions and the theory of vortex formation in superconductivity.[6] We assume that the molecules in a liquid are correlated, and at temperature T, the state of the liquid can be described by a dielectric function. We further assume that in the presence of an intense optical field greater than the critical field E_C, the molecular interactions in the liquid can be changed, and the system can experience a phase transition. (Field-induced phase transitions have been observed in ferroelectrics.)

We shall begin by discussing the energy of an arbitrary two-phase configuration of the liquid and then go on to discuss a liquid with trapped light filaments. In both cases we assume that the system can exist in two states described by the dielectric constants ϵ_a and ϵ_b. As will be clearer later, the states a and b will be analogous to the superconducting and normal states, respectively. The energy difference between these states (or the condensation energy) is $\Delta\epsilon|E_c|^2/8\pi$, where $\Delta\epsilon=\epsilon_b-\epsilon_a$. We now consider the case of a light beam of uniform intensity $c|E_0|^2/8\pi$ propagating into the liquid medium where both a and b phases exist simultaneously. After the light beam has traveled some distance along the $\hat{z}$ direction, the field distribution in the beam should become stable and invariant with respect to z. We can then conclude that from the minimization of the Gibbs free energy, the fields will concentrate in the high-dielectric-constant b region with field penetration of distance λ into the a region. For simplicity, we assume that the field E_b is constant in the b region and the λ penetration region.[7] The free energy of the system is (assuming a dispersionless medium),

$$F=F_0+(8\pi)^{-1}\{A_b(E_c^2\Delta\epsilon+|E_b|^2\epsilon_{b0}+\tfrac{1}{2}|E_b|^4\epsilon_{b2}) + \lambda l_b(|E_b|^2\epsilon_{a0}+\tfrac{1}{2}|E_b|^4\epsilon_{a2}) + \xi l_b(E_c^2\Delta\epsilon+|E_b|^2\Delta\epsilon)\}, \quad (1)$$

with

$$(A_b+\lambda l_b)|E_b|^2=A|E_0|^2,$$

where F_0 is the free energy arising from all sources other than those we are considering, A_b and A are the cross-sectional areas of the b region and of the beam, respectively, l_b is the perimeter length of the ϵ_b region, ξ is the characteristic length over which the transition from ϵ_a to ϵ_b takes place,[7] and it is assumed for simplicity that $\epsilon_{a2}\approx\epsilon_{b2}\equiv\epsilon_2$. Comparison of Eq. (1) with the free energy for a uniphase in A shows that a two-phase system is energetically favorable if $|E_0|>E_c$ and $\lambda>\xi$. The medium wants to form new walls between the phases. From arguments similar to those used to describe the formation of vortex lines in superconductivity,[6] it is energetically favorable to form circular filaments of radius ξ (field filaments of radius λ).

For the case of n filaments, the free energy of the system can be written as

$$F=F_0+(8\pi)^{-1}\{n\pi\lambda^2(|E_b|^2\epsilon_{a0}+\tfrac{1}{2}|E_b|^4\epsilon_2) + n\pi\xi^2(|E_b|^2\Delta\epsilon+E_c^2\Delta\epsilon)\}, \quad (2)$$

with the constraint $n\pi\lambda^2|E_b|^2=A|E_0|^2$, where E_b is the field in the filament. For a given $|E_0|^2$, the above free energy can be minimized to yield the number of filaments. Thus, $\partial F/\partial n=0$, and we find that

$$n=\frac{A|E_0|^2\epsilon_2^{1/2}}{\pi\lambda\xi E_c(2\Delta\epsilon)^{1/2}}. \quad (3)$$

Note that the minimum n we can have is at $E_0=E_c$. From Eq. (3), the field intensity in each filament can be obtained:

$$\frac{c|E_b|^2}{8\pi}=\frac{c\xi E_c(2\Delta\epsilon)^{1/2}}{8\pi\lambda\epsilon_2^{1/2}}, \quad (4)$$

which is independent of the applied field if λ is only dependent on the characteristics of the medium. The total power contained in each filament is a constant:

$$Q=\frac{c\lambda\xi E_c(2\Delta\epsilon)^{1/2}}{8\epsilon_2^{1/2}}. \quad (5)$$

