Thus it appears that there are three Gaussian regions in the proton-proton interaction.[14]

A comment should be made on the fact that we used I in relating $d\sigma^\dagger/dt$ to $d\sigma/dt$. This is equivalent to adding cross sections rather than amplitudes, and is necessary to obtain the dramatic fit in Fig. 1. The need to add cross sections in this way implies a relationship near 90° between the different spin amplitudes, which we will not study in detail.

I would like to thank Professor G. L. Kane and Professor M. H. Ross for their comments. I am grateful to the Aspen Institute Physics Division for their hospitality this summer.

---

*Work supported by a research grant from the U. S. Atomic Energy Commission.

[1]K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters 10, 376, 543 (1963); 11, 425, 503 (1963); 14, 862 (1965); 15, 45 (1965).

[2]A. R. Clyde, B. Cork, D. Keefe, L. T. Kerth, W. M. Layson, and W. A. Wenzel, in Proceedings of the International Conference on High Energy Physics, Dubna, 1963, edited by A. A. Solomensky (Atomizdat., Moscow, USSR, 1964); A. R. Clyde, thesis, University of California Radiation Laboratory Report No. UCRL-16275, 1966 (unpublished).

[3]G. Cocconi, V. T. Cocconi, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, W. F. Baker, E. W. Jenkins, A. L. Read, and B. T. Ulrich, Phys. Rev. Letters 11, 499 (1963); 12, 132 (1964); Phys. Rev. 138, B165 (1965).

[4]C. W. Akerlof, R. H. Hieber, A. D. Krisch, K. W. Edwards, L. G. Ratner, and K. Ruddick, Phys. Rev. Letters 17, 1105 (1966); Phys. Rev. 159, 1138 (1967).

[5]J. V. Allaby, G. Bellettini, G. Cocconi, M. L. Good, A. N. Diddens, G. Matthiae, E. S. Sacharidis, A. Silverman, and A. M. Wetherall, Phys. Letters 23, 389 (1966); J. V. Allaby et al., Phys. Letters 25B, 156 (1967).

[6]R. Serber, Phys. Rev. Letters 10, 357 (1963); Rev. Mod. Phys. 36, 649 (1964).

[7]A. D. Krisch, Phys. Rev. Letters 11, 217 (1963); Phys. Rev. 135, B1456 (1964).

[8]D. S. Narayan and K. V. L. Sarma, Phys. Letters 5 365 (1963).

[9]J. Orear, Phys. Letters 13, 190 (1964).

[10]A. D. Krisch, in Lectures in Theoretical Physics, edited by Wesley E. Brittin et al. (University of Colorado Press, Boulder, Colorado, 1966), Vol. IX.

[11]$\beta^2 P_\perp{}^2$ should be the correct variable in a diffraction model in which a spherically symmetric interaction probability density is squashed down by a factor γ by the Lorentz transformation. See Ref. 10.

[12]At some low energies the nature of elastic scattering changes and is no longer diffractive. We have set a cutoff at 5 GeV/c and have only considered data above that momentum.

[13]We have calculated the χ^2 for all points in Refs. 3–5 except those in the immediate region of the break, where the interference must be considered. (See Ref. 14.) For these 93 points we got a $\chi^2 = 158$. For Refs. 3 and 4 we used the point-to-point errors quoted, and for Ref. 5 we added to the point-to-point errors 5% of the quoted 10% normalization error.

[14]As pointed out in Ref. 4, there appears to be some destructive interference at the second break.

ASYMPTOTIC SYMMETRY FOR VECTOR MESONS*

S. K. Bose† and R. Torgerson
Department of Physics, University of Notre Dame, Notre Dame, Indiana
(Received 30 August 1967)

Assuming the asymptotic validity of the nonet symmetry we calculate (1) electromagnetic decays of vector mesons, (2) the $\varphi \to K\bar{K}$ decay, and (3) the coupling constants for ω and φ to baryons.