The above equations relate the field in a filament to the size of the filament, and these can be solved if the quantity $\xi E_c(\Delta\epsilon)^{1/2}$ is known. This quantity is a characteristic of the phase transition in the medium. It is possible to obtain a numerical estimate of $\xi E_c(\Delta\epsilon)^{1/2}$ by making the simplifying assumption that the phase transition we are considering is a second-order phase transition. This allows the use of the Landau-Ginzburg equation,[8]

$$(N/2m)[(\hbar/i)\nabla-(e^*/c)\vec{A}]^2\psi+\alpha\psi+\beta|\psi|^2\psi=0, \quad (6)$$

where ψ is a complex, position-dependent order parameter describing the additional induced correlated polarization responsible for the $\Delta\epsilon$ change. We assume that the induced polarization arises from electronic interactions and N, m, and e^* refer to the density, mass, and effective charge of the electron (assuming one interacting electron per molecule). In deriving Eq. (2) we assume a square well approximation for ψ and E_b; $\psi=1$ for $r>\xi$, $\psi<\xi$,[6] and $E=E_b$ for $r<\lambda$, where r is the radial position measured from the center of the filament.

From the equilibrium condition in the absence of the fields, we find[6] that

$$\alpha=-\beta=-E_c^2\Delta\epsilon/4\pi. \quad (7)$$

If the fields are independent of z, then ψ is also independent of z, and both the fields and ψ can be taken to have cylindrical symmetry.

We can use Eq. (6) to give the characteristic relaxation length ξ to describe the variation of ψ from zero at the center of the filament to unity outside:

$$\xi \approx [2\pi\hbar^2 N/m\Delta\epsilon E_c^2]^{1/2}. \qquad (8)$$

For CS_2, $N \approx 10^{22}$ cm^{-3} and we find that $\xi E_c(\Delta\epsilon)^{1/2} = 8.7\times10^{-3}$ esu. Using this value and the experimental values $\lambda \approx 2$ μ and $\epsilon_2 = 1.8\times10^{-11}$ esu in Eqs. (4) and (5) gives the field intensity $c|E_b|^2/8\pi = 1.8\times10^9$ W/cm^2, and the power contained in each filament, $Q = 220$ W. These estimates agree well with experiment.[4] To find ξ, E_c, and $\Delta\epsilon$ separately, a microscopic calculation which considers the molecular interactions in detail is necessary.

It is possible to compute $\Delta\epsilon$ if λ is known by using the Maxwell wave equation

$$\{\nabla_r^2 - k_z^2 + (\omega^2/c^2)[\epsilon_{b0} - \Delta\epsilon\langle|\psi|^2\rangle_{Av} + \epsilon_2|E|^2]\}E(r) = 0, \qquad (9)$$

where $\langle|\psi|^2\rangle_{Av}$ represents the average value of $|\psi(r)|^2$. If we made the simplification $\langle|\psi(r)|^2\rangle_{Av} = \psi_0^2$ for $r<\xi$, $\langle|\psi(r)|^2\rangle_{Av} = 1$ for $r>\xi$, and $E(r) = E_b$ for $r<\xi$ (since the macroscopic field will not vary appreciably over a dimension less than a wavelength),

$$k_z = (\omega/c)(\epsilon_{b0} - \Delta\epsilon\psi_0^2 + \epsilon_2|E_b|^2)^{1/2}. \qquad (10)$$

For $\epsilon_0 \gg \epsilon_2|E|^2$, $\epsilon_2|E|^2$ can be neglected in Eqs. (9) and (10) in first order. The solution of Eq. (9) for $r>\xi$ is the zeroth-order modified Bessel function $K_0(r/\lambda)$, with a characteristic decay length

$$\lambda = (c/\omega)[\Delta\epsilon(1-\psi_0^2)]^{-1/2}. \qquad (11)$$

For CS_2, $\lambda \approx 2$ μ, and at ruby laser frequency we estimate that typically $\Delta\epsilon > 3\times10^{-3}$, which is much larger than $\epsilon_2|E_b|^2 (\approx 2\times10^{-4})$. We expect that typical values should be $\Delta\epsilon \sim 10^{-2}$, $E_c \sim 2\times10^3$ esu, and $\xi \sim 0.4$ μ. A more rigorous treatment of this problem should account for the variation of E and ψ across the phase boundary by solving the coupled equations (6) and (9). This should yield a functional representation for the n stable filaments as obtained from energy considerations. This would be equivalent to the Abrikosov calculation for vortex lines in superconductors.[9]

In the above discussion, the dynamic process to reach the final stable field distribution in the beam has not been considered. It is clearly not important as far as the stable configuration of filaments is concerned. This is analogous to the growing of a crystal, where we are only interested in the final crystal structure and not in the dynamic process of crystal formation. In actual experiments, the incoming beam intensity is often much less than E_c. However, through self-focusing, the beam cross section reduces and the field intensity finally exceeds E_c. The field distribution in the beam then becomes unstable, and filaments would be nucleated by fluctuations in the medium. The intensity in each filament is so high that stimulated scattering processes set in and deplete the laser power in the filament very rapidly.[4] Fluctuations and stimulated scattering processes would probably prevent the field distribution from reaching a stable configuration of filaments, but each filament already formed should have the characteristics of filaments in the final-state configuration.