Following the idea that symmetries may become exact asymptotically at high energies,[1] we discuss properties of vector mesons. For the relevant symmetry scheme we take the nonet model of Okubo,[2] which properly accounts for singlet-octet (ω-φ) mixing.[3]

To postulate asymptotic nonet symmetry we consider a set of nine vector currents $V_{\mu,\beta}{}^{\alpha}(x)$ to be associated with the nine vector mesons. μ is the four-vector index. The indices α and β each take on values 1, 2, and 3. Specifically, φ is coupled to $V_{\mu,}\,3^3$; ω to $(1/\sqrt{2})(V_{\mu,}\,1^1 + V_{\mu,}\,2^2)$; ρ^0 to $(1/\sqrt{2})(V_{\mu,}\,1^1 - V_{\mu.}\,2^2)$; and K^* to $V_{\mu,}\,3^1$. Thus the currents constitute the usual nonet tensor. Note that the upper and lower indices of this mixed tensor may be considered to refer to two commuting SU(3) algebras in a manner discussed in detail by Bose and Sudarshan.[2]

A convenient method of imposing asymptotic symmetry requirements has been given by Das et al.[1] Following these authors we define the propagators

$$\Delta_{\mu\nu,\beta}{}^{\alpha}(q^2) = \int e^{iq\cdot x}\langle 0|T\{V_{\mu,\beta}{}^{\alpha}(x)V_{\nu,\alpha}{}^{\beta}(0)\}|0\rangle d^4x, \quad (1)$$

and assume that in the limit $q^2\to\infty$ the difference between any two propagators vanishes. The spectral representation for these propagators then leads to a superconvergence behavior for the difference of any two spectral functions:

$$0=\int\frac{\rho^{\omega}(\mu^2)-\rho^{\varphi}(\mu^2)}{\mu^2}d\mu^2=\int\frac{\rho^{\omega}(\mu^2)-\rho^{\rho}(\mu^2)}{\mu^2}d\mu^2 =\int\frac{\rho^{\omega}(\mu^2)-\rho^{K^*}(\mu^2)}{\mu^2}d\mu^2. \quad (2)$$

In Eq. (2) $\rho^{\omega}(\mu^2)$ is the spectral function of the propagator with the quantum number of the physical ω, and so on. If we saturate Eq. (2) by taking the contribution of single vector meson states we obtain[4]

$$\frac{G_{\rho}{}^2}{M_{\rho}{}^2}=\frac{G_{\omega}{}^2}{M_{\omega}{}^2}=\frac{G_{\varphi}{}^2}{M_{\varphi}{}^2}=\frac{G_{K^*}{}^2}{M_{K^*}{}^2}, \quad (3)$$

where the leptonic decay amplitudes G are defined by

$$\langle 0|V_{\mu,3}{}^3(0)|\varphi(k)\rangle=(2k_0)^{-1/2}\epsilon_{\mu}(k)G_{\varphi}$$

and similarly for G_{ρ}, G_{ω}, and G_{K^*}.

Weak leptonic decays of vector mesons are unknown at present. To test Eq. (3) we thus turn to the electromagnetic (e.m.) current. It is now a nontrivial problem[5] to write down the e.m. current in the present context. We consider the simplest possibility, which is to retain the "old" SU(3) expression for the current. Thus we do not introduce a unitary singlet[6] current. In our notation we have

$$J_{\mu}{}^{\text{e.m.}}(x)=V_{\mu}{}^{\rho^0}(x)+\tfrac{1}{3}V_{\mu}{}^{\omega}(x)+\tfrac{1}{3}\sqrt{2}V_{\mu}{}^{\varphi}(x). \quad (4)$$

Equations (3) and (4) predict the following relation involving the e.m. decay rates:

$$M_{\omega}\Gamma(\omega\to l\bar{l})=\tfrac{1}{9}M_{\rho}\Gamma(\rho^0\to l\bar{l})=\tfrac{1}{2}M_{\varphi}\Gamma(\varphi\to l\bar{l}). \quad (5)$$

The experimental ratio $R_{\omega}=\Gamma(\omega\to l\bar{l})/\Gamma(\rho\to l\bar{l})$ may be found as follows: The (weighted) average of two determinations[7] of $\Gamma(\rho\to\mu^+\mu^-)/\Gamma(\rho\to\pi^+\pi^-)$ yields $(0.48\pm0.09)\times10^{-4}$. We may combine this with $\Gamma(\rho\to e^+e^-)/\Gamma(\rho\to\pi^+\pi^-)=(0.50\pm0.13)\times10^{-4}$ (obtained in the same way from two very recent experiments).[8] (Our model, as well as any other model known to us, predicts that they should coincide to within 1 %.) Thus[9] $R_{\omega}(\text{expt.})=0.22\pm0.07$ which is to be compared with our prediction 0.11.

From the lowest upper limit known to us for[10] $\Gamma(\varphi\to\mu^+\mu^-)/\Gamma(\varphi\to\text{all})$, namely, 7.4×10^{-4}, we find $R_{\varphi}(\text{expt.})=\Gamma(\varphi\to l\bar{l})/\Gamma(\rho\to l\bar{l})<0.47$. Our prediction is 0.17.

Now consider strong decays. First, notice that the (unitary singlet) baryon current is given by

$$J_{\mu}{}^{B}(x)=(\tfrac{2}{3})^{\frac{1}{2}}V_{\mu}{}^{\omega}(x)-3^{-\frac{1}{2}}V_{\mu}{}^{\varphi}(x). \quad (6)$$

Taking matrix element of $J_{\mu}{}^B$ between vacuum and $K\bar{K}$ states we have

$$\langle K(p)\bar{K}(p')|J_{\mu}{}^{B}(0)|0\rangle =(4p_0p_0')^{-\frac{1}{2}}(p-p')_{\mu}F((p+p')^2); \quad (7)$$

we notice first $F(0)=0$. Now writing an unsubtracted dispersion relation for $F((p+p')^2)$ and retaining the contributions of single vector-meson states we have

$$\frac{G_{\omega}g_{\omega K\bar{K}}}{M_{\omega}{}^2}-\frac{1}{\sqrt{2}}\frac{G_{\varphi}g_{\varphi K\bar{K}}}{M_{\varphi}{}^2}=0. \quad (8)$$

$g_{\omega K\bar{K}}$ and $g_{\varphi K\bar{K}}$ are coupling constants of ω and φ to $K\bar{K}$. Similarly, if the (isoscalar) e.m. form factor of the K meson is dominated by vector mesons we have

$$\frac{G_{\omega}}{3M_{\omega}{}^2}g_{\omega K\bar{K}}+\frac{\sqrt{2}}{3}\frac{G_{\varphi}}{M_{\varphi}{}^2}g_{\varphi K\bar{K}}=\tfrac{1}{2}. \quad (9)$$

From Eqs. (8) and (9) we get

$$(G_{\varphi}/M_{\varphi}{}^2)g_{\varphi K\bar{K}}=1/\sqrt{2}. \quad (10)$$

Equations (3) and (10) together with the current-algebra result[11] $G_{\rho}=M_{\rho}{}^2/g_{\rho\pi\pi}$ gives

$$\left|\frac{g_{\varphi K\bar{K}}}{g_{\rho\pi\pi}}\right|=\frac{1}{\sqrt{2}}\frac{M_{\varphi}}{M_{\rho}}. \quad (11)$$

From Eq. (11) we obtain the rate[9]

$$\Gamma(\varphi\to K^+K^-)=2.6\text{ MeV}.$$

The current experimental value is[12] 1.9 ± 0.5 MeV.

We now consider vector-meson coupling to baryons. Taking matrix elements of Eq. (6) between single-baryon states we now obtain

$$\frac{\sqrt{2}}{\sqrt{3}}\frac{M_\rho}{M_\omega}\frac{g_{\omega B\overline{B}}}{g_{\rho\pi\pi}}-\frac{1}{\sqrt{3}}\frac{M_\rho}{M_\varphi}\frac{g_{\varphi B\overline{B}}}{g_{\rho\pi\pi}}=1. \qquad (12)$$

If the electric form factors of the baryons are dominated by vector mesons we have

$$\frac{1}{3}\frac{M_\rho}{M_\omega}\frac{g_{\omega B\overline{B}}}{g_{\rho\pi\pi}}+\frac{\sqrt{2}}{3}\frac{M_\rho}{M_\varphi}\frac{g_{\varphi B\overline{B}}}{g_{\rho\pi\pi}}=\frac{\gamma}{2}. \qquad (13)$$

From Eqs. (12) and (13) we get

$$\frac{g_{\omega B\overline{B}}}{g_{\rho\pi\pi}}=(\tfrac{1}{2}Y+\sqrt{\tfrac{2}{3}})\frac{M_\omega}{M_\rho}, \qquad (14)$$

$$\frac{g_{\varphi B\overline{B}}}{g_{\rho\pi\pi}}=\left(\frac{1}{\sqrt{2}}Y-\frac{1}{\sqrt{3}}\right)\frac{M_\varphi}{M_\rho}. \qquad (15)$$

Numerically, for $Y=1$,

$$(1/4\pi)g_{\varphi N\overline{N}}{}^2=0.07; \quad (1/4\pi)g_{\omega N\overline{N}}{}^2=4.2.$$

In the literature we find that estimates of $g_{\omega N\overline{N}}{}^2$ are subject to wide fluctuation, varying from 2 to 25.[13] The small value of $g_{\varphi N\overline{N}}$ is consistent, for example, with the suppression of φ production in backward K^-p reactions.[14,15]

The discussion of this paper can be extended to include nine axial-vector currents. The underlying group structure will then be $[SU(3)\otimes SU(3)]_L\times[SU(3)\otimes SU(3)]_R$.[16]

While this paper was being written, two articles utilizing asymptotic SU(3) for vector-meson decays have appeared.[17]

---

*Research supported in part by the U. S. Atomic Energy Commission.