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ANISOTROPIC PROPAGATION AND DAMPING OF ION ACOUSTIC WAVES IN A CURRENT-CARRYING PLASMA

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Since methods of excitation of ion acoustic waves by an external source with a grid[1] or a small coil[2] were published, propagation and damping of ion acoustic waves were studied most conveniently by a number of authors[1-12] who reported collisional damping,[1,8] Landau damping,[3] viscous damping,[4] existence of geometrical cut-off and no cut-off frequencies,[2,5,7,11] effects of electron contribution at a place far from an exciting region,[6] determination of compression coefficient of electron gas,[9,10] interference,[10] behavior near ion plasma frequency,[12] etc. This Letter reports experimental results and qualitative discussions about anisotropic propagation and damping of ion acoustic waves due to an electric current in a hot-cathode mercury discharge of several hundred milliamperes.

The experimental apparatus is shown in Fig. 1. Wave excitation and detection are done by using two grids with their planes normal to an electric current. One grid is used as an exciter, to which bursts of sinusoidal or continuous sinusoidal waves are applied, and the other as a detector. The frequency of the sinusoidal waves is in the range 10-80 kc/sec, well below the ion plasma frequency (~1 Mc/sec). The experiment is mainly made in the A-G_1 region, because, in the K-G_1 region, the electron thermal velocity is far from drifting-Maxwellian distribution and includes a strong beam especially near the cathode.

A typical example of anisotropic propagation and damping is shown in Fig. 2. The phase velocity (v_p) along the ion drift direction is larger than that along electron drift (against ion drift) direction and the damping along electron drift direction is smaller than that along ion drift direction. The damping seems to follow two slopes along the distance. The larger

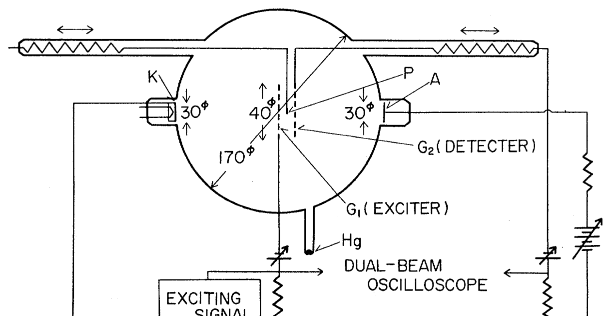

FIG. 1. Experimental apparatus for ion acoustic waves propagating along electron drift. G_2 is used as an exciter and G_1 as detector. A, anode; K, cathode; G_1 and G_2 (movable), grid (10 meshes/in. made of 0.5-mm-diam stainless steel wire); P (movable), probe. All dimensions are in millimeters.

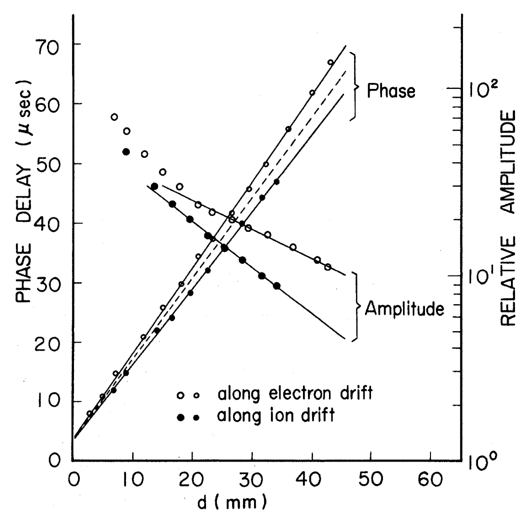

FIG. 2. Phase delay and relative amplitude along electron and ion drifts as a function of the distance (d) from the exciting grid for $f = 45.5$ kc/sec and discharge current = 500 mA. Dashed line ($v_p = 7.34 \times 10^4$ cm/sec) is obtained from $v_p = (\kappa T_e/m_i)^{1/2}$ and $T_e = 13\,000$°K measured by the Langmuir probe.