†On leave of absence from the Centre for Advanced Studies in Theoretical Physics and Astrophysics, Delhi University, New Delhi, 7, India.

[1]G. Costa and A. H. Zimmerman, Nuovo Cimento 46A, 198 (1966); L. K. Pandit, Phys. Rev. Letters 19, 263 (1967); T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967); J. Schechter and G. Venturi, Phys. Rev. Letters 19, 276 (1967); see also S. Weinberg, Phys. Rev. Letters 18, 507 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967).

[2]S. Okubo, Phys. Letters 5, 165 (1963). It may be recalled that Okubo's scheme can be reinterpreted in terms of the representation $(3,3^*)$ of the nonchiral $SU(3)\otimes SU(3)$ group. See S. K. Bose, Phys. Rev. 150, 1231 (1966); S. K. Bose and E. C. G. Sudarshan, Phys. Rev. (to be published).

[3]J. J. Sakurai, Phys. Rev. Letters 9, 108 (1962); S. L. Glashow, Phys. Rev. Letters 11, 112 (1963).

[4]If we would also assume the stronger condition $\int[\rho^\omega(\mu^2)-\rho^\varphi(\mu^2)]d\mu^2=\int[\rho^\omega(\mu^2)-\rho^\rho(\mu^2)]d\mu=\int[\rho^\omega(\mu^2)-\rho^{K^*}(\mu^2)]d\mu^2=0$, then in the same approximation, we reproduce the result of exact symmetry. Hence, we do not assume such a strong condition.

[5]In this connection, see, for example, N. Kroll, T. D. Lee, and B. Zumino, Physics 157, 1376 (1967).

[6]An alternative postulate is to identify $V_{\mu,1}{}^1(x)$ with the e.m. current. Now the φ is not coupled to photons. Equation (5) of the text is replaced by $m_\omega\Gamma(\omega\to l\bar{l})=m_\rho\Gamma(\rho^0\to l\bar{l})$ which is strongly violated by experiment. [Ref. 9; R. C. Chase, P. Rothwell, and R. Weinstein, Phys. Rev. Letters 18, 810 (1967).]

[7]J. K. de Pagter et al., Phys. Rev. Letters 16, 35 (1966); A. Weinstein et al., Phys. Rev. Letters 18, 929 (1967). It is known that the result of de Pagter et al. should be multiplied by $\frac{4}{3}$ as remarked by R. Weinstein, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967) [see also J. J. Sakurai, Phys. Rev. Letters 17, 1021 (1966), footnote 23].

[8]J. G. Asbury et al., to be published; M. A. Azimov et al., to be published.

[9]For the ρ width we use 128 ± 5 MeV; M. Roos, CERN Report No. 67/830/5-Th798, 1967 (unpublished). For the ρ mass we use 776 which is the isospin-triplet average of Roos's values. The other numbers, e.g., $\Gamma(\omega\to l\bar{l})=(1.4\pm0.4)\times10^{-3}$ MeV, come from A. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

[10]Chase, Rothwell, and Weinstein, Ref. 6.

[11]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).

[12]Rosenfeld et al., Ref. 9.

[13]See, for instance, K. Kawarabayashi, Phys. Rev. 134, B877 (1964).

[14]G. W. London et al., Phys. Rev. 143, 1034 (1966); J. Badier et al., in Proceedings of the Oxford International Conference on Elementary Particles, September, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966).

[15]H. Sugawara and F. von Hippel [Phys. Rev. 145, 1331 (1966)] discuss several experimental indications of the smallness of $g_{\varphi N\overline{N}}$.

[16]P. G. O. Freund and Y. Nambu, Ann Phys. (N.Y.) 32, 201 (1965); A. Salam and J. C. Ward, Phys. Rev. 136, B763 (1964).

[17]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 470 (1967). P. P. Divakaran and L. K. Pandit, Phys. Rev. Letters 19, 535 (1967).